Instructor Information

- **Instructor:** Daniel Bauer
- **Office:** Room 1126, Robinson College of Business (35 Broad Street)
- **Office Hours:** By appointment (just send me an email, I am usually in most days)
- **Phone / Email:** 404-413-7490 / dbauer@gsu.edu

Class Information

- **Location / Time:** Classroom South 205 / Thursday, 4:30 pm - 7:00 pm
- **Website:** D2LBRIGHTSPACE – https://gsu.view.usg.edu
- **Prerequisites:** MAS and MRM students should follow the program guides at http://robinson.gsu.edu/mas/ and http://robinson.gsu.edu/mrm/. In general, you should have a background in calculus (differential, integral, and multivariate), mathematical statistics, and finance. (If you do not have FI8000, you will need to do some extra reading early on, particularly on financial options and review the Binomial option pricing model.) Experience with a spreadsheet such as Excel is required, including basic macro programming skills. Web-based courses available at Skillsoft, including a variety of courses on macro programming (you need to be familiar with loops, if-else statements, etc.). However, you can also complete the assignments in any programming environment of your choice (e.g., R, C++, or Python). Some programming will be necessary.

Catalog Description

This course introduces students to continuous-time financial models essential for the practice of mathematical risk management. It begins with a discussion of the fundamental mathematical tools from continuous-time stochastic processes including Ito’s formula, change of measure, and martingales. This provides a framework for financial concepts including hedging, complete markets, and incomplete markets. The mathematical tools and financial concepts are applied to the risk management and valuation of financial derivatives based on stocks and bonds, separately, and insurance company liabilities with embedded financial options. The course concludes with a consideration of models that jointly value stocks, bonds and non-traded assets.

Textbooks

You are not required to purchase a textbook – lecture notes will be posted on D2L. Nevertheless, it is strongly advised that you take notes during lecture as there may be ideas presented in the class which are not included in the posted notes. Useful references will be given for certain concepts and further reading. Handouts as well as computer programs will also be posted on Sharepoint.

If you would like to purchase a textbook, I recommend Björk (2009). There are many other financial engineering (Bingham & Kiesel (2004), Hull (2005), Shreve (2004), Wilmott (2013), etc.), but I find Björk (2009) contains most of the important topics from class and beyond – and I like the layout. Another good resource for the elementary ideas behind stochastic calculus is Mikosch (1998).

1See http://technology.gsu.edu/technology-services/it-services/.
Course Objectives

This course introduces students to continuous-time financial models essential for the practice of mathematical risk management and the inevitable mathematical concept: stochastic calculus. The focus is on complete markets and the Black-Scholes model in particular, but generalizations are addressed. Emphasis is put on practical application such as pricing and hedging rather than mathematical rigor. By the end of the course, successful students will be able to:

1. Explain the concepts of arbitrage and replication as well as their implications on derivative pricing.
2. Price and hedge derivative securities using the Binomial Asset Pricing Model.
3. State the properties of Brownian motion and the principles of basic stochastic calculus, in particular Itô’s Formula.
4. Apply the Black-Scholes Model for pricing and hedging and European call and put options.
5. Derive Partial Differential Equations (PDEs) for the value function of European contingent claims for models driven by Brownian motion.
6. Price contingent claims by Monte Carlo simulation or by numerically solving certain PDEs and implement basic approaches within generalized Black-Scholes-type models.
7. Explain the flaws of Black-Scholes. Know where to continue from there.

Methods of Instruction

The course material is presented in lecture form. Weekly homework is assigned to clarify concepts and deepen the understanding. There will be one 90 minute in-class midterm examination and a take-home final project.

Attendance Policy

Attendance is not formally taken. However, it is strongly suggested that students do not miss class as most students will have difficulties completing the assignments without attending the lectures.

Homework

I will collect homework every week, and me and a grader will go briefly over every assignment. Every student has to submit her/his own work – group submissions are not allowed. You may discuss the assignments among each other, but every student has to write up the assignment on his own. Students copying from their classmates or from previous years’ assignments will receive a zero score. In addition, the student who let others copy from her/his assignment will receive a zero score. There are no exceptions to this rule! Further consequences are possible, including ejection from the program.

Grading Criteria

The mid-term examination is weighted as 35%, homework counts for 25%, and the final project is weighted as 40% of the final grade in the course. Make-up examinations are offered only under extraordinary circumstances. Students who miss examinations should contact me immediately. Grades will be awarded on a +/- basis, and the following guaranteed scale applies. Grades may be moved upward based on difficulty, but not downward:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Score</th>
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<tbody>
<tr>
<td>A+</td>
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<tr>
<td>A</td>
<td>95</td>
</tr>
<tr>
<td>A-</td>
<td>90</td>
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<tr>
<td>B+</td>
<td>85</td>
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<tr>
<td>B</td>
<td>80</td>
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<tr>
<td>B-</td>
<td>75</td>
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<tr>
<td>C+</td>
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<td>D</td>
<td>50</td>
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<td>F</td>
<td>&lt; 50</td>
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Remarks

- Students exhibiting disruptive behavior, including talking, sleeping, talking on cell phones or disturbing other students will be asked to leave.
- Please advise the instructor if you have a documented disability that needs to be accommodated.
- As members of the academic community, students are expected to recognize and uphold standards of intellectual and academic integrity. See the University's policy on Academic Honesty (Section 409, http://www2.gsu.edu/~wwfhb/sec409.html) for details.

