Selling Hopes: Managerial Compensation and Overstatement of Growth Options*

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1. Introduction

The standard solution to the agency problem between the shareholders and the managers is to add stock-based components to the compensation package of the latter. In a static agency model, the optimal contract exhibits more powerful incentives (higher exposure to the stock price), when the firm value is more sensitive to the managerial effort, since in this case the benefit of aligned incentives is higher. In this paper we show that in a dynamic setting this traditional solution of equity-based compensation in its standard form could significantly amplify the agency problem and result in destruction of value. The most damage is likely to occur precisely in the environment where the static model prescribes it.

While we believe that the phenomenon is widespread, for illustrative purposes we focus our attention on the 1990's Hi-Tech boom.[1] This period was characterized by expectations

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[1] We are not interested in dot.com companies, but rather in firms with sales, profits, and real investment options.

of high growth rates, stemming from the optimistic hopes on the adoption of Information Technology, yet at the same time by extremely high uncertainty around these expectations. The managers were facing very high powered stock-price-based incentives: the possibility of an IPO or an acquisition, or just due to generous options awards. As always, the managers were much better informed about the firm's prospects than the outside investors, which created an extreme adverse selection problem. A firm with perceived high growth options was priced many times higher than a firm with similar operating performance, but with lower perceived growth options. Given the managers's high powered incentives, it is obvious they had incentives to present theirs as high growth firms. Was this strategy feasible?

We claim that any new technology environment, and the 1990's environment in particular, has an important, yet somewhat overlooked feature, that makes this strategy feasible, and perhaps quite widely used. In any company pursuing new technologies the border between the investments and cost is blurred. Recall, for example, that most of the investments in R&D are expensed. The same programmers and engineers could be classified as either working on R&D projects, or on product "manufacturing" and support. The management has much leeway in deciding on the allocation of these costs. Given that the industry is young and the uncertainty is immense, the analysts had not yet figured out how to separate the costs from the investments, as they do when analyzing firms in the "old economy".[2] Being able to hide the investments allows the manager to pretend to be a high growth-high investment firm. But the pretense does not come cheap: the manager has to consistently deviate from the optimal investment profile, and in particular it has to underinvest in profitable projects to maintain the appearances, until at some point the firm can no longer conceal its true state.

The goal of this paper is to present an extremely stylized model that captures the main intuition described above in the most parsimonious way, and then test whether its implications are consistent with the data. This calls for several highly simplifying assumptions, which only leave the core intuition. For example, modeling a dynamic signaling game would generate multiplicity of equilibria, and obfuscate the main message of the paper. Instead, we present a tractable model with several prediction that fits the extant evidence from the 1990's.

We start by assuming that all technological firms start with a high rate of growth of investment opportunities, and at some point mature and move to a lower rate of growth. The future growth rate of the investment opportunities of they firms determines with certainty its future earnings for every investment policy. The manager knows the time at which the change occurs, but the market does not. When the change takes place, the manager has two options, tell the truth, or continue to behave as if nothing had happened. The former policy leads to an immediate decline in the stock price. The latter does not, but forces the manager to follow a sub-optimal investment strategy to maintain the pretense. We assume that as long as the reported earnings over time are consistent with the high growth rate, the manager keeps his job; the first deviation constitutes a "default" and the manager is

[2]In fact we would claim that even firms in the old economy are subject to this phenomenon, as long as they have significant investments in R & D. The difference is in the degree of information asymmetry between the managers and the outside investors.

fired. Clearly, managers are rarely fired for deviating from the promised growth rate, but the important point is that they must suffer some cost of deceit, and by imposing such severe cost, we bias the model against finding our main result. These assumptions collapse the dynamic game into a simple static reporting game, in which the manager chooses whether to tell the truth or to conceal, and the market prices the stock accordingly over time. After the report the manager's strategy is straightforward: keep the "default" as distant as possible, by choosing the appropriate investment strategy.

The main results are as follows: if the manager is compensated based on his reported earnings, the only pure strategy equilibrium is separating, in which the manager always tells the truth. The same is true when the range of the possible growth rate values is narrow. In these cases the threat of future repercussions is strong enough to deter the small short-term gains from overstating the growth options.

Whenever the manager has a large stock-price-based component in his compensation, and the range of possible growth rates is large, the equilibrium is pooling for almost all parameter. In this equilibrium, managers of low growth firms sell false hopes to investors, and underinvest in profitable projects to maintain the pretense. Notice that firms that fall into this category of high uncertainty, are also likely to be the

Our model generates several empirical predictions that are consistent with the empirical evidence. In particular, our results suggest that the importance of meeting analysts' forecasts may have less to do with estimating the current cash flows of the firm, than with the firm's ability to maintain the cash flows growth pattern that is consistent with the perceived growth rate. This explains why meeting the analysts forecasts increases the value of the firm, while a small downward deviation from these forecasts may result in massive stock price reactions. Barth et al (1999) show that PE ratios of firms increase quite significantly in the number of periods during which their earnings grow. While the authors do not stress this point, this is consistent with the evolution of beliefs about the future growth rates, as in our model. They also show that a decline in earnings (regardless of the analysts expectations), significantly reduces the PE ratios, which implies that the effect is more than proportional to the level of earnings; indicating again a possible adjustment of beliefs about the growth rate. Similarly, Skinner and Sloan (2002) show that a decline in the firm value following a failure to meet the analysts' forecasts is much more pronounced in high growth firms, which is also consistent with the predictions of the model.

Graham et al. (2004) present the results of an extensive survey among managers of various companies. Most managers state that they would forego a positive NPV project if it causes them to miss the earnings target; High tech firms are much more likely to do so. They also are much more likely to cut R&D and other discretionary spending to meet the target. High tech firms believe more strongly that missing earnings target introduces uncertainty and raises red flags about the company. All of these findings are consistent with the assumptions and predictions of the model.[3]

[3]However, more specific empirical work (in progress) is required to test the theory formally.

This paper has some of the same assumptions as Miller and Rock (1985), who study the effects of dividends announcements on the value of the firm. We also assume that the manager has a significant informational advantage over the investors, and that investors do not observe the true earnings or the true investment, but must rely on aggregate announcements for their valuation. However, the emphasis of our paper is quite different in two dimensions: we focus on the beliefs about the future growth rates, rather than about the current earnings levels; and we link them to the incentives of the managers.

Our story presents a picture of a firm, that is temporarily overvalue, and destroys value to maintain this appearance. A similar scenario has been recently described by Jensen (2004) as follows:

> "Like taking heroin, manning the helm of an overvalued firm feels great at first. ....you are on TV, investors love you, your options are going through the roof, and the capital markets are wide open. But as heroin users learn, massive pain lies ahead."

He then proceeds to claim that a significant stock-based compensation component makes it virtually inconceivable to expect such a manager to reduce the valuation right away. He writes:

> "Adding equity based pay in the context of overvalued equity is like throwing gasoline on a fire. Thus, the substantial increase in option holdings by managers in the decade of the 1990s exacerbated the problems of overvalued equity."

The motivation of our paper is closely related to Jensen's work, yet our analysis differ from his in several respects. First, we endogenize the overvaluation problem itself, which in our model stems from the fact that managers of low growth firms manipulate their investment policy to convince the market in the high growth potential of their firms. They benefit today from prices that are based on false hopes of investors. Equity-based compensation induces managers to overstate growth option which makes equity of their firms overvalued.

Second, Jensen stresses the behavioral origin of overvaluation; investors are essentially fooled by managers. In our model the valuation of every firm is correct in expectations. Our model does not predict overvaluation of the market on average, since we assume that the true distribution of growth rates in the population of firms is common knowledge. This assumption is not crucial for our results: in fact, if the aggregate growth rate of the sector is not known either, we could easily imagine scenarios where the market overall is overvalued as well (see Zeira 1999). In fact, we argue informally towards the end of the paper that in such a case we can predict the end of the bubble by looking at the number of firms whose cash flows can no longer support their claim for high growth rates. However, if the stock becomes overvalued for whatever exogenous reasons, and the basic conditions of our model apply (as they did in the late 1990's), we show that the managers will not correct the market, but instead will take value-destroying actions to maintain this impression. The logic is the same as in our model, where the overvaluation is endogenous.

Third, we assume that the analyst provides an unbiased price of the stock, which the market adopts, and do not affect the managerial decisions beyond that. In contrast, Jensen argues that analysts are part of the agency costs of overvalued equity since they put pressure on managers to manipulate cash flows. Our goal is to show that when the border between investments and operational costs is blurred, equity can be overvalued even when analysts are unbiased. If the analysts are biased,[4] the detrimental effect may be amplified.

Fourth, Jensen emphasizes the fraudulent behavior, while we do not allow it. The managers in our model can be fired for investing suboptimally, but it is assumed as a technical simplification, and clearly they cannot be charged for fraud. Adding an additional dimensions of fraudulent manipulations to the manager's arsenal would definitely increase the scope of the problem, and may explain the Enrons and Worldcoms of the world as well. In the paper we focus on managers of regular Hi Tech firms that pretended to be more "exciting" than they actually were, but did not break any laws.

Finally, our model has numerous empirical predictions, described above, that are not found in Jensen (2005) work. We also predict that firms are likely to grow through acquisitions (this is not modeled), due to two results in the model: first, if the equity is overvalued, it is a cheap currency to acquire other firms. Second, an acquired firm, unlike an internal project, is definitely considered an investment - the manager can credibly commit to that. This makes this form of growth advantageous from the point of view of a manager who overstates his growth options. Finally, as stated above, we argue informally that this model can be used to explain market bubbles associated with new technologies, but the exploration of this topic is left for future research.

Overall, the two papers are highly complementary, and conclude that contrary to the static model prescription, providing managers with high-powered short-run incentives based on the stock price may be dangerous, because the stock price accumulates the beliefs about the uncertain future. The manager can use deceptive or even fraudulent practices that destroy value to maintain the pretense of a bright tomorrow.

Jensen (2005) suggests that firms "...must refuse playing the earning management game..." by adopting compensation plans that do not give incentives to manipulate earnings. Our model makes more specific predictions. We show that compensating managers proportional to the reported earnings significantly reduces the incentives to engage in value destroying activities to support the inflated expectations. However, most earnings-based compensation schemes take the form of bonuses conditional on reaching a target, which may also induce earnings manipulations. This result is consistent with the recent move to expense executive options, which would make them much less attractive. According to Jensen (2004): "[This] move ... has some logic to it, but if it goes too far we will have difficulty attracting the best qualified people to these critically important jobs, and their incentives will not be as

[4]See for example Bartov *et al.* (2002), Hutton (2003), or a survey of this literature in Guttman (2004).

effective. So we must be careful lest we go too far in this dimension".

An alternative solution that comes out of the model, is when the high-powered incentives are needed for motivation, they must be non-vested for a longer periods of time. Most top managers' cash needs are much lower than the value of their annual compensation. If their stock-based compensation is deferred, their incentives to overstate growth options by destroying value are significantly reduced. One has to take into account the possibility of reneging on the part of the firm, still this solution should be considered carefully.

The rest of the paper is organized as follows. Section 2 presents the model setup and analyzes the benchmark case of full information. The asymmetric information case is presented in Section 3. Section 4 presents the possible equilibria. Section 5 contains a discussion and possible extensions. Section 6 concludes.

2. The Model

Our economy is populated by a continuum of entrepreneurs. New projects become available, and each entrepreneur is randomly assigned one of them. All projects have similar characteristics in terms of investment opportunities. That is, they all initially offer a high growth rate G of investment opportunities. As the firm grows, the rate of growth in investment opportunities may decline. For simplicity, we assume that there is a constant intensity λ determining the shift of growth rate in investment opportunities to a lower level $g < G$. When the decline takes place, however, is private information of the manager of the firm. The choice the manager faces is then whether he/she wants to declare the worsening of underlying investment opportunities or not. The decision depends crucially on whether his/her compensation is tied to stock prices, or earnings.

2.1. Investment Opportunities

In our model the manager operates in the full certainty environment. However the prospects of the firm are his private information. More specifically, given a level of capital K_t, we assume that the firm earnings Y_t are given by

$$Y_t = \begin{cases} zK_t & \text{if} \quad K_t \leq J_t \\ zJ_t & \text{if} \quad K_t > J_t \end{cases} \tag{1}$$

where z is the rate of return on capital, and J_t is an upper bound that depends on the characteristics of the technology itself, operational costs, product demand and so on. In other words, the Leontief technology specification (1) implies constant return to scale up to the upper bound J_t, and then zero return for $K_t > J_t$. This simple specification of a decreasing return to scale technology allows us to conveniently model the growth rate in

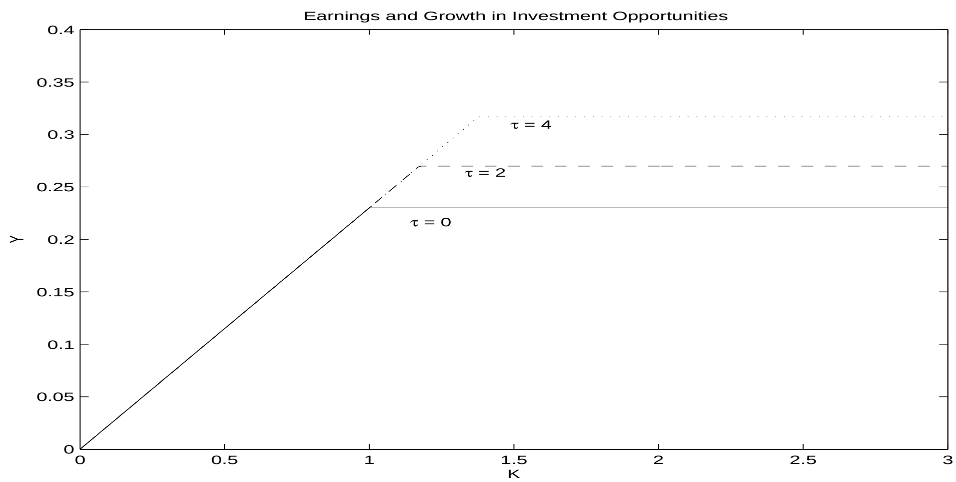

Figure 1: Growth in Investment Opportunities: This figure reproduces the earnings profile Y_t as a function of capital K_t, for three different time periods $t = 0$, $t = 1$ and $t = 2$.

profitable investment opportunities, whose different dynamics across firms will be the driving force of our model.

More specifically, we assume that the upper bound J_t in (1) grows according to

$$\frac{dJ_t}{dt} = \tilde{g} \tag{2}$$

where $\tilde{g}$ is a constant. The combination of (1) and (2) captures our idea of growing investment opportunities. In fact, given that the technology displays constant returns to scale up to J_t, it is optimal to keep the capital at the level J_t if the return on capital, z, is sufficiently high. This implies that it will be optimal for the firm to keep investing in the technology, and the growth rate of these investments depends on the growth rate $\tilde{g}$. Figure 1 illustrates the growth rate in investment opportunities.

We assume that the firm does not retain earnings, thus the annual dividends of the firm equals its operating profits derived from it stock of capital, K_t, less the investment it chooses to make, I_t. Formally, given the form of the technology in (1) we have

$$D_t = z \min\left(K_t, J_t\right) - I_t \tag{3}$$

Finally, the existing stock of capital depreciates at the rate of δ.

2.2. Firm Life Cycle

We assume that when the firm is born, say at time τ, it enjoys a relatively high growth rate in investment opportunities. However, at some random time τ^* the firm matures, and the growth rate in investment opportunities slow down. More specifically, we assume that

$$\tilde{g} = \begin{cases} G & \text{for} \quad \tau \leq t \leq \tau^* \\ g & \text{for} \quad t > \tau^* \end{cases} \tag{4}$$

The time of firm maturity, and therefore of the shift to a lower growth, is random. For simplicity, we assume that τ^* is exponentially distributed, with probability density function given by

$$f(\tau^*) \sim \lambda e^{-\lambda(\tau^* - \tau)}$$

That is, in every instant dt there is a constant probability λ that a shift from G to g occurs. The assumption of a life-cycle of firm characterized by high initial growth followed by a slow down is consistent with the extant empirical evidence.

3. Benchmark case: Symmetric Information

Consider first the (benchmark) case in which the manager and shareholders have symmetric information. In order to maximize the firm value, the manager must invest to its fullest potential, that is, sufficiently to keep $K_t = J_t$ for all t. To find the optimal investment policy, notice first that the capital evolution equation is given by :

$$\frac{dK_t}{dt} = I_t - \delta K_t, \tag{5}$$

From (2), we have that the target level of capital, J_t, is given by

$$J_t = \begin{cases} e^{G(t-\tau)} & \text{for} \quad \tau \leq t \leq \tau^* \\ e^{G(\tau^*-\tau)+g(t-\tau^*)} & \text{for} \quad t > \tau^* \end{cases} \tag{6}$$

Imposing $K_t = J_t$ for every t and using (5) we find that the optimal investment policy is

$$I_t = \begin{cases} (G+\delta)e^{G(t-\tau)} & \text{for} \quad \tau \leq t \leq \tau^* \\ (g+\delta)e^{G(\tau^*-\tau)+g(t-\tau^*)} & \text{for} \quad t > \tau^* \end{cases} \tag{7}$$

We can then determine the dividend stream of a firm that fully invests by using (3), which is given by:

$$D_t = zK_t - I_t = \begin{cases} D_t^G = (z-G-\delta)e^{G(t-\tau)} & \text{for} \quad \tau \leq t \leq \tau^* \\ D_t^g = (z-g-\delta)e^{G(\tau^*-\tau)+g(t-\tau^*)} & \text{for} \quad t > \tau^* \end{cases} \tag{8}$$

Figure 2 plots the dynamics of the optimal dividend path for a firm with a high growth in investment opportunities until τ^*, and a low growth afterwards. As the figure shows, the slow down in investment opportunities makes it optimal to decrease the investment rate, which in turn increases dividend payouts.

We assume $z > G+\delta$, so that the amount of dividends paid is always positive. Notice that (8) shows that the rate of increase in dividends equals the rate of increase in the investment opportunities, $\tilde{g}$.

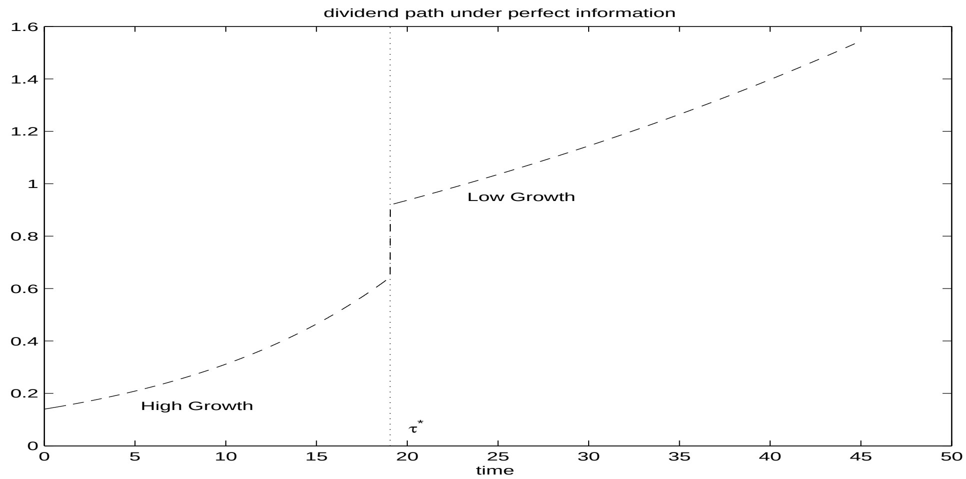

Figure 2: A dividend path under perfect information. We use the following parameters: $r = .12$,$z = .23$,$g = 0.02$,$G = .08$,$\delta = .01$,$\lambda = 1/15$.

3.1. The value of the firm

In order to compute the value of the firm, we must determine the appropriate discount rate that investors use to discount future cash flows. We assume that the discount rate does not depend on the growth rate (the same level of systematic risk) and is given by r. To ensure a finite value to the firm stock price, we assume $r > G + \lambda$. In addition, we assume $z > r + \delta$, that is, the return on capital is sufficiently high to compensate for the rate of return on stock r and depreciation δ. This assumption implies that it is economically optimal for investors to provide capital to the company and produce up to its fullest potential, as determined by the Leontief technology described in (1). Later on we also assume that the degree to which the managers have an incentive to misrepresent the growth rate of their firms is independent of the systematic risk of the economy, and does not affect it.[5]

Proposition 1: *Under perfect information: (a) The value of the firm at time $t \geq \tau^*$ is*

$$P_t^g = \int_t^\infty e^{-r(s-t)} D_s^g ds = \left(\frac{z-g-\delta}{r-g}\right) e^{G(\tau^*-\tau)+g(t-\tau^*)}. \tag{9}$$

(b) The value of the firm at time $t < \tau^*$

$$P_t = E_t\left[\int_t^{\tau^*} e^{-r(s-t)} D_s^G ds + e^{-r(\tau^*-t)} P_{\tau^*}^g\right] \tag{10}$$

$$= \frac{(z-G-\delta)}{r+\lambda-G} e^{G(t-\tau)} + \lambda\left(\frac{z-g-\delta}{(r-g)(r+\lambda-G)}\right) e^{G(t-\tau)} \tag{11}$$

[5]Each manager's decision cannot affect the systematic risk of the economy, however, in equilibrium the latter will depend on the aggregate behavior.

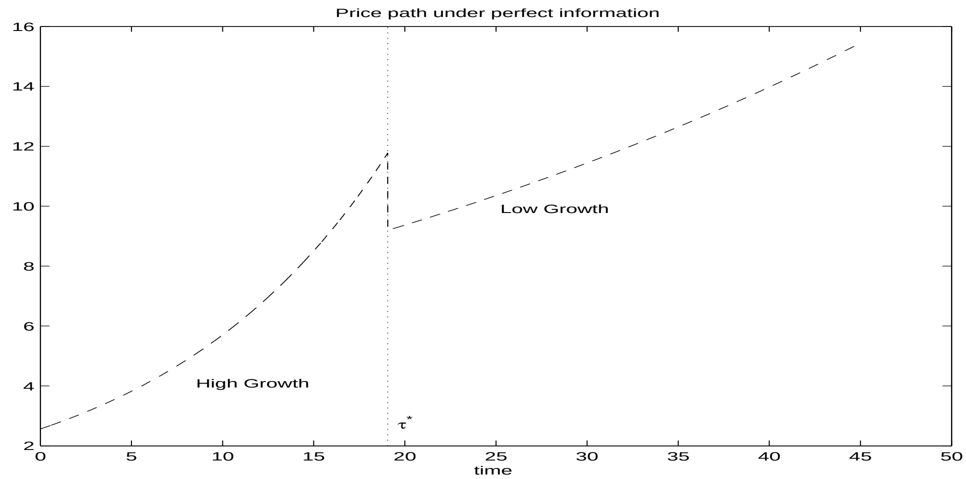

Figure 3: A price path under perfect information. We use the following parameters: $r = .12$,$z = .23$,$g = 0.02$,$G = .08$,$\delta = .01$,$\lambda = 1/15$.

Under full information, the price of stock will jump at time τ^*. The size of the jump is

$$\Delta = P^g_{\tau^*} - P_{\tau^*-} \;=\; -\frac{e^{G(\tau^*-\tau)}}{(r-g)(r+\lambda-G)}(z-r-\delta)(G-g)$$

Assuming that $z > (r + \delta)$, at time τ^* a decrease in price results. This effect feeds back into the utility function of the manager, whose compensation depend on stock price performance. This effect, if strong enough, will prevent the manager from revealing the shift in fundamentals. Figure 3 plots the price path corresponding to the dividend path in Figure 2, under perfect information.

4. Asymmetric Information

Consider now the case in which the decline in investment opportunities is private information of the manager. Investors are assumed to be unable to see inside the firm, and, in particular, the investment activity of the firm. This is assumption is realistic in many industries, and especially the rapidly growing new industries, as the market (i.e. the analyst) does not know how to distinguish between investments and costs. Indeed, in many industries, and in particular in high tech, almost all the R&D expenditures are expensed rather than capitalized; and are labeled as such at the discretion of the management. In mature R&D-intensive industries rules of thumb had been established that yield reasonable estimates of investments in R&D; in newly developing industries such rules had not yet crystallized. We assume, therefore, that the market/analyst has to base the valuation of the firm's prospects only on the dividend stream D_t, for $t > \tau$.

The manager at time τ^* has two choices: reveal the shift in fundamentals or conceal it.

In the case the manager reveals the shift in fundamentals, the price drops to P_{τ^*} given in equation (9) for $t = \tau^*$. If the manager decides to conceal the true growth rate of investment opportunities, it must elaborate an investment strategy that enables the firm to pay the dividend stream D_t^G that is consistent with the high growth firm. Intuitively, this strategy cannot be supported forever, as investment opportunities are not growing fast enough, and thus at some point in the future the manager will have to default, in the sense that its dividend D_t will not meet expectations that are consistent with a high growth firm. We denote by T^{**} this time of "default".

Shareholders know that the firm when born at τ has a high growth of investment opportunities G. Since it is costly to monitor the investment strategy of the firm every t, shareholders use cash flow information to assess whether the firm at any time t is a G firm or a g firm. So long the firm is G type, they expect a dividend with dividend described in (8). However, they also monitor the manager by executing audits at the times in which the dividend policy changes. During these audits, the whole history of investments is made public information. Our assumption that shareholders conduct an internal audit every time there is a change in dividend policy may appear extreme, but it matches the lack of randomness in dividend realizations in our setting. In a more realistic settings in which dividend realizations are random around a mean, the equivalent assumption is to allow shareholders to conduct an audit when large changes in dividend policy occur.

4.1. Manager Preferences

We assume the manager receives a performance based compensation w_t at the time he/she leaves the company. For simplicity, we assume that in any instant dt there is a constant probability $\alpha\ dt$ that the manager leaves the company. Let s^* denote the random time of departure. The manager may also leave the company voluntarily, in particular, right before being fired due to an investment strategy that was not optimal for shareholders. More specifically, at the time of a shift τ^* the manager has to take a decision: He/she can either change the investment and dividend policy to reflect the slower growth in investment opportunities and thus maximize value for the shareholders, or he/she can elaborate a suboptimal investment strategy that enables the firm to pay a dividend stream that is consistent with a fast growing company. Given the slower growth in investment opportunities, such investment strategy cannot be kept up indefinitely. We let T^{**} denote the time this strategy must be abandoned. At this time, the manager must admit the firm needs more capital to continue operations, a contingency that would have not occurred if the optimal investment strategy was followed. As a consequence, the manager is fired. Since the manager knows this fact, he will leave the firm one instant before default.

Let β be the time discount rate of the manager. Assuming linear utility for the manager – as we shall see, after the decision at τ^* there is essentially no risk left – his/her expected utility at time t is given by

$$U_t = E_t \left[\int_t^{\min(T^{**}, s^*)} e^{-\beta(u-t)} w_u du \right]$$

For the managers, there are two types of uncertainty: One is the time τ^* at which investment opportunity growth declines; and the second is the time s^* at which he will leave the firm. After τ^*, only the time of his departure from the firm is uncertain. The time T^{**} of voluntary departure is chosen by the manager to maximize his/her utility: If he/she follows the optimal strategy, $T^{**} = \infty$, as the dividend strategy can continue indefinitely. A finite T^{**} only occurs in the case in which the manager wants to conceal the change in investment opportunity growth. Thus, for $t \geq \tau^*$, an application of integration by parts shows that the manager's utility is given by (see Appendix):

$$U_t = \int_t^{T^{**}} e^{-(\alpha+\beta)(s-t)} w_s ds \qquad (12)$$

We consider two types of compensation schemes: One in which compensation is tied to stock price performance, and another in which compensation is tied to cash flow performance, i.e. earnings levels. More specifically,

$$w_t = \begin{cases} \eta P_t & \text{for stock based compensation} \\ \eta D_t & \text{for earnings based compensation} \end{cases}$$

where η is a constant. We further assume that $\alpha + \beta - G > 0$, which is required to keep the total utility of the manager finite.

5. Optimal Investments under Incentive Contracts

We start from the case in which the manager's compensation is stock based. In order to solve for the optimal investment policy, we go backward on the decision tree. In particular, decisions have only to be taken at time τ, when the firm is born, and at time τ^*, when the firm matures.

Let K_{τ^*} be the amount of capital available at time τ^*. The choice of the manager at this point is to either change the dividend payout to reflect the new status of the firm, or keep dividends that are consistent with a high growth of the company. The following Lemma characterizes the optimal policy rule in case the manager opts for concealing the true state of the economy.

Lemma 1: *Conditional on the decision to conceal the true state at τ^*, the manager optimal investment policy is to maximize the time to "default" T^{**}.*

The proof is intuitive: Conditional on his decision to conceal the true state g, the manager must provide a dividend stream that is consistent with the G state. The first time the firm does not deliver the promised dividends, shareholders performs an audit and the manager is fired, as for some time $[\tau^*, t]$ the manager has not acted optimally. However, as long as he delivers, the price of the stock simply reflects the present value of future cash flows conditional on the information of analysts, and it is independent of the manager actions. Thus, whether his pay is based on dividend performance or price performance, his utility is fixed as long as he holds the job. Since he loses his job in case of "default", he would like to delay the "default" as much as he can. We must now calculate the optimal investment strategy that leads to the longest time to "default".

5.1. Time to "default"

Time to "default" is the main driving force in this model. Lemma 1 shows that once the manager chooses to conceal the true state g at τ^*, he will maintain the associated dividend stream for as long as he can, since this will delay the reprisal and allow him to exercise more shares. Thus the time to "default" is the maximal time the manager can maintain the appearances without violating any constraints. The following Proposition characterizes the maximal time to "default" T^{**} and the optimal investment strategy:

Proposition 2: *Let K_{τ^*-} be the capital available at time τ^*. A manager of the firm with the rate of growth g, but pretending to be G, chooses to invest all of his initial capital stock, i.e. $K_{\tau^*} = K_{\tau^*-}$. For $t > \tau^*$ the optimal investment, given by*

$$I_t = z \ \min\left(K_t, e^{G(\tau^*-\tau)+g(t-\tau^*)}\right) - (z - G - \delta)\, e^{G(t-\tau)}, \tag{13}$$

satisfies

$$\frac{dK}{dt} = z \ \min(K_t, e^{G(\tau^*-\tau)+g(t-\tau^*)}) - \delta K_t - (z - G - \delta)\, e^{G(t-\tau)} \tag{14}$$

Default time T^{**} is determined by the condition $K_{T^{**}} = 0$.

Proposition 3: *Default time $h^{**} = T^{**} - \tau^*$ is independent of the time the firm matures, τ^*.*

Figure 4 presents an illustration of the optimal investment path for a g-firm pretending to be a G firm after τ^*, and it compares it to the optimal investment policy under full information for a specific set of parameters. The top panel presents the capital dynamics while the bottom panel presents the investment dynamics.

A few comments are in order: First, the optimal capital dynamics initially has $K_t > J_t$. This implies that the pretending firm is investing in *negative* NPV projects initially: The amount $K_t - J_t$ has a zero return but it is depreciating at the rate δ. Intuitively, note that the g firm has to pay lower dividends when it pretends to be G than under its own optimal

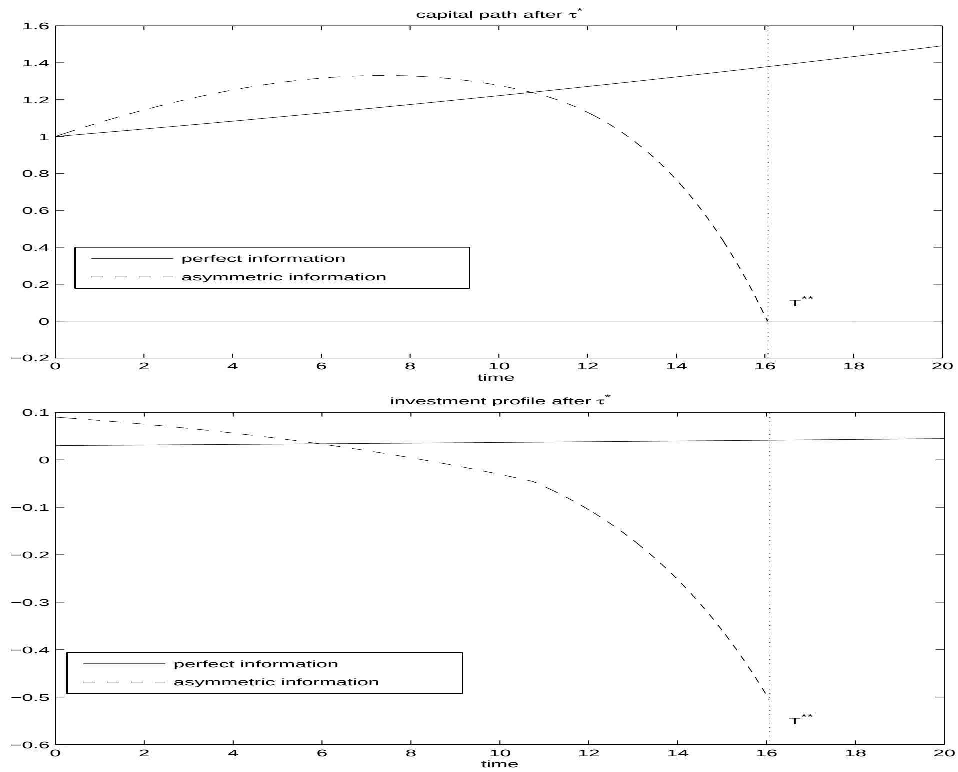

Figure 4: Capital dynamics (top panel) and investment dynamics (bottom panel) for a g firm pretending to be a G firm (dashed line), relative to its full information counterpart (solid line). The vertical dotted line denotes "default" time T^{**}. The following parameters are used: $r = .12$,$z = .23$,$g = 0.02$,$G = .08$,$\delta = .01$, $\lambda = 1/15$.

investment dynamics (see Figure 2). The extra cash is (suboptimally) invested in negative NPV projects as a reserve of value to postpone as much as possible the default time T^{**}.

Second, the pretending firm engages in large disinvestment for higher t: the cash raised in disinvestment is used to pay the large dividends of the growing firm. The firm can support this state until it runs out of capital K_t.

Finally, for the parameter used in our examples, T^{**} turns out to be very large, between 13 and 25 years. While this time to "default" seems excessively high, one must recall that it is based on very restrictive assumption that the market cannot distinguish at all between the investments and costs. If the market can partially distinguish between the two – e.g. the market can spot the large disinvestment required by the g firm – the time to "default" is likely to become significantly lower.

The above defines the optimal policy for the manager in equilibrium, thus would be used by the analyst in pricing the stock.

5.2. Pricing functions

For simplicity of presentation we assume that manager's compensation is not coming out of the value of the firm - these contracts are settled elsewhere.[6] This is a simplifying assumption that does not alter the basic intuition of the model, but makes the pricing functions very transparent.

We have shown that the pretending firm will "default" at T^{**}. The post-default valuation of the firm is straightforward since the information is complete. The difference is that the firm has no money to invest, thus has to borrow the investment amount $J_{T^{**}} = e^{G(\tau^*-\tau)+g(T^{**}-\tau^*)}$:

$$P^L_{T^{**}} = \int_{T^{**}}^{\infty} D^g_t e^{-r(t-T^{**})} dt - J_{T^{**}} = e^{G(\tau^*-\tau)+g(T^{**}-\tau^*)} \left(\frac{z-r-\delta}{r-g} \right) \tag{15}$$

We now turn to the price of the stock for $t < T^{**}$. In this exercise, we ignore the price of the risk associated with the lack of information, and only focus on the expected value. This could be justified by the fact that this informational risk is uncorrelated with the market risk, or other systematic risk factors, thus can be disregarded by a well-diversified investor. The pricing formula for $t < T^{**}$ is then

$$P_t = E_t \left[\int_t^{T^{**}} e^{-r(s-t)} D^G_s ds + e^{-r(T^{**}-t)} P^L_{T^{**}} \right] \tag{16}$$

This formula can be compared with the analogous formula under perfect information

[6] For example, the manager is just selling shares to outside investors, which has no impact on firm valuation. We must add that including compensation into the firm valuation would not change our results qualitatively, but would significantly complicate the exposition.

(10): As it can be seen, the formula is identical with the only difference that the switch time τ^* is replaced by the (later) T^{**}, and the price $P^g_{\tau^*}$ is replaced with the (much lower) price $P^L_{T^{**}}$. Our assumptions allow us to obtain analytical solutions:

Proposition 4: *Under asymmetric information, (a) if $t > \tau + h^{**}$, the value of the stock is*

$$P_t = e^{G(t-\tau)}A \tag{17}$$

where

$$A = \frac{(z-G-\delta)}{(r+\lambda-G)} + \lambda e^{-(G-g)h^{**}}\left(\frac{z-r-\delta}{(r-g)(r+\lambda-G)}\right) \tag{18}$$

(b) If $t \in [\tau, \tau+h^{**})$, then

$$P_t = (z-G-\delta)e^{G(t-\tau)}\frac{1-e^{-(r-G)(\tau+h^{**}-t)}}{(r-G)} \tag{19}$$

$$+e^{r(t-\tau)}e^{(G-r)h^{**}}\frac{(z-G-\delta)}{(r+\lambda-G)} + \lambda e^{r(t-\tau)}e^{(g-r)h^{**}}\left(\frac{z-r-\delta}{(r-g)(r+\lambda-G)}\right) \tag{20}$$

The pricing formula (17) should be compared with the pricing formula (11) in the case of perfect information. The first term is identical. The second term is smaller, as it takes into account two additional effects: First, even if default has not been declared yet, it may be possible that the true investment opportunities are growing at a lower rate g for the last h^{**} periods. The adjustment $e^{-(G-g)h^{**}} < 1$ takes into account this possibility. Secondly, at default, the firm will need to borrow capital to resume operations, a fact that show in the numerator of the second term (the smaller $z-r$ in the case of asymmetric information replaces $z-g$ in the case of symmetric information).

Figure 5 plots the dynamics of dividends $D(t)$ under the asymmetric information case, in which the agent chooses to conceal the true state. Figure 6 plots the corresponding dynamic of prices P_t. The figures also show the dividends and the prices that would realize, was the manager to choose to invest according to shareholders preferences at τ^*.

How does asymmetric information affect the level of prices? Figure 7 plots the price path under a "concealing" equilibrium, and the price path under a perfect information equilibrium. As it can be seen, rational investors decrease the value of prices in a "concealing" equilibrium, as they know that the manager will invest suboptimal when a shift to a new state g will occur.

The behavior of prices at T^{**} reflects a typical pattern of prices observed in the market. Specifically, the fact that "bad news are really bad news": failing to meet dividend expectations, even by a small amount, results in a large decrease in the stock price.

Although our model is very stylized, the basic intuition that meeting expectations / failing to meet expectations are strong signals of the true type of the firm is likely to hold in

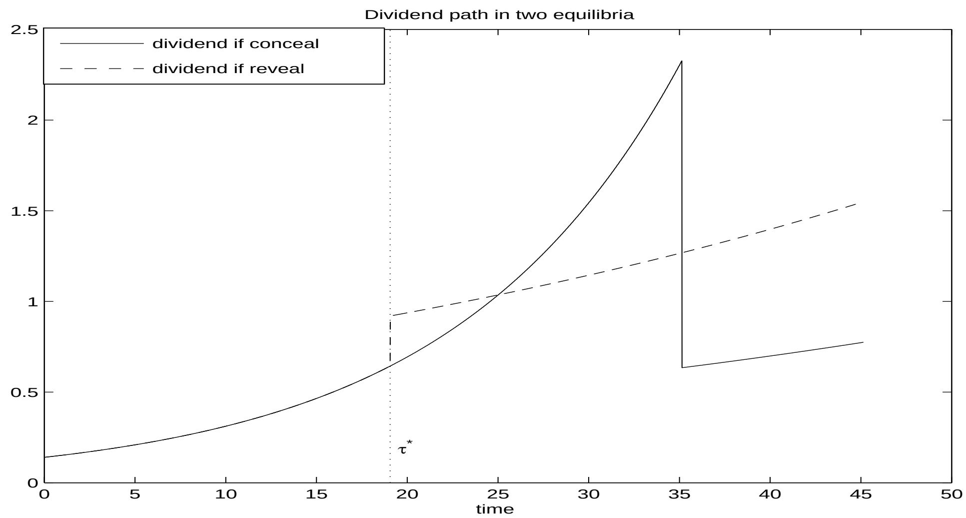

Figure 5: Dividend dynamics. The vertical dotted line denotes time τ^* of the growth change from G to g. The following parameters are used: $r = .12$,$z = .23$,$g = 0.02$,$G = .08$,$\delta = .01$, $\lambda = 1/15$, $\alpha = .1$, $\beta = .1$.

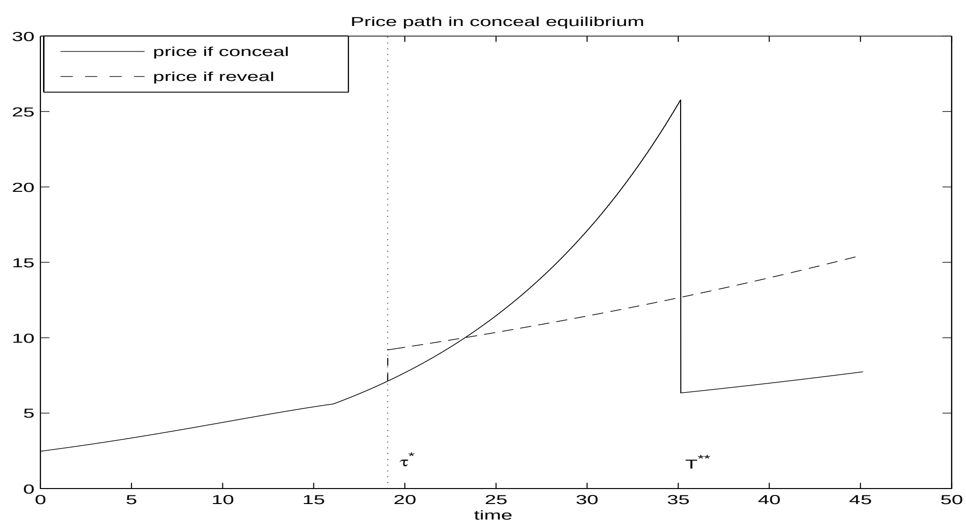

Figure 6: Price dynamics in a "conceal" equilibrium. The vertical dotted line denotes time τ^* of the growth change from G to g. The following parameters are used: $r = .12$,$z = .23$,$g = 0.02$,$G = .08$,$\delta = .01$, $\lambda = 1/15$, $\alpha = .1$, $\beta = .1$.

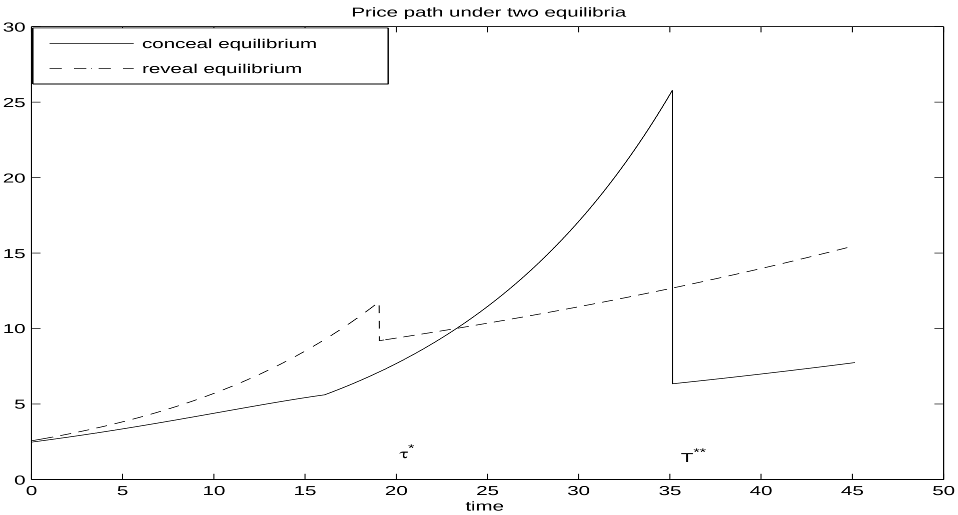

Figure 7: Price dynamics in the two equilibria: Perfect Information and Asymmetric Information. The following parameters are used: $r = .12$,$z = .23$,$g = 0.02$,$G = .08$,$\delta = .01$, $\lambda = 1/15$, $\alpha = .1$, $\beta = .1$.

more general models. Below, we consider a few extensions, including a continuum of types, as well as random changes of type over time.

6. Equilibrium

We finally consider the manager's incentive at time τ^* to conceal or reveal the true state. Under stock based compensation we have

$$\begin{aligned} U_{\tau^*}^{reveal} &= \int_{\tau^*}^{\infty} e^{-(\alpha+\beta)(t-\tau^*)}(\eta P_t^g)dt \\ U_{\tau^*}^{conceal} &= \int_{\tau^*}^{T^*} e^{-(\alpha+\beta)(t-\tau^*)}(\eta P_t)dt \end{aligned}$$

Proposition 5: *Let $\tau^* > \tau + h^{**}$. Then, stock based compensation implies that the two utility functions under "reveal" and "conceal" strategies are*

$$\begin{aligned} U_{\tau^*}^{reveal} &= \frac{e^{G(\tau^*-\tau)}}{\alpha+\beta-g}\left(\frac{\eta(z-g-\delta)}{(r-g)}\right) \\ U_{\tau^*}^{conceal} &= \frac{e^{G(\tau^*-\tau)}}{\alpha+\beta-G}\left(1-e^{-(\alpha+\beta-G)h^{**}}\right)(\eta A) \end{aligned}$$

If $\tau^* < \tau + h^{**}$, the utility function $U_{\tau^*}^{conceal}$ is more complicated, and it is left to the appendix. A conceal equilibrium results if $U_{\tau^*}^{reveal} < U_{\tau^*}^{conceal}$.

Under earnings based compensation we have

$$U^{reveal}_{\tau^*,Div} = \int_{\tau^*}^{\infty} e^{-(\alpha+\beta)(t-\tau^*)}(\eta D^g_t)dt$$
$$U^{conceal}_{\tau^*,Div} = \int_{\tau^*}^{T^*} e^{-(\alpha+\beta)(t-\tau^*)}(\eta D^G_t)dt$$

Proposition 6: *At time τ^*, earnings based compensation implies that the two utility functions under "reveal" and "conceal" strategies are*

$$U^{reveal}_{\tau^*,Div} = \frac{e^{G(\tau^*-\tau)}}{\alpha+\beta-g}\left(\eta(z-g-\delta)\right)$$
$$U^{conceal}_{\tau^*,Div} = \frac{e^{G(\tau^*-\tau)}}{\alpha+\beta-G}\left(1-e^{-(\alpha+\beta-G)h^{**}}\right)\left(\eta(z-G-\delta)\right)$$

A conceal equilibrium results if $U^{reveal}_{\tau^*,Div} < U^{conceal}_{\tau^*,Div}$

6.1. Equilibrium Conditions

Corollary 1: *A necessary and sufficient condition for a "reveal equilibrium" under dividend-based compensation is then*

$$\left(\frac{z-G-\delta}{z-g-\delta}\right)\left(1-e^{-(\alpha+\beta-G)h^{**}}\right) < \frac{\alpha+\beta-G}{\alpha+\beta-g} \tag{21}$$

This condition is satisfied if the return to capital net of depreciation is less than or equal to the effective time discount: $z-\delta \leq \alpha+\beta$. This latter sufficient condition is though weak: in fact, even when $z-\delta > \alpha+\beta$, we could not find instances of reasonable parameters in which condition (21) is violated, as we show in the next section. In other words, dividend compensation generates a "reveal equilibrium," and thus an optimal investment decision for managers.

Exactly the opposite occurs under stock-based compensation. In fact, in this case we obtain the following necessary and sufficient condition:

Corollary 2: *Let $\tau^* > \tau + h^{**}$. A necessary and sufficient condition for a "reveal equilibrium" under stock based compensation is*

$$\frac{A\,(r-g)}{(z-g-\delta)}\left(1-e^{-(\alpha+\beta-G)h^{**}}\right) < \frac{\alpha+\beta-G}{\alpha+\beta-g} \tag{22}$$

where the constant A is given in equation (18).

Comparing equation (22) and (21), we see that stock based compensation is more likely to imply a concealing equilibrium (i.e. a violation of (22) if

$$A\,(r-g) > z - G - \delta \tag{23}$$

From equation (18) we see that

$$A\,(r-g) \;=\; \left(\frac{r-g}{r+\lambda-G}\right)(z-G-\delta) + \lambda e^{-(G-g)h^{**}}\left(\frac{z-r-\delta}{r+\lambda-G}\right) \tag{24}$$

which implies that condition (23) is certainly satisfied whenever $G > g+\lambda$, that is, whenever the growth rate in the good state G is sufficiently high compared to the mature growth g. Intuitively, when G is high, the price of stock will be high as well, as it reflects the higher potential growth in dividends. The higher stock price implies a higher compensation for the firm's manager, and thus it generates a greater incentive for him or her to conceal the decrease in g when it happens at τ^*. Indeed, as we shall see below, we find that condition (22) is violated for most parameter configurations, unless G is quite close to g. In this latter case it may be not worth concealing the equilibrium and being fired, as the "kick" on the stock price, and thus on compensation, is small.

7. A Numerical Example

When is it optimal to conceal the change in the growth rate of investment opportunities? Figure 8 plots the indicator function of the concealing equilibrium in the space (g, G). As it can be observed, under Stock Compensation (top panel) even a small differential between the two possible growth states of only 5% is sufficient to induce the manager to conceal the decrease in growth rates g, and pretend that the firm is still enjoying a high growth G. As we know from the previous discussion, this equilibrium is responsbile for the large negative reaction of prices to bad earnings reports. The bottom panel of the figure shows that under Earnings-based compensation, it is never optimal for the manager to conceal the equilibrium. This differential in behavior under the two compensation schemes is consistent with the discussion in the previous section. The intuition, recall, is that under stock based compensation, the manager obtaines a much higher compensation if he conceals, compared to earnings based compensation, because of the multiplier effect that is generated by the present value formula in the discount of future cash flows.

Figure 9 plots the same indicator function of the concealing equilibrium but in the space (z, G), where recall that z is the instantaneous return to capital. We see that "concealing" is a pervasive equilibrium outcome under stock-based compensation, as the indicator function is essentially always equal to 1, under the remaining parameter assumptions. In contrast, the bottom panel reveals that "concealing" is never optimal in the dividend based compensation.

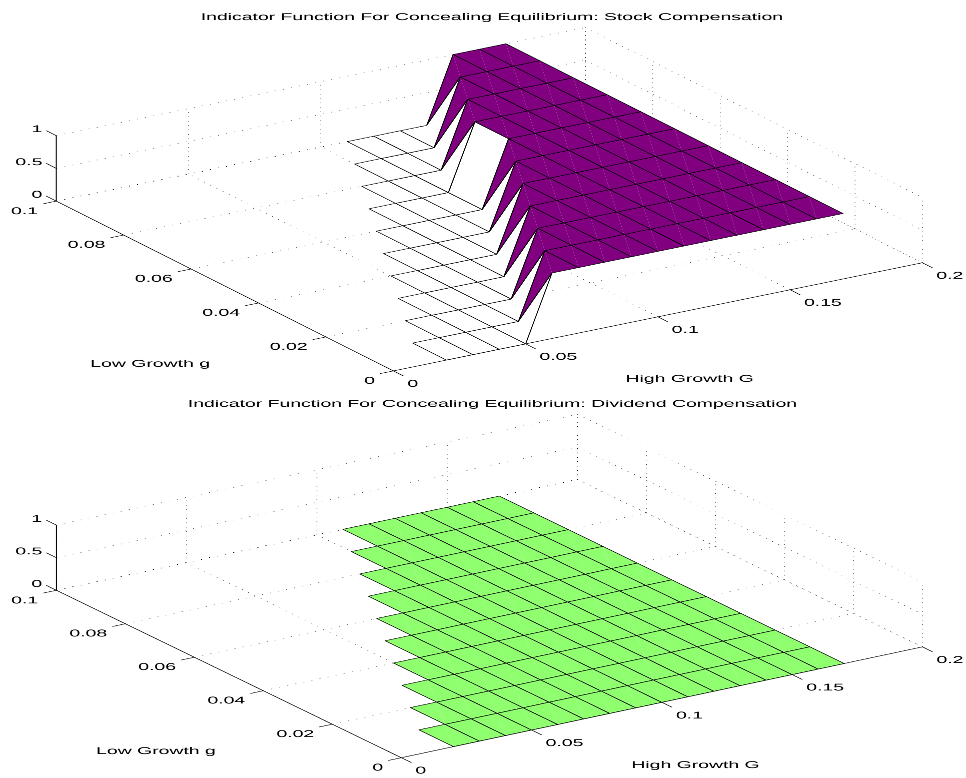

Figure 8: Concealing Equilibrium under Stock Compensation (top panel) and Dividend Compensation (bottom panel). Low growth g ranges between 0 and 10%, while the high growth rate G ranges between g and 17%. The remaining parameters are as follows: $r = .12$, $z = 0.25$, $\delta = .01$, $\lambda = 1/15$, $\alpha = \beta = .1$.

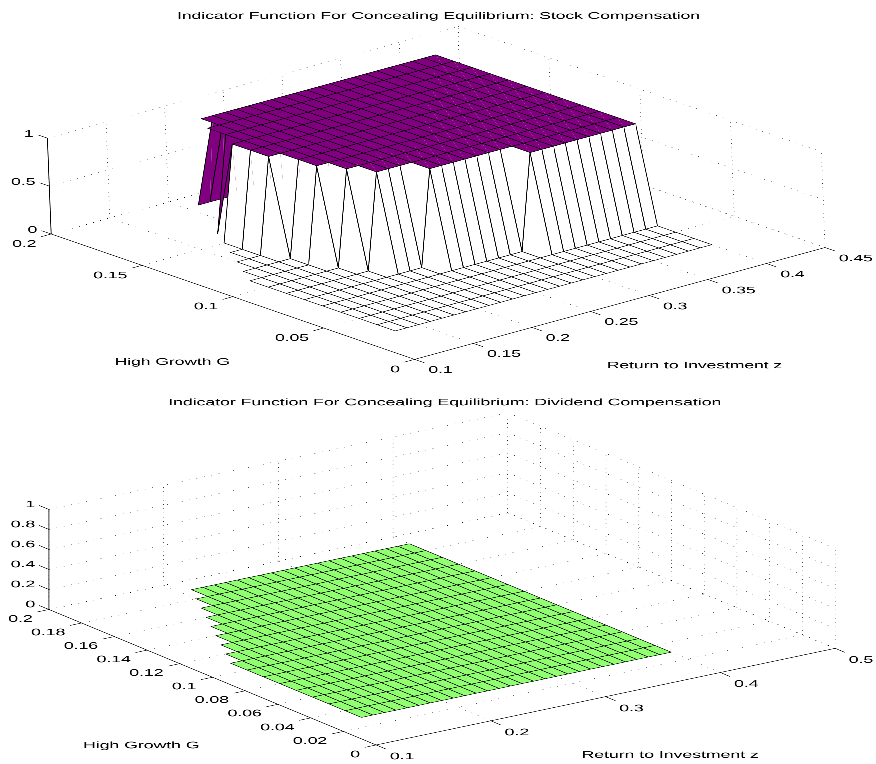

Figure 9: Concealing Equilibrium under Stock Compensation (top panel) and Dividend Compensation (bottom panel). Return to investment z ranges between .13 to .4 and the growth rate in investment opportunities G ranges between .03 and .19. The remaining parameters are as follows: $r = .12$, $g = 0.02$, $\delta = .01$, $\lambda = 1/15$, $\alpha = \beta = .1$.

8. Conclusions

We have shown that in industries where the market cannot distinguish between investment and costs, managers posses a severe information advantage that is very hard to eliminate via accounting standards and regulation. When only the manager knows the future growth rate of the firm, and his compensation is based on the short term stock prices, he has a strong incentive to pretend to be a high growth firm and is willing to invest suboptimally to support this pretense. We show that such behavior can be prevalent in equilibrium.

9. Appendix

Proof of Proposition 1 For any $t > \tau^*$ we have

$$\begin{aligned} P_t^g &= \int_t^\infty D_t e^{-r(s-t)} ds = (z-g-\delta) e^{G(\tau^*-\tau)+g(t-\tau^*)} \int_t^\infty e^{-(r-g)(s-t)} ds \\ &= \left(\frac{z-g-\delta}{r-g}\right) e^{G(\tau^*-\tau)+g(t-\tau^*)}. \qquad (25) \end{aligned}$$

In particular, at $t = \tau^*$ we have

$$P_{\tau^*} = \left(\frac{z-g-\delta}{r-g}\right) e^{G(\tau^*-\tau)}. \qquad (26)$$

Thus, for $t < \tau^*$, we have

$$P_t = E_t\left[\int_t^{\tau^*} e^{-r(s-t)} D_s ds + e^{-r(\tau^*-t)} P_{\tau^*}\right] \qquad (27)$$

where $D_s = (z-G-\delta)e^{G(s-\tau)}$. By using integration by parts, we find

$$\begin{aligned} P_t &= \int_t^\infty e^{-(r+\lambda)(\tau^*-t)} D_{\tau^*} + \lambda e^{-(r+\lambda)(\tau^*-t)} P_{\tau^*} d\tau^* \qquad (28) \\ &= \int_t^\infty (z-G-\delta) e^{-(r+\lambda)(\tau^*-t)} e^{G(\tau^*-\tau)} + \lambda \left(\frac{z-g-\delta}{r-g}\right) e^{-(r+\lambda)(\tau^*-t)} e^{G(\tau^*-\tau)} d\tau^* \qquad (29) \\ &= \frac{(z-G-\delta)}{r+\lambda-G} e^{G(t-\tau)} + \lambda \left(\frac{z-g-\delta}{(r-g)(r+\lambda-G)}\right) e^{G(t-\tau)} \qquad (30) \end{aligned}$$

(31)

Proof of Utility Function 12.

The result stems from the properties the exponential distribution. In particular:

$$U_t = E\left[\int_t^{\min(T^{**},s)} e^{-\beta(u-t)} w_u du\right]$$

$$
\begin{aligned}
&= E\left[\int_t^{\min(T^{**},s)} e^{-\beta(u-t)} w_u du | s < T^{**}\right] \Pr\left(s < T^{**}\right) \\
&\quad + E\left[\int_t^{\min(T^{**},s)} e^{-\beta(u-t)} w_u du | s > T^{**}\right] \Pr\left(s > T^{**}\right) \\
&= E\left[\int_t^{s} e^{-\beta(u-t)} w_u du | s < T^{**}\right] \Pr\left(s < T^{**}\right) \\
&\quad + E\left[\int_t^{T^{**}} e^{-\beta(u-t)} w_u du | s > T^{**}\right] \Pr\left(s > T^{**}\right)
\end{aligned}
$$

From the properties of the exponential density function, we have

$$
\begin{aligned}
& E\left[\int_t^{s} e^{-\beta(u-t)} w_u du | s < T^{**}\right] \Pr\left(s < T^{**}\right) \\
&= \left[\int_t^{T^{**}} \int_t^{s} e^{-\beta(u-t)} w_u du \frac{\alpha e^{-\alpha(s-t)}}{1 - e^{-\alpha(T^{*}-t)}} ds\right] \left(1 - e^{-\alpha(T^{**}-t)}\right) \\
&= \left[\int_t^{T^{**}} \left(\int_t^{s} e^{-\beta(u-t)} w_u du\right) \times \left(\alpha e^{-\alpha(s-t)}\right) ds\right]
\end{aligned}
$$

Integrating by parts leads to

$$
\begin{aligned}
& \left[\int_t^{T^{**}} \left(\int_t^{s} e^{-\beta(u-t)} w_u du\right) \times \left(\alpha e^{-\alpha(s-t)}\right) ds\right] \\
&= \int_t^{T^{**}} e^{-\beta(u-t)} w_u du \times \left(1 - e^{-\alpha(T^{**}-t)}\right) \\
&\quad - \int_t^{T^{**}} e^{-\beta(s-t)} w_s \left(1 - e^{-\alpha(s-t)}\right) ds \\
&= \int_t^{T^{**}} e^{-\beta(u-t)} w_u du - \int_t^{T^{**}} e^{-\beta(u-t)} w_u du \times e^{-\alpha(T^{**}-t)} \\
&\quad - \int_t^{T^{**}} e^{-\beta(s-t)} w_s ds + \int_t^{T^{**}} e^{-(\alpha+\beta)(s-t)} w_s ds \\
&= -\int_t^{T^{**}} e^{-\beta(u-t)} w_u du \times e^{-\alpha(T^{**}-t)} \\
&\quad + \int_t^{T^{**}} e^{-(\alpha+\beta)(s-t)} w_s ds
\end{aligned}
$$

Similarly, conditional on $s > T^{**}$, the utility will be $\int_t^{T^{**}} e^{-\beta(u-t)} w_u du$ (there is no other source of uncertainty here). So,

$$
E\left[\int_t^{T^{**}} e^{-\beta(u-t)} w_u du | s > T^{**}\right] \Pr\left(s > T^{**}\right) = \left(\int_t^{T^{**}} e^{-\beta(u-t)} w_u du\right) e^{-\alpha(T^{**}-t)}
$$

So, overall

$$
\begin{aligned}
U_t &= E\left[\int_t^{\min(T^{**},s)} e^{-\beta(u-t)} w_u du\right] \\
&= -\int_t^{T^{**}} e^{-\beta(u-t)} w_u du \times e^{-\alpha(T^{**}-t)} + \int_t^{T^{**}} e^{-(\alpha+\beta)(s-t)} w_s ds \\
&\quad + \left(\int_t^{T^{**}} e^{-\beta(u-t)} w_u du\right) e^{-\alpha(T^{**}-t)} \\
&= \int_t^{T^{**}} e^{-(\alpha+\beta)(s-t)} w_s ds
\end{aligned}
$$

QED.

Proof of Proposition 2: The dividend stream corresponding to a G is:

$$D_t^G = (z - G - \delta)e^{G(t-\tau)}. \tag{32}$$

After τ^*, the manager of the g firm must reproduce this pattern for as long as possible, i.e. as long as $K_t > 0$. Recall that for an arbitrary investment policy the dividend is:

$$D_t = z \min(K_t, J_t) - I_t, \tag{33}$$

where $J_t = e^{G(\tau^*-\tau)+g(t-\tau^*)}$, as given in (6). Since the capital evolution is given by the differential equation

$$dK_t/dt = I_t - \delta K_t,$$

it follows that the investment policy I_t that generates the required dividend stream D_t^H in (32) subject to (33) for at least some time, has to solve (14). Given the continuity of the solution of an ODE in its initial condition, $K_t = f(K_{\tau^*})$, ODE (14) implies that the amount of capital available at time t, K_t, is monotonically increasing in its starting value K_{τ^*}. Since T^{**} is determined by the condition $K_{T^{**}} = 0$, it follows from Lemma 1 that the optimal choice of the manager of a g firm is to invest as much as possible in the technology at τ^*. Given a maximum amount $K_{\tau^*} = K_{\tau^*-}$ of capital available, this amount is the optimal solution. The path for investments follows from $I_t = z \min(K_t, J_t) - dK_t/dt$ and equation (14). **QED**.

Proof of Proposition 3: The amount of capital at time τ^* is $K_{\tau^*} = J_{\tau^*} = e^{G(\tau^*-\tau)}$. As it can be seen, this amount enters the optimal solution (13) and (14) in a multiplicative fashion. Indeed, we can rewrite (13) and (14) by pulling $e^{G(\tau^*-\tau)}$ as a common factor on the right hand sides, obtaining

$$
\begin{aligned}
I_t &= e^{G(\tau^*-\tau)} \left(z\ \min\left(K_t/e^{G(\tau^*-\tau)}, e^{g(t-\tau^*)}\right) - (z - G - \delta)\, e^{G(t-\tau^*)}\right), \\
\frac{dK}{dt} &= e^{G(\tau^*-\tau)} \left(z\ \min(K_t/e^{G(\tau^*-\tau)}, e^{g(t-\tau^*)}) - \delta K_t/e^{G(\tau^*-\tau)} - (z - G - \delta)\, e^{G(t-\tau^*)}\right)
\end{aligned}
$$

Since the initial condition of the ODE is $K_{\tau^*} = e^{G(\tau^*-\tau)}$, we can define new variables $\tilde{K}_t = K_t/e^{G(\tau^*-\tau)}$ and $\tilde{I}_t/e^{G(\tau^*-\tau)}$. We then have that these latter two variables satisfy

$$\begin{aligned} \tilde{I}_t &= z \min\left(\tilde{K}_t, e^{g(t-\tau^*)}\right) - (z - G - \delta)\, e^{G(t-\tau^*)}, \\ \frac{d\tilde{K}}{dt} &= z \min(\tilde{K}_t, e^{g(t-\tau^*)}) - \delta \tilde{K}_t - (z - G - \delta)\, e^{G(t-\tau^*)} \end{aligned}$$

with initial condition $\tilde{K}_{\tau^*} = 1$. This $\tilde{K}_{T^{**}} = 0$ if and only if $K_{T^{**}} = 0$. That is, the distance between T^{**} and τ^* is independent of the capital accumulated up to τ^*, and thus default time is independent of τ^* itself. **Q.E.D.**

Proof of Proposition 4: Derivation of Pricing Formula in case of Asymmetric Information.

Consider first the case in which $t > \tau + h^{**}$. If default has not been observed at time t, then a shift cannot have occurred before $t - h^{**}$. In other words, the conditioning event is that $\tau^* > t - h^{**}$. Recalling that τ^* has an exponential distribution, we have that default time $T^{**} = \tau^* + h^{**}$ conditional on no default by time t has the following conditional distribution:

$$\begin{aligned} F_{T^{**}}(t'|\tau^* > t - h^{**}) &= Pr\left(T^{**} < t'|\tau^* > t - h^{**}\right) \\ &= Pr\left(\tau^* < t' - h^{**}|\tau^* > t - h^{**}\right) \\ &= \frac{e^{-\lambda(t-h^{**}-\tau)} - e^{-\lambda(t'-h^{**}-\tau)}}{e^{-\lambda(t-h^{**}-\tau)}} \\ &= 1 - e^{-\lambda(t'-t)} \end{aligned}$$

That is, the default time T^{**} has still the exponential distribution

$$f(T^{**}|\text{no default by } t) = \lambda e^{-\lambda(T^{**}-t)}$$

The value of the firm at time t is then equal to the present value of dividends D_t^G until default (at time T^{**}), plus the value present value $P_{T^{**}}^L$ at default. That is

$$\begin{aligned} P_t &= E_t\left[\int_t^{T^{**}} e^{-r(s-t)} D_s^G ds + e^{-r(T^{**}-t)} P_{T^{**}}^L|\text{no default by } t\right] \\ &= \int_t^{\infty} e^{-(r+\lambda)(T^{**}-t)} D_{T^{**}}^G + \lambda e^{-(r+\lambda)(T^{**}-t)} P_{T^{**}}^L dT^{**} \\ &= \int_t^{\infty} e^{-(r+\lambda)(T^{**}-t)} (z - G - \delta) e^{G(T^{**}-\tau)} dT^{**} \\ &\quad + \int_t^{T^{**}} \lambda e^{-(r+\lambda)(T^{**}-t)} e^{G(T^{**}-h^{**}-\tau)+g(h^{**})} \left(\frac{z - r - \delta}{r - g}\right) dT^{**} \end{aligned}$$

$$
\begin{aligned}
&= e^{G(t-\tau)}\frac{(z-G-\delta)}{(r+\lambda-G)} \\
&\quad +\lambda e^{G(t-h^{**}-\tau)+g(h^{**})}\left(\frac{z-r-\delta}{(r-g)(r+\lambda-G)}\right) \\
&= e^{G(t-\tau)}\frac{(z-G-\delta)}{(r+\lambda-G)} \\
&\quad +\lambda e^{G(t-\tau)}e^{-(G-g)(h^{**})}\left(\frac{z-r-\delta}{(r-g)(r+\lambda-G)}\right)
\end{aligned}
$$

If $t < \tau + h^{**}$, then the conditional distribution of T^{**} is zero in the range $[t, \tau + h^{**}]$: Indeed, even if a shift occurred at time τ, there would be no default before $\tau + h^{**}$. Thus, we have

$$
f(T^{**}) = \lambda e^{-\lambda(T^{**}-(\tau+h^{**}))}1_{T^{**}>\tau+h^{**})}
$$

We then obtain

$$
\begin{aligned}
P_t &= E_t\left[\int_t^{T^{**}} e^{-r(s-t)}D_s^G ds + e^{-r(T^{**}-t)}P^L_{T^{**}}|\text{no default by } t\right] \\
&= \int_t^{\tau+h^{**}} e^{-r(s-t)}D_s^G ds \\
&\quad +e^{-r(\tau+h^{**}-t)}E_t\left[\int_{\tau+h^{**}}^{T^{**}} e^{-r(s-(\tau+h^{**}))}D_s^G ds + e^{-r(T^{**}-(\tau+h^{**}))}P^L_{T^{**}}|\text{no default by } t\right] \\
&= \int_t^{\tau+h^{**}} e^{-r(s-t)}D_s^G ds \\
&\quad e^{-r(\tau+h^{**}-t)}\int_{\tau+h^{**}}^{\infty} e^{-(r+\lambda)(T^{**}-(\tau+h^{**}))}D^G_{T^{**}} + \lambda e^{-(r+\lambda)(T^{**}-(\tau+h^{**}))}P^L_{T^{**}}dT^{**} \\
&= \int_t^{\tau+h^{**}} e^{-r(s-t)}(z-G-\delta)e^{G(s-\tau)}ds \\
&\quad e^{-r(\tau+h^{**}-t)}\int_{\tau+h^{**}}^{\infty} e^{-(r+\lambda)(T^{**}-(\tau+h^{**}))}(z-G-\delta)e^{G(T^{**}-\tau)}dT^{**} \\
&\quad +e^{-r(\tau+h^{**}-t)}\int_{\tau+h^{**}}^{T^{**}} \lambda e^{-(r+\lambda)(T^{**}-(\tau+h^{**}))}e^{G(T^{**}-h^{**}-\tau)+g(h^{**})}\left(\frac{z-r-\delta}{r-g}\right)dT^{**} \\
&= (z-G-\delta)e^{G(t-\tau)}\frac{1-e^{-(r-G)(\tau+h^{**}-t)}}{(r-G)} \\
&\quad +e^{-r(\tau+h^{**}-t)}e^{G(\tau+h^{**}-\tau)}\frac{(z-G-\delta)}{(r+\lambda-G)} \\
&\quad +\lambda e^{-r(\tau+h^{**}-t)}e^{G(\tau+h^{**}-h^{**}-\tau)+g(h^{**})}\left(\frac{z-r-\delta}{(r-g)(r+\lambda-G)}\right) \\
&= (z-G-\delta)e^{G(t-\tau)}\frac{1-e^{-(r-G)(\tau+h^{**}-t)}}{(r-G)}
\end{aligned}
$$

$$+e^{-r(\tau+h^{**}-t)}e^{Gh^{**}}\frac{(z-G-\delta)}{(r+\lambda-G)}$$
$$+\lambda e^{-r(\tau+h^{**}-t)}e^{gh^{**}}\left(\frac{z-r-\delta}{(r-g)(r+\lambda-G)}\right)$$

Q.E.D

Proof of Proposition 5: At time τ^* the manager must decide whether he/she wants to reveal the change in growth rate or not. Given that any change in dividend policy results in an audit, we know from Lemma 1 that if the manager decides to conceal the new growth rate, he/she will choose an investment strategy to maximize the default time T^{**} given in Proposition 2. In this case, the price path will be given by the one described in equations (20) and (17) until T^{**}. The behavior of prices after T^{**} does not matter. If instead the manager decides to reveal the new state, the price path after τ^* is given by P_t^g in equation (9). Thus, we must simply compare two utilities.

$$\begin{aligned} U_{\tau^*}^{reveal} &= \int_{\tau^*}^{\infty} e^{-(\alpha+\beta)(t-\tau^*)}(\eta P_t^g)dt \\ U_{\tau^*}^{conceal} &= \int_{\tau^*}^{T^*} e^{-(\alpha+\beta)(t-\tau^*)}(\eta P_t)dt \end{aligned}$$

Note that after τ^* there is no longer any uncertainty on the price path, as discussed in Lemma 1. We can compute the value of the two utilities exactly

$$\begin{aligned} U_{\tau^*}^{reveal} &= \int_{\tau^*}^{\infty} e^{-(\alpha+\beta)(t-\tau^*)}(\eta P_t^g)dt \\ &= \int_{\tau^*}^{\infty} e^{-(\alpha+\beta)(t-\tau^*)}\left(\eta\frac{z-g-\delta}{r-g}\right)e^{G(\tau^*-\tau)+g(t-\tau^*)}dt \\ &= \left(\eta\frac{(z-g-\delta)}{(r-g)}\right)e^{G(\tau^*-\tau)}\int_{\tau^*}^{\infty} e^{-(\alpha+\beta-g)(t-\tau^*)}dt \\ &= \frac{e^{G(\tau^*-\tau)}}{(\alpha+\beta-g)}\left(\frac{\eta(z-g-\delta)}{(r-g)}\right) \end{aligned}$$

Similarly, assume that $\tau^* > \tau + h^{**}$, so that for this calculation only price (17) applies. Then

$$\begin{aligned} U_{\tau^*}^{conceal} &= \int_{\tau^*}^{T^{**}} e^{-(\alpha+\beta)(t-\tau^*)}(\eta P_t)dt \\ &= \int_{\tau^*}^{T^{**}} e^{-(\alpha+\beta)(t-\tau^*)}e^{G(t-\tau)}(\eta A)dt \end{aligned}$$

$$
\begin{aligned}
&= (\eta A)e^{G(\tau^*-\tau)} \int_{\tau^*}^{T^{**}} e^{-(\alpha+\beta-G)(t-\tau^*)}dt \\
&= \frac{(\eta A)}{(\alpha+\beta-G)} e^{G(\tau^*-\tau)} \left(1 - e^{-(\alpha+\beta-G)h^{**}}\right) \\
&= (\eta A)e^{G(\tau^*-\tau)} \left(\frac{1 - e^{-(\alpha+\beta-G)h^{**}}}{\alpha+\beta-G}\right)
\end{aligned}
$$

If $\tau^* < \tau + h^{**}$, then the utility function uses the pricing function (20), yielding (ADD)

Proof of Proposition 6: At time τ^* the manager must decide whether he/she wants to reveal the change in growth rate or not. Given that any change in dividend policy results in an audit, we know from Lemma 1 that if the manager decides to conceal the new growth rate, he/she will choose an investment strategy to maximize the default time T^{**} given in Proposition 2. In this case, the dividend path is D_t^G until T^{**}. The behavior after T^{**} does not matter. If instead the manager decides to reveal the new state, the dividend path after τ^* is given by D_t^g. Thus, we must simply compare two utilities.

$$
\begin{aligned}
U_{\tau^*}^{reveal} &= \int_{\tau^*}^{\infty} e^{-(\alpha+\beta)(t-\tau^*)}(\eta D_t^g)dt \\
U_{\tau^*}^{conceal} &= \int_{\tau^*}^{T^*} e^{-(\alpha+\beta)(t-\tau^*)}(\eta D_t^G)dt
\end{aligned}
$$

We can compute the value of the two utilities exactly

$$
\begin{aligned}
U_{\tau^*}^{reveal} &= \int_{\tau^*}^{\infty} e^{-(\alpha+\beta)(t-\tau^*)}(\eta D_t^g)dt \\
&= \int_{\tau^*}^{\infty} e^{-(\alpha+\beta)(t-\tau^*)} \left(\eta(z-g-\delta)\right) e^{G(\tau^*-\tau)+g(t-\tau^*)}dt \\
&= (\eta(z-g-\delta))e^{G(\tau^*-\tau)} \int_{\tau^*}^{\infty} e^{-(\alpha+\beta-g)(t-\tau^*)}dt \\
&= \frac{(\eta(z-g-\delta))}{(\alpha+\beta-g)} e^{G(\tau^*-\tau)}
\end{aligned}
$$

Similarly, the utility under the "conceal" strategy is

$$
\begin{aligned}
U_{\tau^*}^{conceal} &= \int_{\tau^*}^{T^*} e^{-(\alpha+\beta)(t-\tau^*)}(\eta D_t^G)dt \\
&= \int_{\tau^*}^{T^*} e^{-(\alpha+\beta)(t-\tau^*)}(\eta(z-G-\delta)e^{G(t-\tau)}dt \\
&= (\eta(z-G-\delta)e^{G(\tau^*-\tau)} \int_{\tau^*}^{T^*} e^{-(\alpha+\beta-G)(t-\tau^*)}dt \\
&= \frac{(\eta(z-G-\delta)}{(\alpha+\beta-G)} e^{G(\tau^*-\tau)} \left(1 - e^{-(\alpha+\beta-G)h^{**}}\right) \\
&= (\eta(z-G-\delta))e^{G(\tau^*-\tau)} \left(\frac{\left(1 - e^{-(\alpha+\beta-G)h^{**}}\right)}{(\alpha+\beta-G)}\right)
\end{aligned}
$$