Home Bias and High Turnover: Dynamic Portfolio Choice with Incomplete Markets

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Abstract

Why do investors trade a lot in foreign assets and hold so little of them in their portfolios? This paper shows that both observations can arise naturally in the presence of nondiversifiable nontraded consumption risk when each country specializes in production, preferences exhibit consumption home bias, and asset markets are incomplete. Using a general equilibrium two-country, two-sector (tradable and nontradable) model of the world economy with production I show that low diversification occurs because variations in relative prices (i) increase the riskiness of foreign returns; and (ii) facilitate risk-sharing across countries. Large and volatile capital flows are necessary to take advantage of international risk premia differentials that occur in response to productivity changes. I characterize the optimal portfolio holdings, the evolution of the investment opportunity set, the risk premium, and the dynamics of capital flows using a new methodology for solving a dynamic general equilibrium model with incomplete markets.

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Introduction

Investors around the world allocate most of their portfolios to domestic assets despite the apparent diversification gains to be had from holding foreign assets. The potential welfare gains from international diversification and investors' unwillingness to diversify is a long-standing puzzle in international finance – termed home bias. Another aspect of the home bias puzzle concerns portfolio flows. A number of studies document that cross-border equity flows by domestic residents are large in magnitude and volatile, suggesting that investors do try to take advantage of diversification opportunities abroad.[2] It seems as though investors trade too much *and* hold too little of their portfolios in foreign assets. This paper develops a model that can reconcile these two seemingly contradictory observations. It generates a bias in country portfolios towards domestic assets together with large and variable international asset flows.

The key feature that distinguishes this paper from earlier research on home bias is its analysis of a model with endogenously incomplete asset markets. Specifically, my analysis is based on a set of primitive assumptions regarding investors access to equity markets rather than an assumption about the degree of risk-sharing achieved in equilibrium. The novelty of my approach is that the degree of risk-sharing is determined endogenously as part of the competitive equilibrium of the model I study. Consequently, I can address the fundamental question of why investors hold most of their portfolios in domestic assets when it appears, a priori, that international diversification could bring welfare gains by facilitating greater risk-sharing.[3] Addressing this question is the central task of this paper.

My analysis uses a two-country general equilibrium model with two sectors: a tradable sector and a nontradable sector. Both sectors are subject to stochastic productivity shocks. Each country specializes in the production of its tradable good. Household preferences are defined over the consumption of three goods: a domestic nontradable and a basket of domestic and foreign tradable goods. Households finance these consumption expenditures by trading in equities issued by tradable firms in both countries, equities indexed to domestic nontradable production, and bonds. In this setting, home bias arises as households try to hedge the fluctuations in their nontradable consumption even though productivity changes are independent across sectors and countries.

The intuition for this result is the following. When preferences are complementary in the consumption of a nontradable and a basket of tradable goods, any increase in nontradable consumption must be accompanied by an increase in tradable consumption, both local and foreign. Thus, households allocate a larger share of their portfolios to tradable assets whose payoffs are high when their nontraded consumption is high. When, in addition, local and imported tradable goods are imperfect substitutes, variations in the relative price of the imported good perform two roles: First they correlate negatively with the relative supply of foreign

[2]For example, based on U.S. balance of payments data, U.S. holdings of foreign assets accounted for 18% of U.S. GDP in 2003. At the same time, according to the International Financial Statistics (IMF), U.S. annual gross capital flows amounted to almost 6% of GDP in 2003. Tesar and Werner (1994, 1995) provide detailed evidence on high volume and high turnover rate of foreign equity investments in a sample of OECD countries. Warnock (2002) updates these statistics using the results of comprehensive benchmark surveys of foreign equity holdings for U.S. and Canada. Lewis (1999) provides a survey of literature on home bias in equities and consumption, as well as on welfare gains from diversification.

[3]Of course, the presence of home bias does not in itself imply that risk-sharing is incomplete. For example, Heathcote and Perri (2004) present a model in which the competitive equilibrium is Pareto efficient and portfolios exhibit home bias.

dividends; second, they increase the variability of foreign returns in the eyes of domestic households. The first effect decreases the relative value of foreign equity payouts to domestic households. The second effect makes the return on foreign asset riskier. The combination of these two effects inclines households to increase their holdings of domestic equity and skew their portfolios against foreign assets. Variations in the terms of trade are also associated with significant changes in the risk-premia on tradable assets. As households adjust their portfolios in response to the shifts in expected excess returns, they generate large and volatile international capital flows.

This paper builds on a large literature that studies the importance of non-traded risk for international risk-sharing. One source of non-traded risk is nondiversifiable labor income risk. Baxter and Jermann (1997) show that when labor income correlates positively with capital income, investors take large short positions in domestic assets. Engel and Matsumoto (2004) reverse this finding in a new open economy macro model. They show that in the presence of sticky prices, the returns to labor and capital may become negatively correlated in response to productivity shocks, leading to a home bias in investors' portfolios.

In this paper, I focus on non-traded risk that arises due to the presence of a large fraction of goods that can not be traded internationally. Stockman and Dellas (1989) started this literature by studying an endowment economy in which investor preferences are separable in tradable and nontradable consumption. They find that asset holdings are constant and the equilibrium portfolio shares are equally split between home and foreign tradable equity, while domestic nontradable equity is completely held by domestic investors. Tesar (1993) relaxes the separability in preferences assumption. She shows that when tradable and nontradable goods are complementary in consumption, the deviations from an equally-weighted tradable portfolio towards a home biased tradable portfolio will be welfare-improving if domestic nontradable productivity is more strongly correlated with domestic than foreign tradable productivity. Pesenti and van Wincoop (1996) derive an analogous result in a partial equilibrium framework and confirm it empirically using a sample of 14 OECD countries. They find, however, that only a fraction of observed home bias can be accounted for by the presence of nontradable consumption fluctuations.

Serrat (2001) examines the implications of a richer preference specification. He solves for the optimal portfolios in an endowment general equilibrium model in which preferences are defined over a nontradable good and a basket of home and foreign tradable goods. In his framework domestic investors are the sole owners of local nontraded assets, while the home bias in traded portfolios arises under conditions analogous to those in Tesar (1993) and Pesenti and van Wincoop (1996). Kollmann (2005a) revisits Serrat's model and shows that when the consumption aggregator is Cobb-Douglas and asset markets are complete, the optimal tradable portfolio remains equally-weighted, while the nontradable portfolio split becomes indeterminate. This result is reminiscent of Baxter, Jermann and King (1998). They argued that home bias in tradable equity cannot arise in a static economy with complete markets and international trade in claims to tradable and nontradable goods.

This paper is also related to the literature on the role of relative prices in international risk-sharing. Cole and Obstfeld (1991) studied a two-country economy with complete markets in which each country endowments are specialized and preferences are defined over the consumption of both goods. They showed that when preferences are symmetric Cobb-Douglas or separable, any variation in relative endowments induces an exactly off-setting change in relative price. As a result, any portfolio ensures perfect risk-sharing across countries. Heathcote and Perri (2004) extend this analysis by introducing production. In their framework each country specializes in production of a final good that uses both local and imported intermediate inputs. As in Cole and Obstfeld (1991), changes in relative prices facilitate pooling of risks across countries. Kollmann (2005b) generates portfolio home bias in an endowment economy with home bias in consumption. Uppal (1993) reaches an opposite conclusion in a complete market general equilibrium setting with shipping costs. He shows that observed portfolios can not be justified by the consumption bias towards domestic goods. On contrary, investors that are more risk-averse than a log investor will prefer foreign stocks. The reason for his result is that the real exchange rate is negatively correlated with foreign returns making returns on foreign assets less risky than domestic returns in the eyes of home investors.

The model presented here extends this literature on home bias along three dimensions. First, the optimal portfolios are studied in a general equilibrium framework with incomplete asset markets. As noted above, existing research focuses primarily on complete markets equilibria, which in combination with isoelastic utility implies that portfolio holdings are time-invariant. Relaxing this assumption allows me to explore the dynamics of international portfolio holdings as well as properties of capital flows and factors driving them. However, finding an equilibrium of a model with incomplete markets is challenging. In such economy any shifts in the distribution of wealth affect the dynamics of interest rates and asset returns, which in turn determine the variations in risk-premia and investors' portfolios. This requires that wealth in each country is included in the state vector, which immediately leads to a number of technical difficulties.[4] The equilibrium of this model is obtained by using a new methodology developed in Evans and Hnatkovska (2005).

Second, I assume that households preferences are specified in terms of a nontradable good and a basket of tradable goods; one produced locally and one that must be imported from abroad. Preferences are assumed to be nonseparable in the consumption of all three goods. Home bias arises in my model under the plausible parametrization of inter- and intratemporal elasticities.

The third dimension concerns production. I introduce production in the tradable sector. This allows for tradable dividends to be endogenously determined by firms in order to accommodate the changes in the demand for tradable goods. As a result, in my model the changes in relative prices arise from endogenous shifts in both demand and supply. Furthermore, I do not assume constant returns to scale in production. When constant returns to scale are present, the dynamics of wealth become monotonically related to the physical capital stock. This means that aggregate wealth in the economy becomes proportional to the capital stock. It also means that the expected returns on risky assets and their variances become equivalent to the corresponding moments of the return on capital. In this setting expected returns are constant and changes in the risk-free rate are the only source of variability in the investment opportunity set. In my model, variations in equity returns govern the variability of households' wealth and risk premia, which, in turn, are the key determinants of households' portfolios.

[4] The challenges relate to the dimensionality of the state vector, its conditional heteroskedasticity, and the nonstationarity of the wealth processes.

My analysis produces the following main findings:

1. Market incompleteness in my model is quantitatively important. I find that the restrictions on asset ownership significantly impede international risk-sharing in equilibrium.

2. The portfolios of households in each country exhibit a significant degree of home bias. Moreover, the degree of bias is comparable with the bias in the international positions of typical U.S. investors.

3. The portfolios of households are volatile, conditionally mean-variance efficient, and guarantee a risk-adjusted expected portfolio return in excess of its unconditional equivalent.

4. The equilibrium bond and equity flows between countries are large and variable. There is nothing inherently contradictory between home bias in portfolio holdings and a large amount of international asset trade. In fact, the flows generated by the model are larger than their empirical counterparts.

5. International asset flows are driven by differentials in risk premia arising from the different perception of productivity changes by local and foreign households.

The paper is organized as follows. The next section sets up the model. Section 2 describes the equilibrium behavior of households and firms. Section 3 develops an intuition for home bias. The solution and calibration of the dynamic model are described in section 4. Section 5 analyses the equilibrium properties of the model. Section 6 concludes.

1 The Model

The world consists of two symmetric countries: home (H) and foreign (F). In what follows, I will refer to home country as the US and to the foreign country as the UK. Each country is populated by a continuum of identical households who supply their labor inelastically to domestic firms. Two types of firms exist in each country: (i) firms that specialize in production of a tradable (T) good, and (ii) firms that receive an endowment of a nontradable (N) good. Both firm types issue equity on the domestic stock market.

1.1 Firms

A continuum of tradable good firms in each country is perfectly competitive. A representative US firm owns all of its capital stock, K_t^{H}, and produces output, Y_t^{H}, according to

$$Y_t^{\text{H}} = Z_t^{\text{H}} \left(K_t^{\text{H}}\right)^{\theta},$$

where Z_t^{H} denotes the exogenous state of productivity. The output of traded goods in the UK, Y_t^{F}, is given by an identical production function using capital, K_t^{F}, and productivity Z_t^{F}. Hereafter, I use the term tradables to refer to traded goods produced by US and UK firms.

At the beginning of each period, firms observe the state of world productivity and decide how to allocate their output between investment and consumption goods. Output allocated to consumption is supplied competitively to US and UK households and the proceeds are used to finance dividend payments to the owners of the firm's equity. Output allocated to investment adds to the stock of physical capital available for production next period. I assume that firms allocate output to maximize the value of the firm to its shareholders.

Let P_t^{H} denote the ex-dividend price of equity providing a claim to US tradable dividends, D_t^{H}, during period t. P_t^{H} and D_t^{H} are both denominated in units of the US tradable good. The number of shares issued is normalized to one, so that the value of the firm in period t is given by $P_t^{\text{H}} + D_t^{\text{H}}$. US firms producing tradables choose investment, I_t^{H}, to solve

$$\max_{I_t^{\text{H}}} \left(P_t^{\text{H}} + D_t^{\text{H}}\right),$$

subject to

$$I_t^{\text{H}} = K_{t+1}^{\text{H}} - (1-\delta)\, K_t^{\text{H}}, \qquad \text{and} \qquad D_t^{\text{H}} = Y_t^{\text{H}} - I_t^{\text{H}},$$

where $\delta > 0$ is the depreciation rate on physical capital.

UK firms producing tradables solve an analogous problem. I assume that the US tradable is numeraire, and use Q_t^{F} to denote the relative price of the UK tradable. If P_t^{F} is the price of a share providing a claim to UK tradable dividends, D_t^{F} (both measured in terms of UK tradables), and the number of UK shares issued is also normalized to one, then the problem facing UK firms can be written as:

$$\max_{I_t^{\text{F}}} \left(P_t^{\text{F}} + D_t^{\text{F}}\right),$$

subject to

$$I_t^{\text{F}} = K_{t+1}^{\text{F}} - (1-\delta)\, K_t^{\text{F}}, \qquad \text{and} \qquad D_t^{\text{F}} = Y_t^{\text{F}} - I_t^{\text{F}}.$$

Production in the US and UK nontradable sectors does not require capital. Outputs of nontraded goods in the US and UK (hereafter nontradables), denoted by Y_t^{N} and $\hat{Y}_t^{\text{N}}$, are produced by

$$Y_t^{\text{N}} = \kappa Z_t^{\text{N}}, \qquad \text{and} \qquad \hat{Y}_t^{\text{N}} = \kappa \hat{Z}_t^{\text{N}},$$

where $\kappa > 0$ is a constant, and Z_t^{N} and $\hat{Z}_t^{\text{N}}$ denote the period$-t$ state of nontradable productivity in the US and UK respectively. Nontradables can only be consumed by domestic households. The proceeds are paid out as dividends to domestic households. The number of shares issued by the representative nontradable firm is normalized to unity. As a result, $D_t^{\text{N}} = Y_t^{\text{N}}$, and $\hat{D}_t^{\text{N}} = \hat{Y}_t^{\text{N}}$. The ex-dividend price of a share in the representative US and UK firm, measured in terms of nontradables, is P_t^{N} and $\hat{P}_t^{\text{N}}$ respectively.

The productivity processes in the tradable and nontradable sectors, summarized in $z_t \equiv [\ln Z_t^{\text{H}}, \ln Z_t^{\text{F}}, \ln Z_t^{\text{N}}, \ln \hat{Z}_t^{\text{N}}]'$, is governed by an AR(1) process:

$$z_t = a z_{t-1} + e_t,$$

where e_t is a (4×1) vector of i.i.d. normally distributed, mean zero shocks with covariance Ω_e.

1.2 Households

Each country is populated by a continuum of households, whose preferences are defined over the consumption of two goods: a composite tradable and a domestic nontradable. The preferences of US households are represented by

$$\mathbb{U}_t = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}^{\mathrm{T}}, C_{t+i}^{\mathrm{N}}), \tag{1}$$

where $0 < \beta < 1$ is the discount factor, and $U(.)$ is a concave sub-utility function defined over the consumption of a basket of tradable consumption, C_t^{T}, and US nontradable consumption, C_t^{N} :

$$U(C^{\mathrm{T}}, C^{\mathrm{N}}) = \tfrac{1}{\phi} \ln \left[\lambda_{\mathrm{T}}^{1-\phi} (C^{\mathrm{T}})^{\phi} + \lambda_{\mathrm{N}}^{1-\phi} (C^{\mathrm{N}})^{\phi} \right],$$

with $\phi < 1$. λ_{T} and $\lambda_{\mathrm{N}} = 1 - \lambda_{\mathrm{T}}$ are the weights the household assigns to tradable and nontradable consumption respectively. The elasticity of substitution between tradable and nontradable consumption is $(1-\phi)^{-1} > 0$. Preferences for UK households are similarly defined in terms of the consumption of a basket of tradables and UK nontradables, $\hat{C}_t^{\mathrm{T}}$ and $\hat{C}_t^{\mathrm{N}} : \hat{U}_t \equiv \hat{U}(\hat{C}_t^{\mathrm{T}}, \hat{C}_t^{\mathrm{N}})$.

The tradable consumption basket in the US, C_t^{T}, is given by a CES aggregator over tradables produced in the US and UK:

$$C^{\mathrm{T}} = \left[\lambda_{\mathrm{H}}^{1-\rho} (C^{\mathrm{H}})^{\rho} + \lambda_{\mathrm{F}}^{1-\rho} (C^{\mathrm{F}})^{\rho} \right]^{\frac{1}{\rho}},$$

where $\rho < 1$. The weights that households assign to the consumption of two tradable goods are λ_{H} and $\lambda_{\mathrm{F}} = 1 - \lambda_{\mathrm{H}}$ respectively. The elasticity of substitution between the two tradable goods is given by $(1-\rho)^{-1}$. The tradable consumption bundle in the UK is defined symmetrically in terms of US tradables, $\hat{C}^{\mathrm{H}}$, and UK tradables, $\hat{C}^{\mathrm{F}}$, as

$$\hat{C}^{\mathrm{T}} = \left[\hat{\lambda}_{\mathrm{H}}^{1-\rho} (\hat{C}^{\mathrm{H}})^{\rho} + \hat{\lambda}_{\mathrm{F}}^{1-\rho} (\hat{C}^{\mathrm{F}})^{\rho} \right]^{\frac{1}{\rho}}.$$

Households finance their consumption expenditures by holding an array of financial assets. In each country households can allocate their wealth to equity providing claims to US and UK tradable dividend streams, $\{D_t^{\mathrm{H}}\}$ and $\{D_t^{\mathrm{F}}\}$. I will use A^{H} and A^{F} to denote US holdings of tradable equity issued by US and UK firms, respectively. Households also have access to an international bond market. B_t represents the bond holdings of US households in period t, while R_t^1 denotes the prevailing risk-free rate measured in terms of US tradables. The flow budget constraint for US households can now be expressed as

$$\begin{aligned} &C_t^{\mathrm{H}} + Q_t^{\mathrm{F}} C_t^{\mathrm{F}} + Q_t^{\mathrm{N}} C_t^{\mathrm{N}} + P_t^{\mathrm{H}} A_t^{\mathrm{H}} + Q_t^{\mathrm{F}} P_t^{\mathrm{F}} A_t^{\mathrm{F}} + Q_t^{\mathrm{N}} P_t^{\mathrm{N}} A_t^{\mathrm{N}} + \frac{1}{R_t^1} B_t \\ &\leq (P_t^{\mathrm{H}} + D_t^{\mathrm{H}}) A_{t-1}^{\mathrm{H}} + Q_t^{\mathrm{F}} (P_t^{\mathrm{F}} + D_t^{\mathrm{F}}) A_{t-1}^{\mathrm{F}} + Q_t^{\mathrm{N}} (P_t^{\mathrm{N}} + D_t^{\mathrm{N}}) A_{t-1}^{\mathrm{N}} + B_{t-1}, \end{aligned} \tag{2}$$

where Q_t^{F} and Q_t^{N} denote the relative price of UK tradables and US nontradables, respectively. It proves convenient to rewrite the budget constraint in equation (2) in terms of wealth, W_t (measured in terms of US tradables), as

$$W_{t+1} = R_{t+1}^{\mathrm{W}} (W_t - C_t^{\mathrm{H}} - Q_t^{\mathrm{F}} C_t^{\mathrm{F}} - Q_t^{\mathrm{N}} C_t^{\mathrm{N}}), \tag{3}$$

where R^{W}_{t+1} is the (gross) return on wealth between period t and $t+1$, given by

$$R^{\text{W}}_{t+1} = R^1_t + \alpha^{\text{H}}_t(R^{\text{H}}_{t+1} - R^1_t) + \alpha^{\text{F}}_t(R^{\text{F}}_{t+1} - R^1_t) + \alpha^{\text{N}}_t(R^{\text{N}}_{t+1} - R^1_t). \tag{4}$$

The returns on US and UK tradable equity are R^{H}_{t+1} and R^{F}_{t+1}, while R^{N}_{t+1} is the return on US nontradable equity. (Recall that all returns are measured in terms of US tradables.) The shares $\alpha^{\text{H}}_t, \alpha^{\text{F}}_t$ and α^{N}_t denote the fraction of wealth held by US households in US and UK tradable equity and US nontradable equity. The budget constraint facing UK households is similarly written as

$$\begin{aligned}
\hat{W}_{t+1} &= \hat{R}^{\text{W}}_{t+1}\left(\hat{W}_t - \hat{C}^{\text{H}}_t - Q^{\text{F}}_t\hat{C}^{\text{F}}_t - \hat{Q}^{\text{N}}_t\hat{C}^{\text{N}}_t\right), \\
\hat{R}^{\text{W}}_{t+1} &= R^1_t + \hat{\alpha}^{\text{H}}_t(R^{\text{H}}_{t+1} - R^1_t) + \hat{\alpha}^{\text{F}}_t(R^{\text{F}}_{t+1} - R^1_t) + \hat{\alpha}^{\text{N}}_t(\hat{R}^{\text{N}}_{t+1} - R^1_t),
\end{aligned}$$

where $\hat{\alpha}^{\text{H}}_t, \hat{\alpha}^{\text{F}}_t$ and $\hat{\alpha}^{\text{N}}_t$ denote the shares of wealth allocated by UK households into equity issued by US and UK tradable firms, and UK nontradable firms. $\hat{R}^{\text{N}}_{t+1}$ denotes the return on UK nontradable assets measured in terms of US tradables.

To complete the description of the economy, we need to relate the equity returns $\{R^{\text{H}}_{t+1}, R^{\text{F}}_{t+1}, R^{\text{N}}_{t+1}, \hat{R}^{\text{N}}_{t+1}\}$ to equity prices and dividends:

$$R^{\text{H}}_{t+1} = \frac{P^{\text{H}}_{t+1} + D^{\text{H}}_{t+1}}{P^{\text{H}}_t} \qquad R^{\text{F}}_{t+1} = \frac{Q^{\text{F}}_{t+1}}{Q^{\text{F}}_t}\frac{P^{\text{F}}_{t+1} + D^{\text{F}}_{t+1}}{P^{\text{F}}_t}, \tag{5}$$

$$R^{\text{N}}_{t+1} = \frac{Q^{\text{N}}_{t+1}}{Q^{\text{N}}_t}\frac{P^{\text{N}}_{t+1} + D^{\text{N}}_{t+1}}{P^{\text{N}}_t} \qquad \hat{R}^{\text{N}}_{t+1} = \frac{\hat{Q}^{\text{N}}_{t+1}}{\hat{Q}^{\text{N}}_t}\frac{\hat{P}^{\text{N}}_{t+1} + \hat{D}^{\text{N}}_{t+1}}{\hat{P}^{\text{N}}_t}. \tag{6}$$

In this economy, the terms of trade are given by the relative price of UK tradables, Q^{F}_t, so the terms of trade improves for the US when Q^{F}_t falls. The real exchange rate, $\mathcal{E}_t$, is defined as the relative price of foreign to domestic consumption:

$$\mathcal{E}_t = \hat{Q}_t / Q_t,$$

where Q_t and $\hat{Q}_t$ are the US and UK price indices given by

$$Q_t = \left(\lambda_{\text{T}}(Q^{\text{T}}_t)^{\frac{\phi}{\phi-1}} + \lambda_{\text{N}}(Q^{\text{N}}_t)^{\frac{\phi}{\phi-1}}\right)^{\frac{\phi-1}{\phi}} \quad \text{and} \quad \hat{Q}_t = \left(\hat{\lambda}_{\text{T}}(\hat{Q}^{\text{T}}_t)^{\frac{\phi}{\phi-1}} + \hat{\lambda}_{\text{N}}(\hat{Q}^{\text{N}}_t)^{\frac{\phi}{\phi-1}}\right)^{\frac{\phi-1}{\phi}}. \tag{7}$$

Q^{T}_t and $\hat{Q}^{\text{T}}_t$ are the price indices of tradable consumption in the US and UK:

$$Q^{\text{T}}_t = \left(\lambda_{\text{H}} + \lambda_{\text{F}}(Q^{\text{F}}_t)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \quad \text{and} \quad \hat{Q}^{\text{T}}_t = \left(\hat{\lambda}_{\text{H}} + \hat{\lambda}_{\text{F}}(Q^{\text{F}}_t)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}. \tag{8}$$

2 Equilibrium

This section describes the optimality conditions for households and firms, presents the market clearing conditions, and defines the competitive equilibrium.

The first-order conditions for US households are given by

$$Q_t^{\text{F}} = (\partial U_t/\partial C_t^{\text{F}})/(\partial U_t/\partial C_t^{\text{H}}), \tag{9a}$$

$$Q_t^{\text{N}} = (\partial U_t/\partial C_t^{\text{N}})/(\partial U_t/\partial C_t^{\text{H}}), \tag{9b}$$

$$1 = \mathbb{E}_t\left[M_{t+1}R_t^1\right], \tag{9c}$$

$$1 = \mathbb{E}_t\left[M_{t+1}R_{t+1}^{\text{H}}\right], \tag{9d}$$

$$1 = \mathbb{E}_t\left[M_{t+1}R_{t+1}^{\text{F}}\right], \tag{9e}$$

$$1 = \mathbb{E}_t\left[M_{t+1}R_{t+1}^{\text{N}}\right], \tag{9f}$$

where $M_{t+1} = \beta(\partial U_{t+1}/\partial C_{t+1}^{\text{H}})/(\partial U_t/\partial C_t^{\text{H}})$ is the intertemporal rate of substitution (IMRS) between the consumption of US tradables in period t and $t+1$ and $U_t = U(C_t^{\text{T}}, C_t^{\text{N}})$. The first two equations define the relative prices of imported tradable and local nontradable consumption as ratios of their respective marginal utilities to the marginal utility of the numeraire, US tradables. The remaining equations are the standard pricing equations for the bond and equity. The first-order conditions for UK households are symmetric.

All returns that appear in the pricing equations in (9d)-(9f) are functions of equity prices, relative prices of goods, and the dividends paid by firms. Nontradable dividends are exogenous, while tradable dividends are chosen optimally by firms. Recall that US tradable firms choose period-t investment to maximize $P_t^{\text{H}} + D_t^{\text{H}}$. Using the return definition in (5), equation (9d) can be rewritten as $P_t^{\text{H}} = \mathbb{E}_t[M_{t+1}(P_{t+1}^{\text{H}} + D_{t+1}^{\text{H}})]$. The US tradable firm's problem can now be represented by

$$\mathcal{T}_t(Z_t^{\text{H}}, K_t^{\text{H}}) = \max_{I_t^{\text{H}}} \left\{D_t^{\text{H}} + \mathbb{E}_t\left[M_{t+1}\mathcal{T}_{t+1}(Z_{t+1}^{\text{H}}, K_{t+1}^{\text{H}})\right]\right\},$$

subject to

$$D_t^{\text{H}} = Z_t^{\text{H}}\left(K_t^{\text{H}}\right)^{\theta} - I_t^{\text{H}} \qquad \text{and} \qquad I_t^{\text{H}} = K_{t+1}^{\text{H}} - (1-\delta)\,K_t^{\text{H}},$$

where $\mathcal{T}_t(.,.)$ denotes the firm's value function. The optimal dividend policy of US tradable firms in period t can be obtained from the first-order condition for this problem:

$$1 = \mathbb{E}_t\left[M_{t+1}R_{t+1}^{\text{K}}\right], \tag{10}$$

where $R_{t+1}^{\text{K}} = [\theta Z_{t+1}^{\text{H}}(K_{t+1}^{\text{H}})^{\theta-1} + (1-\delta)]$. The dividend issued by UK firms producing tradables is similarly determined from

$$1 = \mathbb{E}_t[\hat{M}_{t+1}\hat{R}_{t+1}^{\text{K}}], \tag{11}$$

with $\hat{R}_{t+1}^{\text{K}} = (Q_{t+1}^{\text{F}}/Q_t^{\text{F}})[\theta Z_{t+1}^{\text{F}}(K_{t+1}^{\text{F}})^{\theta-1} + (1-\delta)]$, where $\hat{M}_{t+1}$ is the IMRS between the consumption of US tradables in period t and $t+1$ by UK households given by $\hat{M}_{t+1} = \beta(\partial\hat{U}_{t+1}/\partial\hat{C}_{t+1}^{\text{H}})/(\partial\hat{U}_t/\partial\hat{C}_t^{\text{H}})$.

In equilibrium, households' and firms' decisions must also satisfy the market clearing conditions. By normalizing the household and firm populations in each country to unity, the output and consumption of tradables and nontradables can be obtained as the output and consumption of the representative firms and households. In the nontradable sector, consumption is equal to output, which is also equal to the dividends paid by nontradable firms to their shareholders:

$$\begin{aligned} C_t^{\text{N}} &= D_t^{\text{N}} = Y_t^{\text{N}}, \\ \hat{C}_t^{\text{N}} &= \hat{D}_t^{\text{N}} = \hat{Y}_t^{\text{N}}. \end{aligned}$$

I assume that the tradables produced in each country are distinct, but can be costlessly transported internationally. Thus, in equilibrium, the world demand for each good must be equal to its corresponding production minus the amounts set aside for investment. For the tradable good produced in the US, market clearing requires that

$$C_t^{\text{H}} + \hat{C}_t^{\text{H}} = Y_t^{\text{H}} - I_t^{\text{H}} = D_t^{\text{H}},$$

while for the tradables produced in the UK, market clearing requires

$$C_t^{\text{F}} + \hat{C}_t^{\text{F}} = Y_t^{\text{F}} - I_t^{\text{F}} = D_t^{\text{F}}.$$

Market clearing in financial markets is equally straightforward. Nontradable equity must be allocated domestically, so that $A_t^{\text{N}} = 1$ and $\hat{A}_t^{\text{N}} = 1$. The equity issued in US firms producing tradables (normalized to unity) must be held by US and UK households, so $A_t^{\text{H}} + \hat{A}_t^{\text{H}} = 1$. Similarly, all tradable equity issued by UK firms must be split by the households in the two countries, so that $A_t^{\text{F}} + \hat{A}_t^{\text{F}} = 1$. Finally, bonds are in zero net supply, so $B_t + \hat{B}_t = 0$, where $\hat{B}_t$ denotes the bond holdings of UK households.

An equilibrium in this economy consists of a set of goods' prices $\{Q^{\text{F}}, Q^{\text{N}}, \hat{Q}^{\text{N}}\}$, asset prices $\{P^{\text{H}}, P^{\text{F}}, P^{\text{N}}, \hat{P}^{\text{N}}\}$, and a risk-free rate R^1, such that all markets clear when tradable firms optimally choose investment, and households optimally choose their consumption and their portfolios taking goods' and asset prices as given.

3 Home Bias

In this section, I characterize the optimal consumption and portfolio decisions of households. I then derive the conditions under which there is home bias in equity holdings.

3.1 Decision Rules

The optimal consumption rules of US households are easy to obtain once we recognize that, when preferences are logarithmic, the optimal consumption-wealth ratio is constant. Combining this result with households' first-order conditions in (9a)-(9b) and taking logs gives equilibrium consumption as functions of wealth, relative goods' prices and preference parameters:

$$c_t^{\text{H}} = \ln(\lambda_{\text{T}}\lambda_{\text{H}}) + \ln(1-\beta) + w_t - \tfrac{\phi}{\phi-1}q_t + \left(\tfrac{\phi}{\phi-1} - \tfrac{\rho}{\rho-1}\right)q_t^{\text{T}}, \tag{12a}$$

$$c_t^{\text{F}} = \ln(\lambda_{\text{T}}\lambda_{\text{F}}) + \ln(1-\beta) + w_t - \tfrac{\phi}{\phi-1}q_t + \left(\tfrac{\phi}{\phi-1} - \tfrac{\rho}{\rho-1}\right)q_t^{\text{T}} - \tfrac{1}{1-\rho}q_t^{\text{F}}, \tag{12b}$$

$$c_t^{\text{N}} = \ln(\lambda_{\text{N}}) + \ln(1-\beta) + w_t - \tfrac{\phi}{\phi-1}q_t + \tfrac{1}{\phi-1}q_t^{\text{N}}. \tag{12c}$$

Hereafter, lowercase letters denote the natural log of the uppercase counterpart (e.g., $c_t^{\text{H}} \equiv \ln C_t^{\text{H}}$).

To understand how portfolio shares are determined, consider the first-order conditions for US households in (9d)-(9f). These equations can be rewritten in log-linear form as

$$\mathbb{E}_t r_{t+1}^{\chi} - r_t^1 + \tfrac{1}{2}\mathbb{V}_t\left(r_{t+1}^{\chi}\right) = -\mathbb{CV}_t\left(m_{t+1}, r_{t+1}^{\chi}\right), \tag{13}$$

where r_{t+1}^{χ} is the log return for equity $\chi = \{\text{H,F,N}\}$, r_t^1 is the log risk free rate, and $m_{t+1} \equiv \ln M_{t+1}$ is the log IMRS for US households. $\mathbb{V}_t(.)$ and $\mathbb{CV}_t(.)$ denote the variance and covariance conditioned on period$-t$ information. The left hand side of (13) is the equity risk premium on asset χ. Equation (13) shows that under an optimal consumption and portfolio plan, the risk premium is equal to the covariance of that asset return with the log IMRS for US households.[5] This covariance can be decomposed further using the fact that m_{t+1} is perfectly negatively correlated with the log of household wealth, w_{t+1}. The risk premium in (13) can now be written as the covariance between wealth and the corresponding asset return:

$$\mathbb{E}_t r_{t+1}^{\chi} - r_t^1 + \tfrac{1}{2}\mathbb{V}_t\left(r_{t+1}^{\chi}\right) = \mathbb{CV}_t\left(w_{t+1}, r_{t+1}^{\chi}\right). \tag{14}$$

This is a standard result. It says that households must choose their portfolios so that the expected excess returns match the covariance of returns with wealth.

The right-hand-side of (14) can also be approximated as

$$\gamma^{\text{H}}\mathbb{CV}_t\left(c_{t+1}^{\text{H}}, r_{t+1}^{\chi}\right) + \gamma^{\text{F}}\mathbb{CV}_t\left(q_{t+1}^{\text{F}} + c_{t+1}^{\text{F}}, r_{t+1}^{\chi}\right) + \left(1 - \gamma^{\text{H}} - \gamma^{\text{F}}\right)\mathbb{CV}_t\left(q_{t+1}^{\text{N}} + c_{t+1}^{\text{N}}, r_{t+1}^{\chi}\right), \tag{15}$$

with

$$\gamma^{\text{F}} \equiv \left[\tfrac{1}{\lambda_{\text{H}}}\left(Q^{\text{T}}\right)^{\frac{\rho}{\rho-1}} - 1\right]\gamma^{\text{H}}, \qquad \text{and} \qquad \gamma^{\text{H}} \equiv \left[\tfrac{1}{\lambda_{\text{T}}\lambda_{\text{H}}}\left(Q^{\text{T}}\right)^{\frac{\rho}{\rho-1} - \frac{\phi}{\phi-1}}\left(Q\right)^{\frac{\phi}{\phi-1}}\right]^{-1},$$

where Q^{T} and Q are the steady state values of Q_t^{T} and Q_t (see equations 7 and 8 above). Equation (15) implies that the equity risk premium on asset χ depends on a weighed average of three covariances, each between the log equity return and a corresponding log consumption: local and imported tradable and local nontradable. Detailed derivations of (13) -(15) and other mathematical details of the model are presented in the Appendix.

To completely characterize the consumption and portfolio rules of US households, we need to specify the process for wealth and the distribution of returns. The process for US household wealth is obtained by log-linearizing the budget constraint in (3):

$$\Delta w_{t+1} = r_{t+1}^{\text{W}} + \ln\beta. \tag{16}$$

To characterize the log return on wealth, r_{t+1}^{W}, let $\mathbf{r}_t = [\; r_t^{\text{H}} \quad r_t^{\text{F}} \quad r_t^{\text{N}} \;]'$ denote the vector of log-returns on equity holdings in US and UK tradables, and US nontradables. Further, let Θ_t represent the variance-covariance matrix of $\mathbf{r}_{t+1}$ conditional on period$-t$ information, and $\boldsymbol{\alpha}_t = [\; \alpha_t^{\text{H}} \quad \alpha_t^{\text{F}} \quad \alpha_t^{\text{N}} \;]'$ denote the vector

[5] Excess returns are adjusted by the addition of one half times the return variance, a Jensen's inequality term, to account for the fact that I am working with log returns.

of portfolio shares chosen by US households. Then, following Campbell et. al. (2003), r^{W}_{t+1} can be approximated by

$$r^{\mathrm{W}}_{t+1} = r^1_t + \boldsymbol{\alpha}'_t \left(\mathbf{r}_{t+1} - r^1_t\right) + \tfrac{1}{2}\boldsymbol{\alpha}'_t \left(diag\left(\Theta_t\right) - \Theta_t \boldsymbol{\alpha}_t\right). \tag{17}$$

The dynamics of UK household wealth and its return are identified in a similar way.

The optimal portfolio policies of US households can now be easily obtained from (14) by substituting for wealth from the budget constraint (16). Solving the resulting system of equations for $\boldsymbol{\alpha}_t$ gives

$$\boldsymbol{\alpha}_t = \Theta_t^{-1}\left[\mathbb{E}_t \mathbf{r}_{t+1} - r^1_t + \tfrac{1}{2} diag\left(\Theta_t\right)\right]. \tag{18}$$

Equation (18) states that the share of wealth allocated into each asset is proportional to the vector of conditional log-Sharpe ratios. In this model, portfolio and consumption choices are myopic. Households consume a constant fraction of wealth every period and invest the remaining wealth according to the risk premia on the available assets, scaled by the inverse of the variance-covariance matrix of returns. Recall that the reciprocal of the coefficient of relative risk aversion that usually enters the myopic portfolio rule is unity when preferences are logarithmic. The intertemporal hedging demand of households in this model is zero, which implies that log-utility investors do not adjust their consumption-wealth ratios in response to (expected) shifts in $\mathbf{r}_{t+1}$ and r^1_t. The intuition for this result is standard: with log utility the income effects arising from the changes in investment opportunity set are exactly offset by the substitution effects.

The demand for equities and bonds by US and UK households are obtained from the optimal portfolio shares in (18) and the process for wealth in (16) as

$$\begin{array}{lll} & \text{US households} & \text{UK households} \\ \text{US tradable equity:} & A^{\mathrm{H}}_t = \alpha^{\mathrm{H}}_t W^{\mathrm{C}}_t / P^{\mathrm{H}}_t, & \hat{A}^{\mathrm{H}}_t = \hat{\alpha}^{\mathrm{H}}_t \hat{W}^{\mathrm{C}}_t / P^{\mathrm{H}}_t, \\ \text{UK tradable equity:} & A^{\mathrm{F}}_t = \alpha^{\mathrm{F}}_t W^{\mathrm{C}}_t / Q^{\mathrm{F}}_t P^{\mathrm{F}}_t, & \hat{A}^{\mathrm{F}}_t = \hat{\alpha}^{\mathrm{F}}_t \hat{W}^{\mathrm{C}}_t / Q^{\mathrm{F}}_t P^{\mathrm{F}}_t, \\ \text{nontradable equity:} & A^{\mathrm{N}}_t = \alpha^{\mathrm{N}}_t W^{\mathrm{C}}_t / Q^{\mathrm{N}}_t P^{\mathrm{N}}_t, & \hat{A}^{\mathrm{N}}_t = \hat{\alpha}^{\mathrm{N}}_t \hat{W}^{\mathrm{C}}_t / \hat{Q}^{\mathrm{N}}_t \hat{P}^{\mathrm{N}}_t, \\ \text{bonds} & B_t = \alpha^{\mathrm{B}}_t W^{\mathrm{C}}_t R^1_t, & \hat{B}_t = \hat{\alpha}^{\mathrm{B}}_t \hat{W}^{\mathrm{C}}_t R^1_t, \end{array} \tag{19}$$

where $W^{\mathrm{C}}_t \equiv W_t - C^{\mathrm{H}}_t - Q^{\mathrm{F}}_t C^{\mathrm{F}}_t - Q^{\mathrm{N}}_t C^{\mathrm{N}}_t$ and $\hat{W}^{\mathrm{C}}_t \equiv \hat{W}_t - \hat{C}^{\mathrm{H}}_t - Q^{\mathrm{F}}_t \hat{C}^{\mathrm{F}}_t - \hat{Q}^{\mathrm{N}}_t \hat{C}^{\mathrm{N}}_t$ denote period$-t$ wealth net of consumption expenditure, while $\alpha^{\mathrm{B}}_t \equiv 1 - \alpha^{\mathrm{H}} - \alpha^{\mathrm{F}}_t - \alpha^{\mathrm{N}}_t$ and $\hat{\alpha}^{\mathrm{B}}_t \equiv 1 - \hat{\alpha}^{\mathrm{H}}_t - \hat{\alpha}^{\mathrm{F}}_t - \hat{\alpha}^{\mathrm{N}}_t$ are the shares of wealth allocated into bonds. Equations in (19) show that asset demands depend on expected future returns and risk via optimally chosen portfolio shares, $\boldsymbol{\alpha}_t$, accumulated wealth net of consumption, W^{C}_t and $\hat{W}^{\mathrm{C}}_t$, current asset prices (i.e., P^{H}_t, $P^{\mathrm{F}}_t, P^{\mathrm{N}}_t$ and $\hat{P}^{\mathrm{N}}_t$ for equity, and $1/R^1_t$ for bonds), and current goods' prices (i.e., $Q^{\mathrm{F}}_t, Q^{\mathrm{N}}_t$ and $\hat{Q}^{\mathrm{N}}_t$).

3.2 When is Home Bias Optimal?

The equity holdings of US households display home bias if the share of wealth they allocate to equity issued by US firms producing tradables exceeds the share of wealth allocated to the equity issued by UK firms producing tradables: $\alpha^{\mathrm{H}}_t > \alpha^{\mathrm{F}}_t$. Similarly, the portfolios of UK households display home bias if $\hat{\alpha}^{\mathrm{H}}_t < \hat{\alpha}^{\mathrm{F}}_t$. In this model home bias in portfolio shares also implies a home bias in the asset holdings of households. For

US households home bias implies $A_t^{\text{H}} > A_t^{\text{F}}$, while for UK households it implies $\hat{A}_t^{\text{H}} < \hat{A}_t^{\text{F}}$. The discussion in this section will focus on home bias in portfolio shares. I will study home bias in both equity shares and equity holdings in section 5.2 below.

To provide intuition for the existence of equity home bias, consider the portfolio choice in (18). In particular, consider the share of wealth allocated by US households to the equity issued by US tradable firms:

$$\alpha_t^{\text{H}} = \frac{\mathbb{V}_t(r_{t+1}^{\text{F}})\left[\mathbb{CV}_t(w_{t+1}^{\text{H}}, r_{t+1}^{\text{H}}) - \mathbb{CV}_t(r_{t+1}^{\text{N}}, r_{t+1}^{\text{H}})\alpha_t^{\text{N}}\right]}{\mathbb{V}_t(r_{t+1}^{\text{H}})\mathbb{V}_t(r_{t+1}^{\text{F}}) - \mathbb{CV}_t^2(r_{t+1}^{\text{H}}, r_{t+1}^{\text{F}})} - \frac{\mathbb{CV}_t\left(r_{t+1}^{\text{H}}, r_{t+1}^{\text{F}}\right)\left[\mathbb{CV}_t(w_{t+1}^{\text{H}}, r_{t+1}^{\text{F}}) - \mathbb{CV}_t(r_{t+1}^{\text{N}}, r_{t+1}^{\text{F}})\alpha_t^{\text{N}}\right]}{\mathbb{V}_t(r_{t+1}^{\text{H}})\mathbb{V}_t(r_{t+1}^{\text{F}}) - \mathbb{CV}_t^2(r_{t+1}^{\text{H}}, r_{t+1}^{\text{F}})}.$$

This equation determines the optimal share of US tradable equity held by US households for a given nontraded equity share, α_t^{N}. It is also a function of the covariance between wealth and returns in the US and UK. The following proposition allows us to examine the determinants of α_t^{H} in detail.

Proposition 1 *When (i) all uncertainty is resolved after period $t+1$; (ii) countries are symmetric; and (iii) the share of nontradable equity in household portfolios is equal to 1/2, the share of domestic assets in US tradable portfolios is given by*

$$\alpha_t^{\text{H}} = \tfrac{1}{4} + bias$$

with

$$bias = \tfrac{1}{4\sigma_t^2}\left[\beta_t^{\text{HF}}\mathbb{CV}_t(q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}}, \Delta\eta^{\text{N}}) - \mathbb{CV}_t(d_{t+1}^{\text{H}}, \Delta\eta^{\text{N}})\right], \tag{20}$$

where $\sigma_t^2 \equiv \mathbb{V}_t(d_{t+1}^{\text{H}})(1-\rho_{\text{HF}}^2) > 0$, ρ_{HF} is the correlation coefficient between US and UK tradable dividends, $\Delta\eta^{\text{N}} \equiv q_{t+1}^{\text{N}} + d_{t+1}^{\text{N}} - (\hat{q}_{t+1}^{\text{N}} + \hat{d}_{t+1}^{\text{N}})$ denotes the relative value of nontradable endowment in US versus UK, and β_t^{HF} is the conditional beta of the payoff on US tradable equity with respect to the payoff on UK tradable equity:

$$\beta_t^{\text{HF}} \equiv \mathbb{CV}_t\left(d_{t+1}^{\text{H}}, q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}}\right)/\mathbb{V}_t\left(q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}}\right).$$

Condition (i) collapses the dynamics in the model to a static framework. At the start of period t, households and firms learn of the productivity shocks in the tradable and nontradable sectors. Households then decide on their consumption demands, while firms determine their optimal production and dividend policies. Under condition (i), the future dividend-price ratio and future returns for each asset are constant, so all the risk from holding equities from t to $t+1$ comes from unanticipated variations in period $t+1$ dividend payments. Conditions (i) and (ii) imply that bond holdings are zero. In equilibrium, market clearing ensures that holdings of nontradable equity are equal to unity, so all variations in α_t^{N} must be due to changes in wealth and/or the price of nontradable equity. To abstract from these variations, condition (iii) sets α_t^{N} equal to its steady state value of 1/2.[6]

The *bias* term in (20) measures the degree to which US households skew their portfolios towards the equity issued by US firms producing tradables. The bias arises due to the cross-hedging demand for tradable equity, which develops as households seek to insure against the fluctuations in their nontradable consumption. The *sign* of the bias depends on the sign of the terms in brackets, which are determined by the co-movements in the value of dividend streams on different assets.

The *extent* of the bias shown in (20) depends on the relative ability of US versus UK tradable equity in hedging unexpected fluctuations in the relative nontradable endowment in the US.[7] In particular, when the amount of US nontradable risk co-moves more strongly with the payoffs of a particular tradable asset (say, UK tradable equity), the diversification benefit from holding the alternative tradable asset (say, US tradable equity) will be larger. Notice that US households assign weight β_t^{HF} to the hedging ability of the UK tradable asset.[8] From their point of view, the usefulness of the UK equity as a hedge decreases with the volatility of UK asset and increases with the co-movement between its payoffs and the payoffs on US asset (irrespective of its sign).

To disentangle these relations, I next discuss several special cases based on different assumptions about the intratemporal elasticities between consumption goods. All cases are based on the benchmark coefficient of relative risk aversion equal to one.

3.2.1 Case 1: $\frac{1}{1-\phi} \to \infty$ and $\frac{1}{1-\rho} \to \infty$

In this case all goods in the economy (i.e., both types of tradables and nontradables) are perfect substitutes. The law of one price ensures that all relative goods prices are now equal to one, so $q_t^{\mathrm{F}} = 0$, $q_t^{\mathrm{N}} = 0$, and $\hat{q}_t^{\mathrm{N}} = 0$. Under these circumstances, and keeping in mind that productivity shocks are independent across sectors and countries, the *bias* term in (20) disappears, giving $\alpha_t^{\mathrm{H}} = 1/4$. With bond holdings equal to zero, equity portfolio shares within the country must add up to one, which implies $\alpha_t^{\mathrm{F}} = 1/4$. As a result, the optimal tradable portfolios of households in both countries are fully diversified.

3.2.2 Case 2: $\frac{1}{1-\phi} = 1$ and $\frac{1}{1-\rho} = 1$

In this case preferences across all three goods become Cobb-Douglas. When utility is logarithmic they simplify further and become separable. Now, changes in nontradable consumption have no effect on the

[6]It can be shown that the steady state value for α^{N} is proportional to the share of nontraded consumption in the total consumption expenditure: $\alpha^{\mathrm{N}} = \lambda_{\mathrm{N}}^{1-\phi}(C^{\mathrm{N}}/C)^{\phi}$. When tradable and nontradable sectors have equal size, $\lambda_{\mathrm{N}} = 1/2$, giving $\alpha^{\mathrm{N}} = 1/2$.

[7]Recall that nontradable output is only consumed domestically and is produced without capital. Thus, in equilibrium, nontradable dividends and consumption are determined entirely by the exogenous state of productivity in the domestic nontraded sector.

[8]With tradable dividends being determined endogenously, β_t^{HF} provides a convenient indicator for how the tradable productivity shocks are transmitted across countries. In the terms of Cole and Obstfeld (1991), when productivity shocks are "transmitted positively" between countries, so that $CV_t(d_{t+1}^{\mathrm{H}}, q_{t+1}^{\mathrm{F}} + d_{t+1}^{\mathrm{F}}) > 0$, β_t^{HF} is positive; when $CV_t(d_{t+1}^{\mathrm{H}}, q_{t+1}^{\mathrm{F}} + d_{t+1}^{\mathrm{F}}) < 0$, shocks to production are "transmitted negatively" across countries, and β_t^{HF} becomes negative.

marginal utility of tradable consumption, local or imported, and no bias in tradable portfolios arises in equilibrium. The reason is that any increase in nontradable or tradable dividends causes a proportional fall in the corresponding relative price, which makes all the covariances in the numerator of (20) equal to zero.

3.2.3 Case 3: $\frac{1}{1-\rho} \to \infty$

In this case tradables are perfect substitutes. With no transaction costs or other market frictions, the law of one price equalizes the prices of tradables across countries, so that $q_t^{\mathrm{F}} = 0$. The bias term now becomes

$$\frac{1}{4\sigma_t^2}\left[\beta_t^{\mathrm{HF}}\mathbb{CV}_t(d_{t+1}^{\mathrm{F}}, \Delta\eta^{\mathrm{N}}) - \mathbb{CV}_t(d_{t+1}^{\mathrm{H}}, \Delta\eta^{\mathrm{N}})\right], \tag{21}$$

where β_t^{HF} now simplifies to $\mathbb{CV}_t(d_{t+1}^{\mathrm{H}}, d_{t+1}^{\mathrm{F}})/\mathbb{V}_t(d_{t+1}^{\mathrm{F}})$. The expression in (21) allows us to focus on the size and sign of the portfolio bias due to variations in q_t^{N}. There are several possible cases to consider. First, when the elasticity of substitution between nontradable and a basket of tradable consumption, $\frac{1}{1-\phi}$, is equal to one, the variations in relative price and nontradable consumption exactly offset each other and the covariances in (21) will all be equal to zero. This is Case 2 discussed above.

The second case corresponds to the logic developed in Baxter and Jermann (1997). They show that foreign financial assets provide a better insurance against fluctuations in nondiversifiable labor income when the return on labor and capital are positively correlated. In my model households would hold large amounts of foreign equity in their portfolios if $bias < 0$. This situation occurs when the intratemporal elasticity between tradables and nontradables is greater than one so that the relative price is inelastic with respect to the relative supply of the two goods. In this case, each covariance term, including β_t^{HF}, in the *bias* is positive, because tradable sector firms in both countries increase their dividend payoffs when nontradable dividends are high, (i.e., $\partial d^{\mathrm{H}}/\partial(d^{\mathrm{N}} - \hat{d}^{\mathrm{N}}) > 0$ and $\partial d^{\mathrm{F}}/\partial(d^{\mathrm{N}} - \hat{d}^{\mathrm{N}}) > 0$).[9] The *bias* will be negative when changes in the relative value of US nontradable consumption covary strongly with US tradable dividends. Intuitively, when $\frac{1}{1-\phi} > 1$ (which implies that it is also above the intertemporal elasticity of substitution) tradable and nontradable goods are considered substitutes. In this setup, an increase in nontradable endowment lowers the demand for tradable consumption. If US tradable firms are more likely to pay higher dividends than UK firms when those dividends are less valued, US households would bias their portfolios in favor of UK equity.

Finally, when the equilibrium price is elastic with respect to the relative supply of two goods (i.e., when $\frac{1}{1-\phi} < 1$), any change in nontradable dividends is more than offset by the change in the relative price of nontradables, making all covariance terms in (21) negative. The sign of the bias in this case is determined by the sign of

$$\frac{1}{4}\left[-\beta_t^{\mathrm{HF}}\mathbb{CV}_t(d_{t+1}^{\mathrm{F}}, d_{t+1}^{\mathrm{N}} - \hat{d}_{t+1}^{\mathrm{N}}) + \mathbb{CV}_t(d_{t+1}^{\mathrm{H}}, d_{t+1}^{\mathrm{N}} - \hat{d}_{t+1}^{\mathrm{N}})\right]. \tag{22}$$

This expression tells us that the share of wealth allocated into the local tradable asset by US households will be higher relative to the equally-weighted portfolio when the relative amount of US nontradable endowment is more correlated with US rather than UK tradable dividends. Intuitively, when households like to consume

[9] Tradable firms are paying out higher dividends in response to an interest rate increase. The latter makes investment costly and induces firms to decrease their capital stock.

a balanced basket of tradable and nontradable goods (i.e., when the two goods are complements), they will demand assets that provide higher payoffs in terms of tradables when nontradable consumption is high. They will, therefore, hold more of locally issued tradable equity as it can guarantee them such a balanced consumption stream. As above, the sensitivity of US demand towards the hedging ability of UK equity is determined by β_t^{HF}. In the analysis below I focus on the case with $\frac{1}{1-\phi} < 1$.[10]

3.2.4 Case 4: $\frac{1}{1-\phi} < 1$ and $\frac{1}{1-\rho} > 1$

When each country specializes in the production of a distinct tradable good, variations in the terms of trade, q_t^{F}, add another dimension to the portfolio problem. To examine this case, I will focus on the scenario with $\frac{1}{1-\phi} < 1$, so that nontradables and a basket of tradables are complements in consumption. Then, the sign of the bias is given by

$$\tfrac{1}{4\sigma_t^2}\left[-\beta_t^{\text{HF}}\mathbb{CV}_t(q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}}, d_{t+1}^{\text{N}} - \hat{d}_{t+1}^{\text{N}}) + \mathbb{CV}_t(d_{t+1}^{\text{H}}, d_{t+1}^{\text{N}} - \hat{d}_{t+1}^{\text{N}})\right] \tag{23}$$

This expression is analogous to (22), except that q_{t+1}^{F} now appears in the first covariance term. As before, it implies that US portfolios will deviate in favor of US assets when the relative endowment of nontradables is more correlated with the value of US than UK tradable dividends. The intuition here parallels with the one developed in Case 3. In particular, when nontradables and tradables are complements, any increase in nontradable endowment is associated with an increase in the demand for tradable goods. How this higher tradable demand is distributed across the local and imported tradables is determined by two parameters: (i) the degree of consumption home bias, which determines the desirability of domestically produced good relative to foreign produced good; and (ii) the elasticity of substitution between two tradables, which controls the degree of adjustment in the relative price of two tradables necessary to accommodate the changes in their relative demand.

Consider the case when US households assign higher weight to the consumption of US tradables. In response to a positive nontradable shock, they will demand higher consumption of US tradables relative to UK tradables. With the firms in the tradable sectors of the two countries now being specialized, nontradable shock can be treated as a demand shock since it does not affect the relative productivity of the firms. Therefore, both firms will increase their dividend payments to households when nontradable endowment is relatively high. When two tradables are substitutes (i.e., $\frac{1}{1-\rho} > 1$), higher relative demand for US tradables is associated with a fall in the relative price of UK tradables. This leads to a decrease in the value of foreign equity payouts and an increase in the relative value of US equity payouts. US assets thus become more attractive to US households as they allow them to finance higher tradable consumption exactly when the demand for such consumption is high. This provides a basis for home bias in US portfolios.

The dynamics of q_t^{F} also influence the hedging ability of the UK asset. In particular, q_t^{F} affects β_t^{HF} via both the variance of UK payout and it covariance with US payoffs. The sign of β_t^{HF} is determined by the sign of $\mathbb{CV}_t(d_{t+1}^{\text{H}}, q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}})$, which in turn depends on the sensitivity of the relative demand for traded goods to

[10] A number of studies estimate the intratemporal elasticity of substitution between tradable and nontradable consumption to be below one. Corsetti, et.al (2004) find it to be equal to 0.74, Tesar (1993) uses $\frac{1}{1-\phi} = 0.44$.

their relative price. Irrespective of the sign, the size of β_t^{HF} decreases with the riskiness of UK asset. Notice that the variance of this asset can be rewritten as $\mathbb{V}_t(q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}}) = \mathbb{V}_t(q_{t+1}^{\text{F}}) + \mathbb{V}_t(d_{t+1}^{\text{F}}) + 2\mathbb{CV}_t(q_{t+1}^{\text{F}}, d_{t+1}^{\text{F}})$. All of the terms in this equation are endogenously determined. It is, however, easy to see that the volatility of UK asset payoffs is increasing in the variance of the terms of trade and decreases in the covariance between UK tradable dividends and their relative price, as the latter is always negative. The relative contribution of each term to the riskiness of UK assets will again depend on the elasticity of substitution between US and UK tradable goods, $\frac{1}{1-\rho}$. The sensitivity of investors portfolios to changes in this elasticity are explored numerically in section 5.

4 Solving the Model

While the static framework developed in the previous section provides a useful shortcut for developing the intuition for home bias, it relies on the simplifying conditions in Proposition 1. In this section, I describe the solution technique used to find the competitive equilibrium in the full dynamic model and to study the optimal portfolio choices of households.

4.1 Solution Technique

To find the competitive equilibrium for the model, I apply the solution method developed in Evans and Hnatkovska (2005). The procedure starts with a second-order log-linear approximation to the model's equilibrium dynamics. Then I make a conjecture about the form and dynamics of the state vector, x_t, which contains elements that characterize the information set available to households and firms in period t. In particular, I posit that $x_t = [z_t, k_t^{\text{H}}, k_t^{\text{F}}, w_t, \hat{w}_t]$ where $z_t \equiv [\ln Z_t^{\text{H}}, \ln Z_t^{\text{F}}, \ln Z_t^{\text{N}}, \ln \hat{Z}_t^{\text{N}}]'$, $k_t^{\text{H}} \equiv \ln(K_t^{\text{H}}/K^{\text{H}})$, $k_t^{\text{F}} \equiv \ln(K_t^{\text{F}}/K^{\text{F}})$, $w_t \equiv \ln(W_t/W_0)$ and $\hat{w}_t \equiv \ln(\hat{W}_t/\hat{W}_0)$. K^{H} and K^{F} are the steady state values of K_t^{H} and K_t^{F}, while W_0 and $\hat{W}_0$ are initial levels of households wealth.

The next step is to characterize the process for x_t. Nonlinearities in the model make it impossible to describe the dynamics of x_t using just its own lagged values. When households face portfolio choice problems, wealth in period t will depend on the first and second moments of returns conditioned on period–$t-1$ information.[11] In general, these moments will be high-order polynomials in the elements of x_{t-1} (e.g., $w_{t-1}^2, \hat{w}_{t-1}^2, w_{t-1}\hat{w}_{t-1}, ...w_{t-1}^3,$), so elements of x_t will depend on not just x_{t-1} but also elements in $x_{t-1}x_{t-1}'$ and so on. I consider an approximate solution to the model that ignores the impact of third and higher order terms. Under this assumption, I conjecture that the dynamics of x_t can be summarized by

$$x_{t+1} = \Phi_0 + (I - \Phi_1)x_t + \Phi_2\tilde{x}_t + \varepsilon_{t+1}.$$

[11] This relation can be seen from equations (16)-(17). Together they imply that the variability of wealth in period t depends on the realizations of returns during that period. How susceptible wealth is going to be to the unexpected components of the returns is determined by the portfolio allocations chosen in period $t-1$. As a result, uncertainty enters the decisions of households multiplicatively, thus adding another layer of nonlinearity to the model. Note that no redifinition of state vector (i.e., in terms of asset holdings rather than wealth) would allow us to get around this issue. As long as assets in the portfolio are risky and their returns are non i.i.d., the multiplicative nature of uncertainty in this model will remain.

where $\tilde{x}_t \equiv vec\left(x_t x_t'\right)$, and vec is vectorization operator. Here Φ_0, Φ_1, and Φ_2 are matrices of coefficients to be determined. ε_{t+1} is a vector of innovations with zero conditional mean, and conditional covariance that is a function of x_t :

$$\begin{aligned}\mathbb{E}\left(\varepsilon_{t+1}|x_t\right) &= 0,\\ \mathbb{E}\left(\varepsilon_{t+1}\varepsilon_{t+1}'|x_t\right) &= \Omega_0 + \Omega_1 x_t x_t' \Omega_1'.\end{aligned}$$

Let X_t denote the composite state vector that consists of a constant, linear states x_t, and their squares and cross-products, $\tilde{x}_t$. Evans and Hnatkovska (2005) show that the dynamics of X_t can be expressed in a compact form as

$$X_{t+1} = \mathbb{A}X_t + U_{t+1}$$

where $\mathbb{A}$ is a matrix of coefficients in terms of Φ_0, Φ_1, and Φ_2, and U_{t+1} is a mean zero, conditionally heteroskedastic error process with variance-covariance matrix $\mathbb{S}\left(X_t\right)$. Endogenous heteroskedasticity of the innovations to X_t as captured by $\mathbb{S}\left(X_t\right)$ is a generic feature of the model with portfolio choice. It governs the time-variable risk premium, which in turn allows me to pin down the equilibrium portfolio shares in the model.

I can now characterize the decision rules for non-predetermined variables and risk. Specifically, I posit that log-dividends, log-consumption demands and portfolio shares can all be written as linear functions of the state vector X_t. Given these conjectures, I then can use the market clearing conditions to derive the expressions for equilibrium equity prices, relative goods prices and the interest rate as functions of X_t. Finally, I verify that the assumed decision rules are consistent with the first-order conditions of firms and households, given the processes for prices and returns.[12] Complete details of this solution method are presented in Evans and Hnatkovska (2005).

4.2 Calibration

Parameter values for the benchmark calibration of the model are summarized in Table 1. The world economy is constructed as consisting of two symmetric countries, matching the properties of US economy in quarterly data. Most of the preference parameter values are borrowed from Corsetti et. al. (2005). In particular, the value for ϕ is chosen to set the elasticity of substitution between tradable and nontradable consumption to 0.74. The share of tradables and nontradables in aggregate consumption expenditure, λ_{T} and $\lambda_{\text{N}} = (1-\lambda_{\text{T}})$, are set to 0.5 in both countries. These numbers are calculated using OECD STAN database for Industrial Analysis.[13]

Preferences over the consumption of local and imported tradables are calibrated using a range of possible values. Under the baseline calibration, the value of ρ is set to obtain the elasticity of substitution between

[12] An important feature that distinguishes this method from the standard second-order perturbation methods is that it allows for heteroskedasticity in the state vector and models its variance-covariance matrix explicitly as part of the equilibrium. This simultaneous determination of the first and second moments of the state variables is necessary because equilibrium returns in the model do not follow i.i.d. processes.

[13] These numbers are similar to the estimates in Corsetti et. al. who use $\lambda_{\text{T}} = 0.45$ and $\lambda_{\text{N}} = 0.55$.

Table 1. Model Parameters

Preferences	β	λ_{T}	λ_{N}	$1/(1-\phi)$
	0.99	0.5	0.5	0.74
		λ_{H}	$\hat{\lambda}_{\text{F}}$	$1/(1-\rho)$
		0.72	0.72	1.10
Production	θ	δ		
	0.36	0.02		
Productivity	a_{ii}^{H}	a_{ii}^{N}	Ω_e	
	0.78	0.99	0.0001	

US and UK tradables equal to 1.1, consistent with Corsetti et. al. (2005).[14] The model is also solved for a sequence of values for $1/(1-\rho)$ ranging from 1.1 to 115. I also assume that countries exhibit consumption home bias in their preferences by setting $\lambda^{\text{H}} = \hat{\lambda}^{\text{F}} = 0.72$ as estimated by Corsetti, et. al. (2005).

On the production side, the capital share in tradable production, θ, is set to 0.36, and the depreciation rate, δ, is set to 0.02. These values are consistent with the estimates in Backus, Kehoe and Kydland (1995). Each of the four productivity processes (i.e., $\ln Z_t^{\text{H}}, \ln Z_t^{\text{F}}, \ln Z_t^{\text{N}}$, and $\ln \hat{Z}_t^{\text{N}}$) follows an AR(1) process. In order to isolate the role of relative prices in explaining portfolio choices, I assume that productivity changes are independent across sectors and countries. The AR(1) coefficients in the processes for tradable productivity, $\ln Z_t^{\text{H}}$ and $\ln Z_t^{\text{F}}$, are 0.78, while the coefficients for nontradable productivity, $\ln Z_t^{\text{N}}$, and $\ln \hat{Z}_t^{\text{N}}$, are 0.99. Shocks to all four productivity processes have a variance of 0.0001. This specification implies that all shocks have persistent but temporary effects on productivity.

The numerical procedure produces the equilibrium dynamics of the state variables as functions of their past values and their squares and cross-products. The variance-covariance matrix of the state vector is similarly obtained. The solution also provides me with a set of decision rules, all expressed as linear functions of a vector of state variables as well as their second moments. I study the characteristics of the competitive equilibrium by simulating the model over 200 quarters. The statistics reported in the next section are derived from 200 simulations and so are based on 10,000 years of simulated quarterly data in the neighborhood of the equal initial wealth distribution.

5 Results

This section presents the main findings from the numerical solution to the model. I first evaluate the importance of market incompleteness for risk-sharing between countries. Second, I characterize the optimal

[14]This value is also close to the parametrization used in Heathcote and Perri (2002), who estimate the elasticity between domestic and foreign intermediate goods to be 0.9.

portfolio shares and the resulting asset-holdings. Third, I analyze the capital flows and the factors driving them. Finally, I examine the conditional and unconditional moments of the portfolio returns.

5.1 How Important is Market Incompleteness?

As I noted in the introduction, risk-sharing plays a conceptually important role in the determination of portfolio allocations and capital flows. In this subsection, I quantify the degree of risk-sharing that arises in the competitive equilibrium of the model. Recall that households cannot hold equity issued by foreign firms producing nontradables. The question I now ask is: to what extent does this restriction on households' portfolios impede international risk-sharing?

To address this question, I return to the first-order conditions governing households' holdings of nontradable equity:

$$1 = \mathbb{E}_t \left[M_{t+1} R^{\mathrm{N}}_{t+1} \right], \tag{24a}$$

$$1 = \mathbb{E}_t \left[\hat{M}_{t+1} \hat{R}^{\mathrm{N}}_{t+1} \right], \tag{24b}$$

where, as before, M_{t+1} and $\hat{M}_{t+1}$ respectively denote the IMRS for US and UK households. Substituting for M_{t+1} with the identity $M_{t+1} \equiv \hat{M}_{t+1} + M_{t+1} - \hat{M}_{t+1}$ in (24a), we can rewrite the first-order condition for US households as

$$1 = \mathbb{E}_t \left[\hat{M}_{t+1} R^{\mathrm{N}}_{t+1} \right] + \mathbb{E}_t \left[(M_{t+1} - \hat{M}_{t+1}) R^{\mathrm{N}}_{t+1} \right]. \tag{25}$$

Similarly, the first-order condition for UK households in (24b) can be rewritten as

$$1 = \mathbb{E}_t \left[M_{t+1} \hat{R}^{\mathrm{N}}_{t+1} \right] - \mathbb{E}_t \left[(M_{t+1} - \hat{M}_{t+1}) \hat{R}^{\mathrm{N}}_{t+1} \right]. \tag{26}$$

Equations (25) and (26) allow us to quantify the degree of risk-sharing in the model and examine its consequences for households' portfolio choices. To see why, suppose (counter-factually) that the equilibrium in the model permitted complete risk-sharing. In this case $M_t = \hat{M}_t$, so that the second terms on the right-hand-side of (25) and (26) disappear, leaving $1 = \mathbb{E}_t[\hat{M}_{t+1} R^{\mathrm{N}}_{t+1}]$ and $1 = \mathbb{E}_t[M_{t+1} \hat{R}^{\mathrm{N}}_{t+1}]$. Notice that these expressions are the first-order conditions for UK and US households that have access to foreign nontradable equity. Thus, if the equilibrium in the model permitted complete risk-sharing, households would find it optimal not to hold any foreign nontradable equity. Intuitively, if there is complete risk-sharing in a competitive equilibrium in which households are prohibited from holding foreign nontradable equity, households will not benefit from gaining access to the foreign nontradable equity market.

Now consider the case where the competitive equilibrium only permits incomplete risk-sharing. Here M_t and $\hat{M}_t$ will be less than perfectly correlated but it is still *possible* that households would not benefit from gaining access to the foreign market for nontradable equity. So long as the difference between M_t and $\hat{M}_t$ is conditionally uncorrelated with the returns on nontradables, R^{N}_{t+1} and $\hat{R}^{\mathrm{N}}_{t+1}$, the second terms on the right-hand-side of (25) and (26) disappear, again leaving $1 = \mathbb{E}_t[\hat{M}_{t+1} R^{\mathrm{N}}_{t+1}]$ and $1 = \mathbb{E}_t[M_{t+1} \hat{R}^{\mathrm{N}}_{t+1}]$.

This observation suggests a simple way to quantify the impact of incomplete-risk sharing on households' portfolio choices. Namely, I consider the projections of R_t^{N} and $\hat{R}_t^{\text{N}}$ on $M_t - \hat{M}_t$:

$$R_t^{\text{N}} = \mathcal{P}(R_t^{\text{N}}|M_t - \hat{M}_t) + \epsilon_t, \quad (27a)$$

$$\hat{R}_t^{\text{N}} = \mathcal{P}(\hat{R}_t^{\text{N}}|M_t - \hat{M}_t) + \hat{\epsilon}_t. \quad (27b)$$

Clearly, these projections only make sense when M_t and $\hat{M}_t$ are imperfectly correlated. In the equilibrium of my model the correlation between M_t and $\hat{M}_t$ is 0.76, so the projections in (27) can be estimated without difficulty. Now, if the estimated projection coefficients are significantly different from zero, we can reject the hypothesis that the returns on nontradables are orthogonal to $M_t - \hat{M}_t$. And, as a consequence, we can also reject the hypothesis that households would never benefit from gaining access to the market for foreign nontradable equity.[15] This turns out to be the case. Using simulated data from the model, the estimated projection coefficients in (27a) and (27b) are -0.95 and 1.09 respectively. Both of these estimates are highly statistically significant.

We can also use the projections in (27) to think about how households would adjust their portfolio holdings if they were given the opportunity to hold foreign nontradable equity. Although households vary their portfolio shares through time, we can characterize their "average shares" by considering the unconditional version of (24): $1 = \mathbb{E}[M_{t+1}R_{t+1}^{\text{N}}]$ and $1 = \mathbb{E}[\hat{M}_{t+1}\hat{R}_{t+1}^{\text{N}}]$ where $\mathbb{E}[.]$ denotes unconditional expectations.[16] My estimates of (27) imply that $\mathbb{E}[R_{t+1}^{\text{N}}(M_{t+1} - \hat{M}_{t+1})] < 0$ and $\mathbb{E}[\hat{R}_{t+1}^{\text{N}}(M_{t+1} - \hat{M}_{t+1})] > 0$. Combining these inequalities with the unconditional version of (24), gives

$$\mathbb{E}[M_{t+1}\hat{R}_{t+1}^{\text{N}}] > 1 \qquad \text{and} \qquad \mathbb{E}[\hat{M}_{t+1}R_{t+1}^{\text{N}}] > 1.$$

Thus, on average, both US and UK households would like to acquire a long position in foreign nontradable equity given the prevailing behavior of returns. This is not to say that the incentives to take such positions are particularly strong. The correlation between $M_{t+1} - \hat{M}_{t+1}$ and the returns on nontradables are approximately 0.5. Thus, foreign nontradable equity provides at best an imperfect hedge against shocks to $M_{t+1} - \hat{M}_{t+1}$.

5.2 Portfolios

I now turn to the analysis of the equilibrium portfolio rules predicted by the model. Table 2 characterizes the properties of the optimal portfolio shares allocated into different assets by US and UK households. Columns (i) and (iv) contain descriptive statistics on the wealth shares allocated by US households into US and UK tradable equity. Their shares in nontradable equity and bonds are given in columns (v) and (vii). Columns (ii), (iii), (vi) and (viii) include the corresponding numbers for UK households. In both countries households allocate half of their wealth into nontradable equity on average. This number is proportional to the expenditure share of nontraded goods in their consumption baskets. The remaining wealth is split

[15] Although (27) can only be used to test the unconditional moment condition $E[(M_{t+1} - \hat{M}_{t+1})\chi_{t+1}]$ for $\chi = \{R_{t+1}^{\text{N}}, \hat{R}_{t+1}^{\text{N}}\}$, rejecting the null that $E[(M_{t+1} - \hat{M}_{t+1})\chi_{t+1}] = 0$ implies that $E_t[(M_{t+1} - \hat{M}_{t+1})\chi_{t+1}] \neq 0$ for at least some t.

[16] Note that these "average shares" are not the same as the true averages of the portfolio shares determined by the households' first-order conditions because the latter are nonlinear functions of conditioning information.

between holdings of domestic and foreign tradable equity and bonds. It is easy to see that tradable portfolios in both countries exhibit significant home bias with more than 41% of US wealth, and more than 44% of UK wealth being allocated into locally issued stocks on average. Further, the proportion of wealth held in tradable equity by UK households tend to exceed the corresponding share of US households' wealth. Since the shares belonging to the same country must add up to one, this asymmetry is matched by the distribution of bond holdings.

Table 2. Equilibrium Portfolio Shares, % wealth

	T equity issued by US held by HHs from		T equity issued by UK held by HHs from		N equity held by HHs from		bonds held by HHs from	
	US	UK	UK	US	US	UK	US	UK
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
mean	41.46	8.54	44.06	5.95	50.01	49.99	2.59	-2.58
std. dev.	1.43	1.54	1.78	1.60	0.27	0.29	0.04	0.07
min	33.98	-0.55	35.70	-3.14	48.96	48.89	2.40	-2.87
max	50.09	16.82	54.13	14.43	51.21	51.11	2.82	-2.23

Bond holdings in this economy have several interesting features. First, US households tend to be a net lender on average, while UK households tend to be a net borrower. The redistribution of wealth arises in the equilibrium of this model because bonds are denominated in units of US tradables. Therefore, the composition of the tradable consumption basket in each country will determine households' attitude towards bonds. Since consumers in each country value local goods more than imported goods in their consumption, bonds provide a safer mechanism for borrowing and lending for US households than for UK households. The latter find bonds less attractive as any risk-free payoffs they receive need to be converted into UK tradables at the relative price q_t^{F}. This exchange introduces a potential source of riskiness in the bond holdings of UK households. As a result, they prefer to sell bonds to US households and take larger positions in tradable equity. Payoffs on tradable equity are then used to finance the interest payments to US households. Second, the positions taken in bonds by the households in the two countries are relatively large and volatile. These results suggest that when markets are incomplete there is a significant role for bonds as a medium for international risk sharing. This finding runs contrary to the complete markets results in Serrat (2001) and Kollmann (2005a), who find that bonds are redundant in equilibrium.

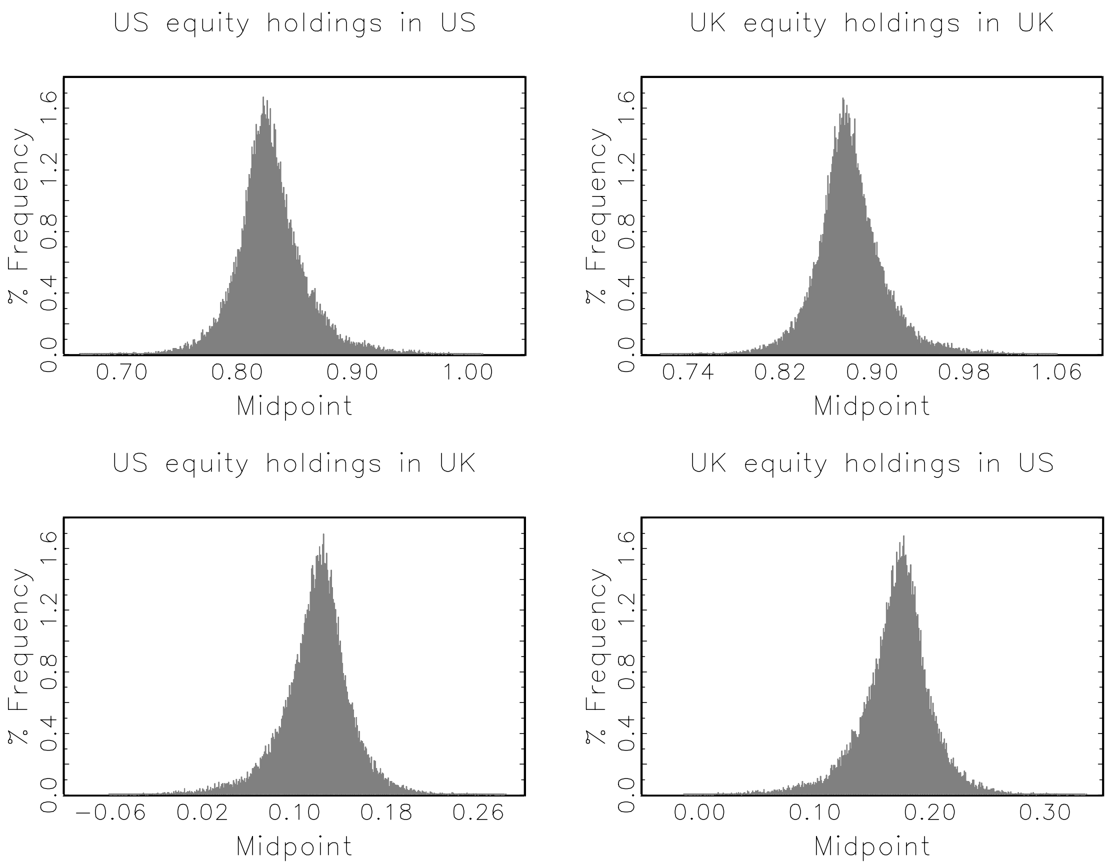

Figure 1. Distribution of portfolio holdings generated by the model

According to the equations in (19), portfolio shares may vary with wealth, equity prices or equity holdings. Generally, any of these factors may be responsible for home bias in tradable equity shares. For example, the distribution of wealth across countries could be skewed enough to generate significant bias in households portfolios. In my model, home bias in portfolio shares is associated with a home bias in equity holdings. Figure 1 shows the distribution of equity holdings $\{A_t^{\text{H}}, A_t^{\text{F}}, \hat{A}_t^{\text{H}}, \hat{A}_t^{\text{F}}\}$ calculated from the optimal portfolio rules. The left hand panel illustrates US households' holdings of equity issued by US and UK firms producing tradables, A_t^{H} and A_t^{F}. UK holdings of equity issued by UK and US tradable firms, $\hat{A}_t^{\text{F}}$ and $\hat{A}_t^{\text{H}}$, are shown in the right hand panel. Two main results stand out: first, portfolio holdings are volatile, second, they are biased towards the equity issued by domestic tradable firms.

Table 3 reports descriptive statistics on the household equity holdings. Columns (i) and (iii) show that on average more that 80% of the equity issued by tradable firms is held by domestic households. Consistent with the visual evidence in Figure 1, equity holdings are also very volatile. In fact, on rare occasions, households take short positions in the equity issued by foreign tradable firms (e.g., $\hat{A}_t^{\text{H}} < 0$ or $A_t^{\text{F}} < 0$). This is an extreme form of home bias.

The model correctly predicts both the direction and the magnitude of portfolio home bias. The *direction* of the bias is attributed to the consumption bias towards domestic tradable goods. When given a choice

Table 3. International Equity Positions

	US T equity		UK T equity	
	US HHs A_t^{H}	UK HHs $\hat{A}_t^{\text{H}}$	UK HHs $\hat{A}_t^{\text{F}}$	US HHs A_t^{F}
	(i)	(ii)	(iii)	(iv)
mean	0.829	0.119	0.881	0.171
std. dev	0.032	0.031	0.031	0.032
min	0.663	-0.062	0.715	-0.014
max	1.014	0.285	1.062	0.337
5th percentile	0.783	0.066	0.835	0.117
95th percentile	0.883	0.166	0.934	0.217

between two tradable equities (US and UK), households increase their holdings of the asset whose payoffs have higher relative value in the states of the world in which the demand for those payoffs is also high. Due to the complementarity in preferences over tradable and nontradable goods, such states of the world occur when the relative domestic nontraded endowment in high. Therefore, when preferences in each country are biased in favor of local tradables, this condition requires that the relative nontraded endowment is correlated with the value of domestic tradable dividends in excess of the corresponding covariance with the value of foreign tradable dividends. As explained in section 3.2.4, in this case households bias their portfolios towards domestic tradable assets.

The *strength* of the portfolio bias is determined by the differential in these covariances, which in turn is critically related to the elasticity of substitution between US and UK tradable consumption, $1/(1-\rho)$. Figure 2 graphs the average equilibrium holdings of tradable equity by US and UK households for a range of elasticities.[17] When tradables are perfect substitutes (i.e., when $1/(1-\rho)=\infty$), and the productivity processes are independent across sectors and across countries, the optimal portfolios of US and UK households are equally split between the equity issued by US and UK firms even in the presence of consumption home bias. As the relative supply of tradables becomes less price elastic, the share of local equity in portfolios initially declines until $1/(1-\rho)=2.5$, but then rises as the elasticity falls further. When $1/(1-\rho)=1.1$ (the baseline parametrization), household holdings of domestic tradable equity are well above their holdings of foreign tradable equity.

[17] These graphs are computed by solving the model for different values for ρ and then simulating each solution 200 times over 200 quarters. The plots are computed from the average values of $A_t^{\text{H}}, A_t^{\text{F}}, \hat{A}_t^{\text{H}}$ and $\hat{A}_t^{\text{F}}$ from these simulations.

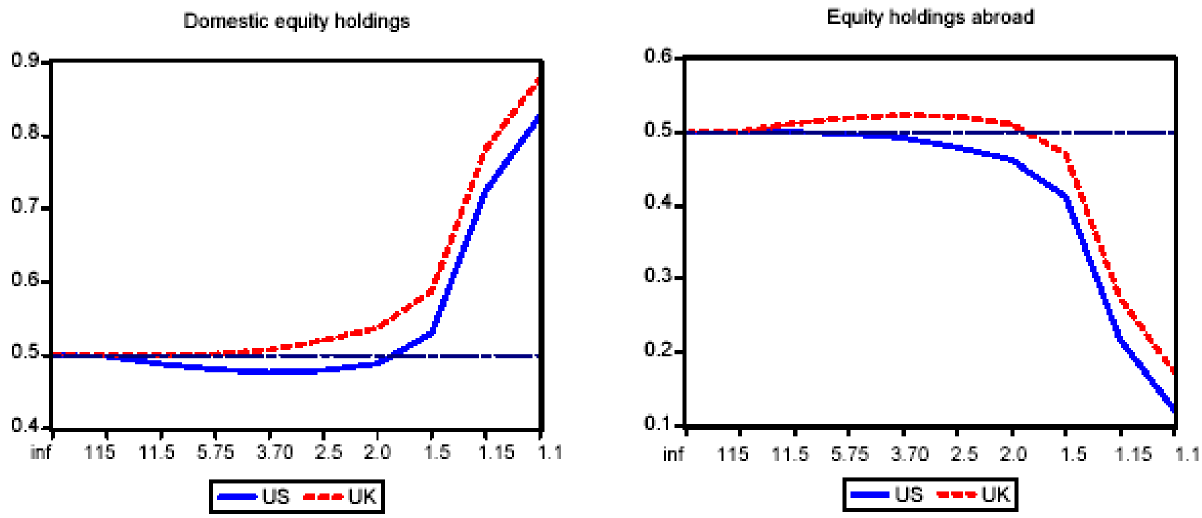

Figure 2. Varying $1/(1-\rho)$, the elasticity of substitution between T's in consumption

To understand this pattern, note that changes in the elasticity mostly affect the behavior of the terms of trade, which in turn influence the hedging capacity of foreign equity. Consider first the case of infinite elasticity of substitution between tradables. Under this parametrization no changes in relative consumption of tradables, arising from nontraded productivity shocks, induce any shifts in the relative price, q^{F}. As a result, the relative value of US to UK dividend payments remains unaffected, and the rationale for home bias disappears. As the elasticity of substitution $1/(1-\rho)$ declines, any changes in UK dividends are counterbalanced by the changes in the relative price. As a result, $\mathbb{CV}_t(q^{\text{F}}_{t+1} + d^{\text{F}}_{t+1}, d^{\text{N}}_{t+1} - \hat{d}^{\text{N}}_{t+1})$ falls relative to $\mathbb{CV}_t(d^{\text{H}}_{t+1}, d^{\text{N}}_{t+1} - \hat{d}^{\text{N}}_{t+1})$, thus strengthening the degree of portfolio home bias.

Optimal portfolio choice also depends on the beta, β^{HF}_t, defined by $\mathbb{CV}_t(d^{\text{H}}_{t+1}, q^{\text{F}}_{t+1}+d^{\text{F}}_{t+1})/\mathbb{V}_t(q^{\text{F}}_{t+1}+d^{\text{F}}_{t+1})$. The covariance between US and UK tradable dividends measures the usefulness of UK equity relative to US tradable equity in hedging shocks to nontradable productivity, while the variance of UK equity payoffs measures its riskiness. The latter can be written as $\mathbb{V}_t(q^{\text{F}}_{t+1}) + \mathbb{V}_t(d^{\text{F}}_{t+1}) + 2\mathbb{CV}_t(q^{\text{F}}_{t+1}, d^{\text{F}}_{t+1})$. A decline in the elasticity is associated with an increase in $\mathbb{V}_t(q^{\text{F}}_{t+1})$ as larger shifts in the terms of trade are necessary to induce changes in the relative consumption of tradables. At the same time, the covariance, $\mathbb{CV}_t(q^{\text{F}}_{t+1}, d^{\text{F}}_{t+1})$, while negative, falls in absolute value. The latter effect dominates when the elasticity is high, so UK equity look less risky to US households. When the elasticity falls below 2.5, the former effect dominates and UK equity appears increasingly risky. The threshold elasticity of 2.5 produces an equilibrium in which investors in both countries choose portfolios that are equally-weighted in the stocks issued by US and UK tradable firms.

In summary, at the baseline value for the elasticity of 1.1, the higher riskiness of foreign tradable equity combined with the reduced incentives for pooling risks lead to home bias. Importantly, the magnitude of the bias is comparable with its empirical counterpart.

5.3 Capital Flows

In the isoelastic economies with complete risk-sharing, portfolio holdings are time-invariant and capital flows are zero.[18] The model developed in this paper generates variable asset holdings and thus allows me to study the properties of capital flows. Table 4 describes the behavior of the bond and equity flows between countries measured relative to stock market capitalization. The bond flows are computed as $\frac{1}{R_t}B_t - B_{t-1}(= -\frac{1}{R_t}\hat{B}_t + \hat{B}_{t-1})$, the change in US "net equity assets" as $Q_t^{\text{F}} P_t^{\text{F}} \Delta A_t^{\text{F}}$ and the change in US "net equity liabilities" as $P_t^{\text{H}} \Delta \hat{A}_t^{\text{H}}$ using the equilibrium portfolio shares and wealth as shown in (19).[19] Columns (i) and (ii) show that both equity flows are large and volatile. To share risks, households have to frequently adjust their positions and these adjustments tend to be large. The size and volatility of bond flows, shown in columns (iii) and (iv), are much below those for equity.

Table 4. International Portfolio Flows, % Market Capitalization

	Δ equity assets (i)	Δ equity liab. (ii)	Δ bonds in US (iii)	Δ bonds in UK (iv)
mean	0.00	0.00	-0.03	0.03
std. dev.	0.38	0.38	0.01	0.01
min	-2.65	-2.77	-0.08	-0.06
max	2.59	2.60	0.03	0.11

Notes: Statistics reported in the table are based on 200 simulations of quarterly series, 200 periods long each. The properties of capital flows are studied relative to stock market capitalization, measured as $P_t^{\text{H}} A_t^{\text{H}} + Q_t^{\text{F}} P_t^{\text{F}} A_t^{\text{F}} + Q_t^{\text{N}} P_t^{\text{N}} A_t^{\text{N}} + \frac{1}{R_t^{\text{I}}} B_t$ in the US and a symmetric expression in the UK.

A perspective on the magnitude of the capital flows can be gained by considering their contribution to the current account. For this purpose, I decompose the current account into the components attributable to bond and equity flows. The results of such a decomposition are presented in Table 5.[20] As the first row indicates, the current account balance has volatility of 0.47% in both economies. Variation in equity holdings contribute the most to the current account variance. Thus, a 1% improvement in the US current account is accompanied by: (i) a 37.8% decrease in US foreign equity assets, which is matched by a financial inflow

[18] Stockman and Svensson (1987) show that in the economy with perfect risk pooling capital flows can arise only due to changes in equity prices, rather than portfolio holdings. Bacchetta and van Wincoop (2000) allow for endowment and preference assymetry to generate nonzero capital flows.

[19] These definitions of capital flows do not take into account capital gains or losses on the existing assets. Instead they allow to focus on the portfolio flows resulting from changes in holdings of different assets.

[20] The decomposition is obtained based on the following identity for US: $CA_t = Q_t^{\text{F}} P_t^{\text{F}} \Delta A_t^{\text{F}} - P_t^{\text{H}} \Delta \hat{A}_t^{\text{H}} + (\frac{1}{R_t} B_t - B_{t-1})$. It expresses US current account as a change in its net foreign equity and bond positions.

of US tradables, (ii) a 36.6% fall in US foreign equity liabilities, accompanied by a corresponding outflow of tradables, and (iii) a 2.2% outflow of resources due to an increase in US bond holdings. These flows in the US are matched by the corresponding changes in the UK financial account. The numbers are not symmetric across countries due to the differences in household bond positions.

Table 5. Variance Decomposition of Current Account (CA)

	US (i)	UK (ii)
std.dev. of CA	0.47%	0.47%
Δ bonds	2.19	2.63
Δ equity assets	-37.82	25.17
$-\Delta$ equity liab.	36.63	-26.79

The directions of the flows are determined by how households adjust their asset holdings in response to changes in the economy's productivity. In particular, the change in US net equity assets can be decomposed using the definition of A_t^{F} in (19) as:

$$Q_t^{\text{F}} P_t^{\text{F}} \Delta A_t^{\text{F}} = \Delta\alpha_t^{\text{F}} W_t^c + \alpha_{t-1}^{\text{F}} \Delta W_t^c - \left[(Q_t^{\text{F}} P_t^{\text{F}} / Q_{t-1}^{\text{F}} P_{t-1}^{\text{F}}) - 1\right] \alpha_{t-1}^{\text{F}} W_{t-1}^c. \tag{28}$$

Equation (28) shows that US households may change their asset position in order to accommodate: (i) shifts in absolute risk premia across countries, as captured by $\Delta\alpha_t^{\text{F}} W_t^c$; (ii) changes in wealth, $\alpha_{t-1}^{\text{F}} \Delta W_t^c$; and (iii) capital gains or losses on the existing portfolios, $[(Q_t^{\text{F}} P_t^{\text{F}} / Q_{t-1}^{\text{F}} P_{t-1}^{\text{F}}) - 1]\alpha_{t-1}^{\text{F}} W_{t-1}^c$. The contribution of each component can be assessed by taking a variance decomposition of $Q_t^{\text{F}} P_t^{\text{F}} \Delta A_t^{\text{F}}$. I find that 99% of variation in the capital flows can be explained by the changes in the risk premia. Changes in wealth and capital gains/losses play only an insignificant role.

Recall from equation (18) that each portfolio share is a weighted average of the risk premia on all assets. Furthermore, equation (15) showed that the risk premium on asset χ could be approximated by

$$\gamma^{\text{H}} \mathbb{CV}_t \left(c_{t+1}^{\text{H}}, r_{t+1}^{\chi}\right) + \gamma^{\text{F}} \mathbb{CV}_t \left(q_{t+1}^{\text{F}} + c_{t+1}^{\text{F}}, r_{t+1}^{\chi}\right) + \left(1 - \gamma^{\text{H}} - \gamma^{\text{F}}\right) \mathbb{CV}_t \left(q_{t+1}^{\text{N}} + c_{t+1}^{\text{N}}, r_{t+1}^{\chi}\right).$$

Variations in the terms of trade, q_t^{F}, affect the risk premium directly via the covariance terms, and indirectly via its impact on the components of consumption; c_t^{H}, c_t^{F} and c_t^{N} (see equation 12). To quantify this dependence, Table 6 summarizes the correlations between the changes in absolute risk premia in each country, measured by the first-difference of respective portfolio share, and the change in q_t^{F}. As columns (i) and (ii) show, an increase in q_t^{F} is associated with an increase in the risk premia on domestic assets and a fall in risk premia on foreign assets in both countries. Recall that higher risk premia on an asset generally implies an increase in the holdings of that asset, while the opposite is true when the risk premia falls. The relation between the risk premia and the change in q_t^{F} is particularly strong if I consider variations in $\Delta\alpha_t^{\chi}$ and Δq_t^{F}

driven solely by nontraded productivity shocks. In this case, as shown in columns (iii) and (iv), the sign of the correlations remains the same, but their magnitude increases dramatically.

Table 6. Correlation between asset risk premia, $\Delta\alpha_t^{\chi}$, and Δq_t^{F}

	US household	UK household	US household N shock only	UK household N shock only
	(i)	(ii)	(iii)	(iv)
US T equity	0.070	-0.075	0.790	-0.831
UK T equity	-0.089	0.093	-0.859	0.876
bonds	0.172	-0.150	0.921	-0.961

To provide some intuition for the results in Table 6, consider a productivity improvement in the US nontradable sector. Higher consumption of nontradables must be accompanied by an increase in the consumption of tradables, which in turn lowers the relative price of UK tradables. Exploiting the substitutability between the consumption of the two tradables, US households import the cheaper good, leading to a current account deficit. At the same time, Table 6 predicts that a fall in q_t^{F} is associated with a lower risk premia on domestic assets in each country and higher risk premia on foreign assets. The latter effect increases US holdings of foreign equity, while the former implies a fall in the US holdings of its domestic stocks. As a result, US foreign equity assets as well as US foreign equity liabilities increase. The resulting net *equity* outflow following the shock implies that US households purchased UK equity in excess of the US equity purchased by UK households, so that $(Q_t^{\mathrm{F}}P_t^{\mathrm{F}}\Delta A_t^{\mathrm{F}} - P_t^{\mathrm{H}}\Delta\hat{A}_t^{\mathrm{H}}) > 0$. To finance the purchases of UK assets and imports of UK goods, US households increase their borrowing, $(\frac{1}{R_t}B_t - B_{t-1}) < 0$. The associated inflow of resources covers the net *equity* outflow and pays for the current account deficit.

An analogous shock in the UK would have the opposite implications for its capital flows and current account. The reason is that an increase in the UK nontradable consumption would lead to a rise in q_t^{F}, which, according to Table 6, is associated with an increases in risk premia on locally issued equity and a decline in the risk premia on equity issued abroad. The UK economy would thus experience a net inflow of equity, since more resources would be flowing into UK by adding to the level of its equity liability to US households, than flowing out of the UK through its purchases of US-issued equity. This implies $(P_t^{\mathrm{H}}\Delta\hat{A}_t^{\mathrm{H}} - Q_t^{\mathrm{F}}P_t^{\mathrm{F}}\Delta A_t^{\mathrm{F}}) < 0$. Combining the net *equity* inflow with an outflow of resources due to the accumulation of bonds, $(-\frac{1}{R_t}\hat{B}_t + \hat{B}_{t-1}) > 0$, we get a current account surplus in the UK.

The outcomes of the tradable shocks can be analyzed in a similar manner. However, in this economy changes in tradable productivity can be almost completely insured with the menu of available assets. The resulting equity and bond flows as well as changes in the current account are very small and are dominated by the capital flows induced by shocks to nontradable productivity.

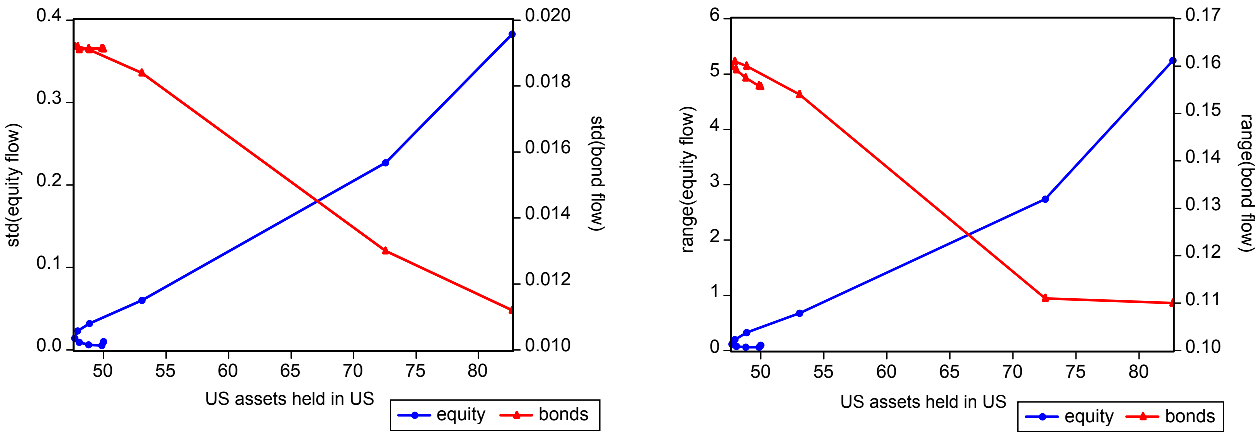

Figure 3. The relation between the degree of home bias and capital flows

Figure 3 examines the relation between capital flows and the degree of home bias. Here I plot the size (captured by the range) and volatility (measured as the standard deviation) of US equity and bond flows against the average equilibrium US holdings of domestic tradable equity, A_t^{H}. Figure 3 clearly shows that equilibria characterized by higher home bias in equity holdings also have larger and more volatile equity flows. At the same time, the size and volatility of bond flows declines as home bias increases. These findings are naturally linked to the role of equity and bonds in international risk sharing.

5.4 Portfolio Characteristics

I now examine the conditional and unconditional moments of portfolio returns and evaluate the importance of dynamic portfolio choice in the model. Figure 4 plots the US mean-variance frontier for the risky assets using the unconditional sample moments of the simulated returns. Only returns on assets available to US households are used to calculate the frontier. Thus, it consists of the set of minimum-variance portfolios, composed of US and UK tradable equity and US nontradable equity, in the return mean-standard deviation space. Point A on the frontier corresponds to the average portfolio implied by the model; the asset weights are computed as the sample averages of simulated portfolio shares. The point corresponding to a fully diversified portfolio is denoted by B. In this portfolio, 50% of wealth is allocated into nontradable equity and 50% of wealth is allocated into tradable equities, with the latter evenly divided between the holdings of stocks issued in the US and UK. Point C corresponds to an autarky portfolio with holdings equally split between US tradable and nontradable equities, while point D characterizes a reverse home biased portfolio, where all wealth is allocated into UK tradable equity. All four portfolios lie on the mean-variance frontier implying that they generate the lowest standard deviation for the given mean return.

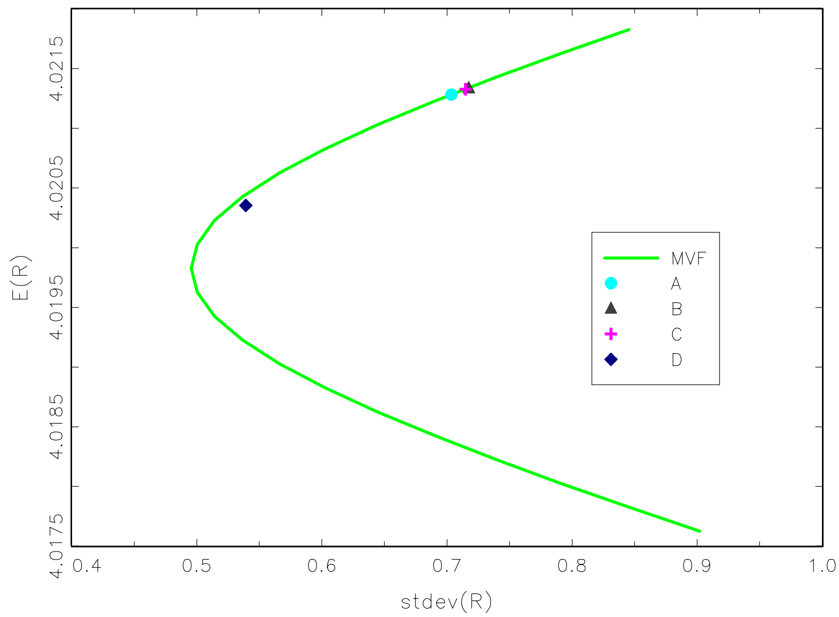

Figure 4. Mean-Variance Frontier and Model Portfolios

Figure 5 shows how the mean-variance frontier shifts as the elasticity of substitution between the local and imported tradable goods changes. Here we see that a decrease in elasticity is associated with an outward shift in the frontier. This is not an unexpected result given that variations in the terms of trade become more significant as the elasticity declines, and thus contribute more to the riskiness of investors portfolios. Figure 5 also shows that for elasticities above 1.5, the average portfolio implied by the model lies inside the frontier. Only portfolios that correspond to elasticities equal to 1.15 and 1.10 fall on the frontier. Recall that these are also the two portfolios that exhibit the largest degree of home bias. Since the frontier is spanned by any two frontier returns, the portfolios that hold most of their weight in any two assets will be on or very close to the frontier. Assets with significant weights on all three equities will be pushed inside the frontier.

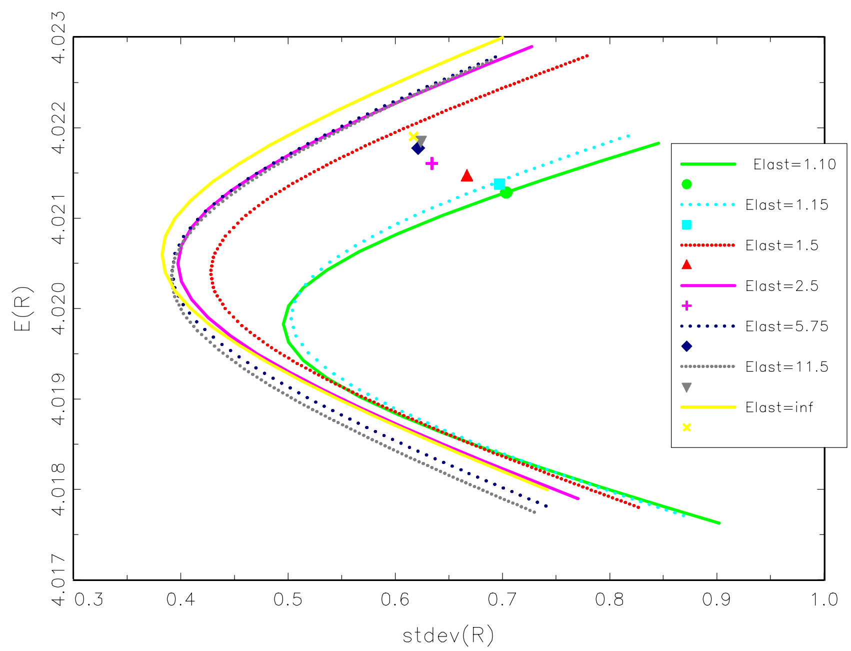

Figure 5. Mean-Variance Frontiers: Varying $\frac{1}{1-\rho}$

Figures 4 and 5 characterize portfolios using unconditional moments of returns. However, throughout this paper I have used conditional moments to describe the decisions of firms and households. In particular, households incorporate the additional information embodied in the conditional moments of returns by constantly adjusting their portfolios. Figure 6 allows us to assess the importance of this conditioning information. As in Figure 4, I plot the frontier implied by the unconditional distribution of returns on US and UK tradable equity and US nontradable equity, and point A that corresponds to the average portfolio implied by the model. Points E, F and G illustrate the importance of conditioning information. According to the model, the conditional first and second moments of optimally invested wealth for US households are

$$\mathbb{E}_t r^{\mathrm{w}}_{t+1} = r^1_t + \tfrac{1}{2}\boldsymbol{\alpha}'_t \Theta_t \boldsymbol{\alpha}_t \qquad \text{and} \qquad \mathbb{V}_t(r^{\mathrm{w}}_{t+1}) = \boldsymbol{\alpha}'_t \Theta_t \boldsymbol{\alpha}_t.$$

Notice that both moments vary through time with changes in the variance-covariance matrix of returns, Θ_t, and the vector of optimally chosen portfolio shares, $\boldsymbol{\alpha}_t$. To assess the importance of these variations, Figure 6 plots the sample average of $\mathbb{E}_t r^{\mathrm{w}}_{t+1}$ and $\mathbb{V}_t(r^{\mathrm{w}}_{t+1})^{1/2}$ as point E, and the sample average of $\mathbb{E}_t r^{\mathrm{w}}_{t+1}$ and $\mathbb{V}(r^{\mathrm{w}}_{t+1})^{1/2}$ as point F, where $\mathbb{V}(r^{\mathrm{w}}_{t+1})$ is the unconditional variance of the return on wealth. Both points lie well above the frontier. Point F lies a little to the right of point E because $\mathbb{V}(r^{\mathrm{w}}_{t+1}) \equiv \mathbb{E}[\mathbb{V}_t(r^{\mathrm{w}}_{t+1})] + \mathbb{V}(\mathbb{E}_t r^{\mathrm{w}}_{t+1})$ and $\mathbb{V}(\mathbb{E}_t r^{\mathrm{w}}_{t+1}) > 0$. To place these results in perspective, I also calculate the minimum-variance portfolio that delivers the expected return on optimally invested wealth (i.e., using conditioning information). This portfolio corresponds to point G, which lies well to the right of E and F. In fact, to achieve the same expected return without the use of conditioning information, households would have to live with 39% more risk. In sum, therefore, conditioning information and dynamic portfolio choice allow households to move significantly beyond the unconditional mean-variance frontier.

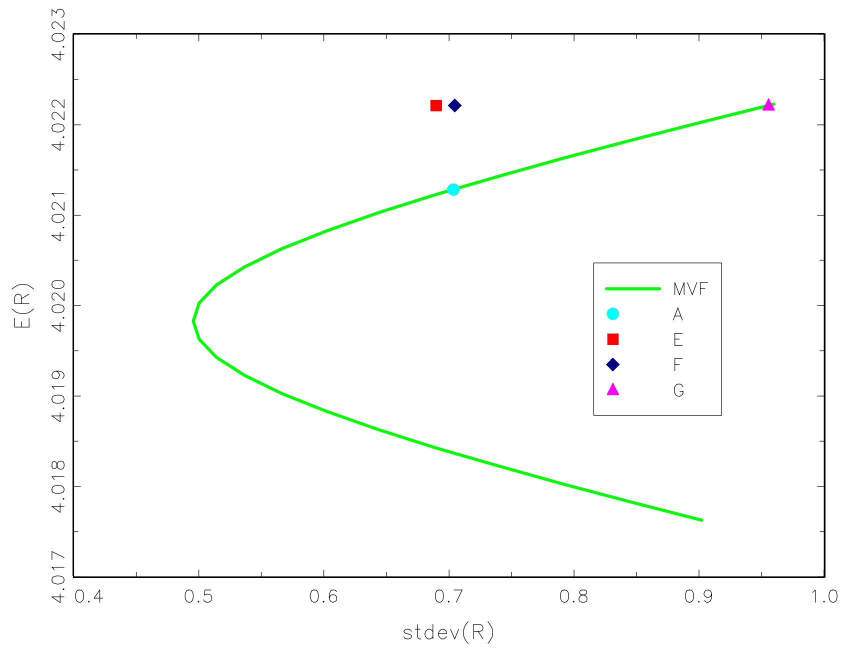

Figure 6. Mean-Variance Frontier: Conditioning Information

We can further study the impact of conditioning information by examining the behavior of the conditional log Sharpe ratio:

$$SR_t = \sum\nolimits_{\chi} RP_t^{\chi} \cdot \alpha_t^{\chi}, \tag{29}$$

where $RP_t^{\chi} \equiv [\mathbb{E}_t r_{t+1}^{\chi} - r_t^1 + \frac{1}{2}\mathbb{V}_t\left(r_{t+1}^{\chi}\right)]/\mathbb{V}_t(r_{t+1}^{\mathrm{W}})^{1/2}$ for $\chi = \{\mathrm{H,F,N}\}$. The log Sharpe ratio provides a convenient way to summarize the risk-adjusted mean return for different portfolios and depends on conditioning information in two ways: (i) through the time-varying conditional moments of asset returns belonging to the investment opportunity set of households; and (ii) through the time-varying portfolio allocations chosen by households. To assess the contribution of each of these components, I use a linearized version of (29):

$$SR_t \simeq const. + \sum\nolimits_{\chi} \left(\bar{\alpha}^{\chi} \cdot RP_t^{\chi} + \overline{RP}^{\chi} \cdot \alpha_t^{\chi} \right), \tag{30}$$

where a bar denotes an unconditional mean. The first component in this approximation, $\bar{\alpha}^{\chi} \cdot RP_t^{\chi}$, allows the conditional moments of returns to change over time while keeping portfolios shares constant. The second component, $\overline{RP}^{\chi} \cdot \alpha_t^{\chi}$, is based on time-varying shares, but sets the conditional moments of returns to their sample averages. I find that 72% of the variation in the conditional log Sharpe ratio is explained by the variation in the portfolio shares, α_t^{χ}, while the remaining 28% are due to changes in the conditional distribution of returns.[21] This finding further highlights the importance of dynamic portfolio choice in the model.

[21] Variance decomposition was calculated from equation (30) as $\mathbb{V}(SR_t) = \bar{\alpha}^{\chi} \sum_{\chi} \mathbb{CV}\left(SR_t, RP_t^{\chi}\right) + \overline{RP}^{\chi} \sum_{\chi} \mathbb{CV}(SR_t, \alpha_t^{\chi})$, where $\mathbb{V}$ and $\mathbb{CV}$ denote the unconditional variance and covariance.

6 Conclusion

This paper reconciles two puzzles in international finance: home bias in equity holdings and high turnover and volatility of international capital flows. These results are developed in a model of international portfolio choice with several distinct features: the production of tradable and nontradable goods in each country is specialized, preferences are nonseparable in the consumption of all goods, and asset markets are incomplete.

Both home bias and international capital flows are driven by the variations in the international relative prices which arise from productivity changes. Low diversification occurs because the variations in the terms of trade affect the hedging ability of foreign assets and the sensitivity of households asset demand to this ability. Large and volatile capital flows occur in response to the international risk-premia differentials which are also driven by the terms of trade movements. Therefore, it is not inconsistent to have home bias in portfolios and significant capital flows. My results also emphasize the importance of market incompleteness and dynamic portfolio choice.

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A Appendix

This appendix provides the derivations of some equations presented in the text.

A.1 Derivation of (13)-(15)

Equation in (13) is obtained by using second-order Taylor series expansion and lognormality of asset returns. First-order condition for bond in equation (9c) can be expressed as

$$1 = \mathbb{E}_t \left[\exp(m_{t+1} + r_t^1) \right] \simeq \exp \left[\mathbb{E}_t \left(m_{t+1} + r_t^1 \right) + \tfrac{1}{2} \mathbb{V}_t \left(m_{t+1} \right) \right].$$

Taking logs on both sides yields a log-linearized version of the consumption Euler equation:

$$0 = r_t^1 + \mathbb{E}_t m_{t+1} + \tfrac{1}{2} \mathbb{V}_t \left(m_{t+1} \right). \tag{A1}$$

First-order condition for asset χ in equation (9) is log-linearized analogously:

$$1 = \mathbb{E}_t \left[M_{t+1} R_{t+1}^{\chi} \right] \simeq \exp \left[\mathbb{E}_t \left(m_{t+1} + r_{t+1}^{\chi} \right) + \tfrac{1}{2} \mathbb{V}_t \left(m_{t+1} + r_{t+1}^{\chi} \right) \right].$$

Again, taking logs and substituting in the bond Euler equation gives expression in (13):

$$\mathbb{E}_t r_{t+1}^{\chi} - r_t + \tfrac{1}{2} \mathbb{V}_t \left(r_{t+1}^{\chi} \right) = -\mathbb{C}\mathbb{V}_t \left(m_{t+1}, r_{t+1}^{\chi} \right). \tag{A2}$$

Next, I characterize m_{t+1}. Recall its definition from section 2: $m_{t+1} \equiv \ln \left(M_{t+1} \right) = \ln \left(\beta \frac{\partial U_{t+1} / \partial C_{t+1}^{\text{H}}}{\partial U_t / \partial C_t^{\text{H}}} \right)$. The US stochastic discount factor then becomes

$$m_{t+1} = \ln \beta + \ln \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\phi} \left(\frac{C_{t+1}^{\text{T}}}{C_t^{\text{T}}} \right)^{\phi - \rho} \left(\frac{C_{t+1}^{\text{H}}}{C_t^{\text{H}}} \right)^{\rho - 1} \right],$$

where $C \equiv \left(\lambda_{\text{T}}^{1-\phi} \left(C^{\text{T}} \right)^{\phi} + \lambda_{\text{N}}^{1-\phi} \left(C^{\text{N}} \right)^{\phi} \right)^{\frac{1}{\phi}}$ denotes the aggregate consumption basket in the US. Substituting in the definitions for C and C^{T}, and collecting the terms, allows to express m_{t+1} as a growth rate of the US consumption expenditures:

$$m_{t+1} = \ln \beta + \ln \left[\frac{C_t^{\text{H}} + Q_t^{\text{F}} C_t^{\text{F}} + Q_t^{\text{N}} C_t^{\text{N}}}{C_{t+1}^{\text{H}} + Q_{t+1}^{\text{F}} C_{t+1}^{\text{F}} + Q_{t+1}^{\text{N}} C_{t+1}^{\text{N}}} \right].$$

When preferences are logarithmic, consumption expenditure is proportional to wealth, so that

$$C_t^{\text{H}} + Q_t^{\text{F}} C_t^{\text{F}} + Q_t^{\text{N}} C_t^{\text{N}} = \left(1 - \beta \right) W_t, \tag{A3}$$

which allows to simplify the expression for m_{t+1} as

$$\begin{aligned} m_{t+1} &= \ln \beta + \ln \left[\frac{\left(1 - \beta \right) W_t}{\left(1 - \beta \right) W_{t+1}} \right] \\ &= \ln \beta - \Delta w_{t+1}. \end{aligned}$$

Substituting this result in (A2) gives equation (14) in text.

Finally, to obtain a representation for risk premium in terms of consumption, as given in equation (15), I log-linearized the consumption rule in (A3).