Detailed Outline:

The material is subdivided into 6 Modules:

Module I: Review of the Binomial Asset Pricing Model

Contents

- What is an arbitrage? Delta-Hedging, state prices, Equivalent Martingale Measure.
- Valuation of contingent claims within the Binomial model (Put-Call parity, European vs. American options)
- The Black-Scholes Formula as a limit in the Binomial Model.

Learning Outcomes

- Explain the concept of no-arbitrage and why it offers a suitable pricing mechanism for derivative securities.
- Be able to price derivative securities within the Binomial Model, and implement it.

Module II: Martingales, Brownian motion, stochastic integration, and Itô’s formula.

Contents

- Sigma-algebras, conditional expectation, filtrations.
- Brownian motion, quadratic variation.
- Stochastic integral, Stochastic Differential Equations (SDEs).
- Itô’s Formula \( dt \cdot dW_t = 0, dW_t \cdot dW_t = dt \).
- Geometric Brownian motion, simulation by Euler-discretization. Closed form solution of Geometric Brownian motion, simulation via the closed form solution.

Learning outcomes

- State the basic definitions and explain why the rules of ordinary calculus do not work here. Be able to apply Itô’s formula.
- Simulate SDEs via Euler discretization.

Module III: The Black-Scholes Model

Contents

- Derive the Black-Scholes PDE via replication arguments.
- Methods to solve the PDE.
- Greeks, Delta-hedging.
Learning outcomes

- Write down the Black-Scholes PDE, and know how to solve it. Interpret the quantities within the PDE.
- Write down the Black-Scholes Formula and be able to derive similar formulas.
- Delta-hedging: Why and how it is done.
- Simulate a GBM-path, determine Deltas for a European call, determine hedging error.

Module IV: Risk-neutral pricing and Markov processes

Contents

- Why does expected discounted payoff not work? What needs to be done: Change of measure. Replace $\mu$ by $r$. Martingale property.
- Theoretical foundation: Radon-Nikodym and Girsanov’s Theorem (basic ideas only).
- Hedging: Representation theorem (basic idea only).
- “Feynman-Kac light”: Derive PDEs from price process. In particular, Black-Scholes PDE.

Learning outcomes

- Explain when and why security prices are martingales. Derive the Black-Scholes PDE from the Martingale property.
- Price “arbitrary European derivatives” via Monte-Carlo simulation.
- Solve expected value of European call for Black-Scholes Formula.

Module V: Simple generalizations and advanced derivatives

Contents

- Dividends / Time-dependent parameters / Multiple assets.
- Implied vs. realized volatilities, Delta-hedging revisited.
- American and Bermudan options. Valuation via nested simulations or PDEs. Basic finite difference schemes.

Learning outcomes

- Know how to apply the generalizations. Derive formulas for options on multiple assets. Simultaneously simulate several stocks.
- Evaluate Bermudan options via nested Monte-Carlo simulations and/or via PDE methods.

Module VI: Advanced Topics

Contents

- Path dependence, exotic options.
- Valuation of non-traded securities. Example: Embedded options in life insurance contracts.
- Problems with Black-Scholes.
- Stochastic volatility (Heston model) and/or Regime-switching models (if there is time).
Learning outcomes

- Be able to price path-dependent options and, particularly, embedded options within life insurance contracts.
- State the problems of Black-Scholes and stochastic volatility. Know where to find information on more advanced models.

<table>
<thead>
<tr>
<th>Date</th>
<th>Module</th>
<th>Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>2016-01-21</td>
<td>II</td>
<td>Björk (2009): Ch. 4+5; Mikosch (1998): Ch. 1-3;</td>
</tr>
<tr>
<td>2016-01-28</td>
<td>II</td>
<td>Mathematically More Advanced:</td>
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<tr>
<td>2016-02-18</td>
<td>III</td>
<td></td>
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<tr>
<td>2016-02-25</td>
<td>IV</td>
<td>Björk (2009): Ch. 10-12; Bingham &amp; Kiesel (2004): Ch. 6;</td>
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<tr>
<td>2016-03-03</td>
<td>IV</td>
<td>Mikosch (1998): Sec. 4.1.</td>
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<td>2016-03-10</td>
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<td>MIDTERM</td>
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<tr>
<td>2016-03-17</td>
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<td>SPRING BREAK</td>
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<td>2016-03-25</td>
<td>IV</td>
<td></td>
</tr>
<tr>
<td>2016-03-31</td>
<td>V</td>
<td>Björk (2009): Ch. 7.6-7.8, 13, 14, 16;</td>
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<td>2016-04-07</td>
<td>V</td>
<td>Hull (2005): Ch. 11.</td>
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<tr>
<td>2016-04-21</td>
<td>VI</td>
<td>Hull (2005): Ch. 17+18, provided material.</td>
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<td>2016-04-27</td>
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<td>Final Project due at 6pm</td>
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<tr>
<td>2016-04-28</td>
<td></td>
<td>Final Project discussion</td>
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</tbody>
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Note that this course syllabus provides a general plan for the course; deviations may be necessary.

References


Other potentially useful resources:
