Aggregate Risk, Bank Competition and Regulation in General Equilibrium

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Abstract

We investigate the optimal design of bank regulation by developing a tractable general equilibrium model of competitive banks who are exposed to idiosyncratic and aggregate risk. A comparison of the autarkic equilibrium allocations with the efficient allocations reveals that the autarkic economy underinvests in production when aggregate risk is below a threshold, but overinvests in production when it is above the threshold. We show how regulatory intervention tools that are central to the debate on bank regulation—capital requirements, liquidity requirements, deposit insurance and bank bailouts financed by taxation—can be used to implement the efficient allocations in a decentralized economy. For a given efficient allocation, there is a range of regulatory policies all of which implement the allocation, but the equilibrium of the regulated economy is unique for a given regulatory policy. The various tools in an optimal regulatory policy must be finely tuned to each other. Capital and liquidity requirements move in opposing directions; an optimal regulatory policy that features a stricter capital requirement has a looser liquidity requirement. When aggregate risk is below a threshold, the efficient allocation can be implemented via deposit insurance and taxation alone. Capital and liquidity requirements are, however, necessary to ensure a unique equilibrium of the regulated economy. When aggregate risk is high, all four regulatory tools are essential components of an optimal regulatory policy. Our results provide qualified support for proponents and opponents of stricter banking regulation. Lower capital requirements for banks could be optimal, but they must be accompanied by stricter liquidity requirements and vice versa.
1 Introduction

The 2007-2008 financial crisis was precipitated by the presence of insufficient liquid reserves and excessive debt levels in the financial system that made banks vulnerable to large aggregate negative shocks. Proponents of stricter bank regulation argue that higher capital requirements for banks reduce their insolvency risk. Opponents argue that higher capital requirements force banks towards costlier equity financing, and hamper banks’s key role in transforming liquid debt securities into less liquid assets. Commentators have also highlighted the importance of imposing adequate liquidity requirements on banks' assets, especially in the presence of aggregate shocks that affect the entire banking system. The debate over bank regulation remains unresolved as exemplified by the fact that the current U.S. government appears to be aggressively moving towards undoing the moves towards stricter bank oversight embodied in the 2010 Dodd-Frank Act and the Basel III agreement. A rigorous evaluation of the pros and cons of bank regulation necessitates a holistic analysis that endogenizes the costs of bank debt and equity, and models the interaction between banks’ assets and liabilities in the presence of aggregate risk.

We investigate the design and efficacy of bank regulation by developing a tractable general equilibrium model of competitive banks. We characterize the equilibria of the autarkic economy in which banks are completely unregulated, and compare the resulting payoff allocations with the set of efficient allocations. The autarkic economy underinvests in production when aggregate risk is below a threshold, but overinvests in production when aggregate risk is above the threshold. We then demonstrate how regulatory intervention tools that are central to the debate on bank regulation—capital and liquidity requirements, deposit insurance and bank bailouts—can be used to implement the efficient allocations in a decentralized economy. For a given efficient allocation, there is, in fact, a range of regulatory policies that implement the efficient allocation that differ in the tightness of the capital requirement and the resulting bank size. Importantly, the various tools in an optimal regulatory policy must be finely tuned to each other with capital and liquidity requirements moving in opposing directions. An optimal regulatory policy that features a stricter capital requirement has a looser liquidity requirement. When aggregate risk is low, the efficient allocation can be implemented via deposit insurance and bailouts alone. Capital and liquidity requirements are, however, necessary to ensure that the efficient allocation is uniquely implemented.
When aggregate risk is high, all four regulatory tools are essential components of an optimal regulatory policy. Taken together, our results provide qualified support for proponents and opponents of stricter banking regulation. Lower capital requirements for banks could be optimal, but they must be accompanied by stricter liquidity requirements and vice versa.

In our general equilibrium model, there is a single consumption/capital good and continuums of households, banks and entrepreneurs. All agents have access to a safe, liquid asset that is in perfectly elastic supply (or a linear storage technology) and provides a constant return per unit of capital invested. Capital in the economy is initially held by the ex ante identical “households.” Each household includes risk-averse “depositors,” who invest in bank deposits, and risk-neutral “equity holders,” who invest in equity issued by banks and firms. Banks raise deposit and equity financing, and provide capital in the form of loans to entrepreneurs, who operate firms with risky, concave production technologies. Firms have identically distributed binomial payoffs that are not, however, independent across all firms as we discuss shortly. Entrepreneurs can obtain capital either via bank loans or by issuing equity to equity holders. Capital and bank loan markets are competitive.

Each bank is exposed to a bank-specific shock, which could stem from bank specialization in geographic areas and/or lending to firms in specific sectors of the economy. A positive (negative) realization of the shock is accompanied by a larger (smaller) proportion of the firms in a bank’s loan pool succeeding with the remaining proportion failing. Banks are, however, also exposed to aggregate risk that leads to correlation among the loan pools of different banks. Specifically, a positive (negative) realization of the aggregate shock causes the loan pools of a proportion of banks to simultaneously succeed (fail). Agents in the economy (including banks) only know the proportion of banks that are exposed to the aggregate shock ex ante, but not their specific identities. As banks lend to firms, the firms in the economy inherit the correlation structure implied by the correlation among banks’ loan pools. Specifically, a positive (negative) realization of the aggregate shock leads to a larger (smaller) proportion of firms succeeding.

We first characterize equilibria of the autarkic economy in which banks are unregulated. In a competitive equilibrium, risk-neutral banks are indifferent between equity and deposit financing, and make zero profits. Further, risk-neutral equityholders are indifferent between bank and firm equity. Hence, the expected returns on bank equity, bank deposits, bank loans and firm equity are equal in equilibrium. There is a continuum of autarkic equilibria that vary from an equilibrium
in which banks are financed purely with debt and the real economy is financed with bank debt and equity, to a “full intermediation” equilibrium in which banks raise all available equity capital and the real economy is financed entirely via bank debt. An equilibrium with greater bank equity capital also features greater bank debt (and, therefore, bank size), lower expected returns on bank deposits and equity, and higher production.

The intuition for the results is as follows. In any equilibrium that features nonzero deposit financing, the expected return on bank deposits strictly exceeds the safe asset return because risk-averse depositors must obtain a risk premium to be induced to invest in banks. As the expected return on bank equity, firm equity and banks loans equals the expected return on deposits, it follows that banks and equity holders do not invest in the safe asset and supply all their capital to firms. Since the capital held by equity holders is invested in firms either directly or indirectly via banks, the total amount of capital invested in firms is determined by the total deposit capital raised by banks. Further, the expected bank loan return equals the marginal return from production in a competitive loan market that, in turn, is determined by the total capital invested in firms. Hence, a competitive equilibrium is fully determined by the total deposit capital raised by banks. Because an increase in the total deposit capital increases the total capital invested in firms, an equilibrium with higher total deposit capital also features higher production. The concavity of firms’ production technology implies that an increase in the total capital invested in firms is associated with a lower marginal return on production and, therefore, lower expected returns on bank loans, deposits and equity. To support an equilibrium with greater total deposit capital and a lower risk premium on bank deposits, banks’ equity buffers must also be higher to lower the risk of bank deposits.

There are three inefficiencies in the autarkic economy. First, consistent with the evidence in Guiso et al. (2002), there is limited market participation as depositors cannot directly invest in productive firms, but only indirectly via banks. Second, depositors cannot diversify their capital across banks and are, therefore, exposed to bank-specific risk. Third, markets are incomplete because there is no traded security contingent on the aggregate shock. As a result, depositors, equity holders and banks cannot internalize the effects of the aggregate shock on banks’ loan portfolios. The combination of the above inefficiencies leads to incomplete sharing of bank-specific risks among depositors as well as imperfect sharing of aggregate risk among depositors and equity holders. Further, limited market participation by depositors as well as the incomplete internalization of
aggregate risk could lead to under- or over-investment of capital in production depending on the level of aggregate risk in the economy.

We derive the set of efficient allocations in the economy by analyzing the problem of a social planner who maximizes the total expected utility of depositors, while ensuring that the other risk-neutral agents in the economy—equity holders, entrepreneurs and banks—receive their autarkic payoffs. For each autarkic equilibrium, therefore, there is a corresponding efficient allocation that maximizes the total expected utility of depositors, while maintaining the other agents’ payoffs at their levels in the autarkic equilibrium. The social planner faces a tradeoff between providing full insurance to risk-averse depositors that protects them against aggregate shocks, and investing as much capital as possible in socially beneficial production. If aggregate risk is below a threshold, it is optimal for the planner to fully insure depositors, while providing equity holders and entrepreneurs with their autarkic payoffs. In this scenario, the planner chooses the investment level to maximize the expected payoff from production, and invests the remaining capital in the safe asset to ensure that depositors are fully insured. If aggregate risk is above a threshold, however, it is socially costly to fully insure depositors so they must bear some aggregate risk. Further, an increase in the aggregate risk causes the social planner to lower investment in production and increase investment in the safe asset to offset the effects of higher aggregate risk on depositors’ expected utility. A comparison of the efficient and autarkic allocations shows that the autarkic economy underinvests in production when aggregate risk is below a threshold, but overinvests in production when aggregate risk is above the threshold.

We examine the implementability of the efficient allocations with the four regulatory tools relevant to the debate on banking regulation; capital requirements, liquidity requirements, deposit insurance and taxpayer-funded bank bailouts. For a given efficient allocation, there is, in fact, a range of regulatory policies that implement the efficient allocation. The regulated equilibria generated by the range of policies differ in the tightness of the capital requirement and the resulting size of banks. Importantly, however, the different regulatory tools must be tuned to each other; a stricter capital requirement is associated with a looser liquidity requirement and less deposit insurance. In contrast with the multiplicity of equilibria in the autarkic economy, the equilibrium of the regulated economy is unique for a given set of regulatory tools so the efficient allocation is uniquely implemented by the regulatory policy. Regulation, therefore, has the additional benefit of
breaking the multiplicity of equilibria in the autarkic economy.

The intuition for the results hinges on the fact that an efficient allocation is determined by the total amounts of capital invested in firms and the safe asset. The efficient allocation can be implemented by having depositors invest their entire capital in banks and imposing a liquidity requirement on banks, which ensures that the total investments in the safe liquid asset and firms are consistent with the efficient allocation. Because the total capital held by equity holders is eventually invested in firms either directly or indirectly via banks, the efficient allocation can be implemented by multiple regulated equilibria that correspond to different levels of equity capital raised by banks. The liquidity requirement ensures that, depending on the total capital that banks raise in a regulated equilibrium, the total amount of capital invested in the safe asset is efficient. Further, given a liquidity requirement, the capital requirement ensures that banks raise the right amount of equity capital, while the deposit insurance and taxation policies guarantee that the payoffs of depositors, equity holders and entrepreneurs are efficient. A stricter capital requirement implies that banks raise more equity capital so less capital is invested directly in firms by equity holders. Therefore, to ensure that the efficient amount of total capital in the economy is invested in production, the liquidity requirement on banks must be more lax. Further, a higher level of equity capital implies that banks are less risky so less deposit insurance is required.

If aggregate risk is below the threshold that ensures the efficiency of full insurance for depositors, the efficient allocation can be implemented via deposit insurance and taxpayer-funded bank bailouts alone. However, adding a capital and liquidity requirement is necessary to ensure that the regulated equilibrium is unique so the efficient allocation is uniquely implemented. Deposit insurance lowers the cost of risk faced by risk-averse depositors in the autarkic economy, while also insuring that more deposit capital is invested in production via banks, thereby mitigating underinvestment in the autarkic economy. If aggregate risk is above the threshold, all four regulatory tools are necessary to implement the efficient allocation. Capital and liquidity requirements work in tandem to mitigate overinvestment in production in the autarkic economy relative to the efficient allocation, while deposit insurance and bank bailouts facilitate optimal risk-sharing among risk-averse depositors, risk-neutral equity holders and entrepreneurs.

Broadly, our results show that there is, indeed, a range of efficient regulatory policies that vary between featuring relatively strict to relatively loose capital requirements. However, the different
tools that constitute a given regulatory policy must be finely tuned to each other. A policy with a strict capital requirement must also have a loose liquidity requirement to ensure that the economy as a whole invests optimally in value-enhancing production and safe, liquid assets that serve to make the banking system less vulnerable to negative aggregate shocks.

By serving as financial intermediaries that provide a conduit for depositors’ capital to flow to the real economy, banks in our framework embody the roles of not just traditional banks, but also “shadow banks” that perform the key intermediation role that traditional banks perform. As we discussed above, the objective of regulation is to optimally balance the tradeoff between maximizing expected production and achieving risk-sharing between risk-averse bank depositors and risk-neutral entrepreneurs and equity holders. In this respect, our results highlight the importance of regulating the traditional and shadow banking sectors within a unified framework.

2 Related Literature

Our paper is related to several lines of literature that study the design and impact of bank regulation from various perspectives (e.g., see Dewatripont and Tirole (1994), Acharya et al. (2010), Hanson, Kashyap and Stein (2011) and Thakor (2014) for surveys). One stream of literature stresses the role of bank capital in inducing banks’ monitoring efforts (Diamond (1984), Giammarino, Lewis, and Sappington (1993), Holmström and Tirole (1997), Allen, Carletti, and Marquez (2011), Mehran and Thakor (2011), Acharya, Mehran, and Thakor (2015)). Another strand of literature considers banks’ liquidity provision role in accepting demand deposits and runs on financial institutions as well as the role of deposit insurance in mitigating bank runs (Bryant (1980), Diamond and Dybvig (1983), Gorton and Winton (1995), Diamond and Rajan (2000), Goldstein and Pauzner (2005), Farhi, Golosov and Tsyvinski (2008), ). A third strand of the literature highlights the role of stricter capital requirements in mitigating shareholder-debt holder agency conflicts due to debt overhang and asset substitution (e.g., see the survey by Santos (2001)).

The aforementioned studies examine partial equilibrium frameworks that focus primarily on either the asset- or liability-side of banks’ balance sheets, and typically take the costs of bank equity and debt capital as exogenous. We contribute to this literature by building a unified, general equilibrium framework that endogenizes the costs of bank equity and debt, and incorporates the in-
teractions between banks’ assets and liabilities. In our model, banks are financial intermediaries that channel depositor capital to productive assets. Further, our model also incorporates non-financial firms who receive financing from banks, and also compete with banks for equity capital. Capital and liquidity requirements as well as deposit insurance balance the tradeoff between achieving optimal aggregate and idiosyncratic risk-sharing among risk-averse depositors, risk-neutral equity holders and entrepreneurs, while also ensuring that capital is invested in risky, but value-enhancing production. The competition between firms and banks for equity capital plays a central role in generating the predicted link between capital and liquidity requirements, that is, an optimal regulatory policy that features a tighter capital requirement must also have a looser liquidity requirement.

Gorton and Winton (1995, 2000) employ general equilibrium frameworks to show that, in the presence of investor demand for liquid financial claims, significantly higher bank capital requirements entail substantial social costs. The studies abstract away from liquidity requirements. Allen and Gale (2004) embed the Diamond-Dybvig (1983) model in a general equilibrium setting with aggregate shocks. They show that, when markets for aggregate risks are incomplete, there may be a role for regulation of liquidity provision. Gale and Ozgur (2005) adapt the Allen-Gale (2004) model to show that a regulatory policy that imposes a minimum capital requirement on banks may be suboptimal, while Gale (2010) shows that increasing capital requirements may not necessarily lower the level of risk in the banking system. Boyd and De Nicolò (2005) show that, when competition among banks in asset and deposit markets is considered, increased bank competition may elevate (or lower) the risk of banks’ portfolio if banks invest directly (or indirectly through a competitive loan market) in risky projects. Repullo and Suarez (2013) develop a general equilibrium model with risk-neutral agents and an exogenous excess cost of bank equity. They focus on comparing various bank capital regulation regimes, while abstracting away from other aspects of bank regulation such as liquidity requirements. DeAngelo and Stulz (2015) show that investor demand for liquidity causes bank debt to endogenously command a liquidity premium so that high bank leverage arises in equilibria even without taxes and other traditional motives for banks to choose high leverage. As in Gorton and Winton (1995, 2000), the social cost of higher capital requirements stems from a reduction in the supply of liquidity. Allen, Carletti and Marquez (2015) develop a general equilibrium model with risk-neutral agents in which the social cost of bank debt stems from exogenous bankruptcy costs, while the benefit arises due to banks’ role as financial intermediaries who channel
depositor capital to productive assets. Allen et al (2015) focus on the effects of deposit insurance and capital requirements, and only study the efficiency properties of the equilibria of their simple model without non-financial firms.\footnote{Moreover, we characterize the set of Pareto efficient allocations via a traditional social planner’s problem, who determines the investment and payoff allocations that maximize depositors’ expected utility subject to risk-neutral equity holders and entrepreneurs receiving at least their autarkic payoffs. In contrast, Allen et al (2015) consider a “hybrid” planning problem where the social planner maximizes depositors’ expected utility, but subject to the the deposit rate being determined via clearing of bank deposit markets. Hence, it is not a priori clear that the allocation they characterize if Pareto efficient.}

We complement the above studies by developing a general equilibrium framework with risk-averse depositors. The benefit of bank debt from the standpoint of depositors arises not due to bank debt being more liquid than other claims (depositors have access to a safe, liquid asset), but due to the \textit{endogenous premium} that banks are able to provide depositors by investing their capital in value-enhancing production. The cost of bank debt stems from the possibility of bank insolvency and the associated utility loss to risk-averse depositors. The tradeoff between the benefit and cost of bank debt leads to a range of autarkic equilibria with nontrivial bank capital structures even in the absence of traditional frictions that lead to nonzero bank leverage. Further, we incorporate aggregate risk and a general, \textit{concave} production technology that play central roles in driving our results. They imply an optimal bank size and level of investment in production, which generates a nontrivial tradeoff between providing insurance to risk-averse depositors that protects them against aggregate shocks, and investing in socially beneficial production. In contrast, the aforementioned studies typically assume that banks have access to linear technologies so that bank size is indeterminate, and the relative sizes of the financial and real sectors are inconsequential. More broadly, we analyze the optimal design of bank regulation in an economy with financial and non-financial firms that are exposed to aggregate risk in which all the main policy tools employed in bank regulation—deposit insurance, capital and liquidity requirements, and taxpayer-funded bank bailouts—play key roles.\footnote{There is a large and growing literature that examines the causes and consequences of financial crises due to bank failures, and their mitigation through monetary and fiscal policies. Our study is not closely related to this literature as we focus on the design of bank regulation using the traditional tools employed by banking regulators; capital and liquidity requirements, deposit insurance and taxpayer-funded bailouts.} As we discussed at the end of Section 1, the fact that banks’ central role in our framework is as financial intermediaries who channel depositor capital to productive firms implies that our model is more broadly applicable to the analysis of the traditional and shadow banking sectors.
3 The Model

Our model has two periods with a single consumption/capital good. We first consider an autarkic economy with unregulated banks, and then introduce regulation in Section 6. At date 0, there is a continuum of mass 1 of financial intermediaries (banks); a continuum of mass $F$ of entrepreneurs (firms) endowed with risky projects; a continuum of mass $D$, of risk-averse “depositors”; and a continuum of mass $E$ of risk-neutral “equity holders.” Each investor—depositor or equity holder—is endowed with one unit of capital. Banks and entrepreneurs are not endowed with any capital. All agents are protected by limited liability. We use lowercase letters to denote choices by individual agents—depositors, equity holders and entrepreneurs—and uppercase letters to denote aggregate variables.

Equity holders, depositors and banks have access to a liquid “safe asset” that is in perfectly elastic supply (or a linear storage technology), and provides a return (per unit of capital invested) $R_f$, that we normalize to 1. Depositors can invest their capital in a portfolio comprising of the safe asset and bank deposits. Equity holders can invest their capital in a portfolio comprising of the safe asset, bank equity, and equity stakes in firms.

At date 0, banks raise capital through equity and deposits. At date 1, banks invest their capital in a portfolio comprising of firms and the safe asset. Entrepreneurs have projects with decreasing returns to scale with the projects’ payoffs occurring at date 2. Capital markets—deposit and equity markets—are competitive, that is, banks and outside equity holders take the returns on equity and deposits as given in making their capital demand and supply decisions. Bank loan markets are also competitive; banks and entrepreneurs take the loan interest rate as given in making their supply and demand decisions, respectively.

3.1 Entrepreneurs/Firms

Each risk-neutral entrepreneur/firm is endowed with a single risky project that has decreasing returns to scale. An investment of $k$ units of capital at date 1 yields a payoff of $\Lambda(k)$ if the project “succeeds” with probability $q \in (0,1)$, or 0 if the project “fails” with probability $1 - q$ at date 2, where $\Lambda(.)$ is strictly increasing, concave and twice continuously differentiable. The projects of different firms are identically distributed (that is, they have the same success probability $q$), but
are not independent as we discuss shortly. We assume that $\Lambda(0) = 0$ and $\Lambda(.)$ satisfies,

$$\Lambda'(0) = \infty, \Lambda'(\infty) = 0$$  \hspace{1cm} (1)

The above Inada conditions ensure the existence of an interior solution to an entrepreneur’s demand for capital.

Entrepreneurs can either obtain capital from banks or equity holders; each entrepreneur either chooses a bank or an equity holder. In other words, the real sector of the economy obtains financing from banks, but also competes with banks for equity capital. The only feasible contract between a bank and a firm is a loan contract defined by the loan return (per unit of capital invested) $R_L$. Banks and entrepreneurs take the loan return, $R_L$, as given in making their capital supply and demand decisions, respectively. Given the payoff structure of a firm’s project, a loan contract is “isomorphic” to an equity contract in that both contracts only pay off upon the project’s success.\(^3\) Further, in a nontrivial interior equilibrium, equity holders are indifferent between supplying their capital to banks and firms, and firms are indifferent between obtaining financing from banks and equity holders. (We provide the simple sufficient condition that ensures an interior equilibrium in Section 4.) To avoid complicating the notation and exposition, therefore, we hereafter assume that the contract between a firm and an equity holder is also defined by the return $R_L$.

Given that firms are protected by limited liability, a firm’s demand for capital from either a bank or an equity holder solves

$$k = \arg \max_{k'}(\Lambda(k') - R_Lk') = \left(\Lambda'\right)^{-1}(R_L)$$  \hspace{1cm} (2)

Hence, the payoff of the firm if its project succeeds is

$$\text{Payoff to Firm if Project Succeeds} = \Lambda(k) - k\Lambda'(k),$$  \hspace{1cm} (3)

which is strictly positive because $\Lambda(0) = 0$, $\Lambda(.)$ is strictly concave and satisfies (1). Hence, for a given loan return, $R_L$, all firms demand the same amount of capital. The revenue of a bank or an equity holder

\(^3\)Alternatively, we can also assume that firms obtain financing from banks via “bank debt” and outside investors via “corporate debt” without altering any of our implications.
equity holder from a single firm if its project succeeds is

\[ \Gamma(k) = R_L k = \Lambda'(k) k \] (4)

3.2 Banks

Each bank, \( m \in [0,1] \), is financed by deposits \( (D_m) \) and equity \( (E_m) \) and it chooses how much to invest in firms \( (L_m) \) and the safe asset \( (S_m) \). A bank’s balance sheet must satisfy the accounting identity:

\[ L_m + S_m = D_m + E_m. \] (5)

Consider a bank \( m \in [0,1] \). The projects of the firms in its loan pool have the following correlation structure. With probability \( p \), which is the same for all banks, a proportion \( \omega_H \) of the projects succeed, and with probability \( 1-p \), a proportion \( 0 \leq \omega_L < \omega_H \leq 1 \) succeed. It follows from (4) that the bank’s total revenue from its loan pool is \( \omega_H L_m R_L \) with probability \( p \), and \( \omega_L L_m R_L \) with probability \( 1-p \). For expositional convenience, we hereafter refer to the two possible states of the bank’s loan pool as “success” and “failure”, respectively. Therefore, the bank’s return on its loan pool is \( \tilde{R}_L = (\omega_H R_L, \omega_L R_L) \).

The potential correlation of the projects of firms in banks’ loan pools captures the fact that banks often develop sector-specific expertise so the probability \( p \) could be viewed as a sectoral shock that differs in general from the success probability \( q \) of an individual firm. By the law of large numbers, we must have

\[ q = p \omega_H + (1-p) \omega_L. \] (6)

As we now discuss, banks could be exposed to economy-wide or aggregate risk so banks’ loan pools are not, in general, independent of each other.

3.3 Aggregate Risk

The loan pools of different banks are also correlated as follows. Specifically, a proportion \( \tau \) of banks is exposed to an aggregate shock so that all these banks’ loans/assets either succeed or fail simultaneously. The shocks faced by the remaining proportion \( 1-\tau \) of banks are idiosyncratic, that is, they are independent across banks. Since an individual bank faces a binomial shock with
success probability $p$, it follows that the loan portfolios of the proportion $\tau$ of banks exposed to the aggregate shock succeed with probability $p$ and fail with probability $1-p$.

By the law of large numbers, following a positive realization of the aggregate shock, which occurs with probability $p$, the loan portfolios of a proportion $\tau + p(1-\tau)$ of banks succeed, while those of a proportion $(1-p)(1-\tau)$ fail. Following a negative realization of the aggregate shock, which occurs with probability $1-p$, a proportion $p(1-\tau)$ succeed, while those of a proportion $\tau + (1-p)(1-\tau)$ fail. Recall that the success (failure) of a bank’s loan pool implies that a proportion $\omega_H(\omega_L)$ of the firms it lends to succeed. It follows that, with probability $p$, a proportion $\gamma_H$ of all the firms that banks lend to succeed, and a proportion $1-\gamma_H$ fail. With probability $1-p$, a proportion $\gamma_L$ of all the firms that banks lend to succeed, and a proportion $1-\gamma_L$ fail. Here,

$$\gamma_H = [\tau + p(1-\tau)]\omega_H + [(1-p)(1-\tau)]\omega_L; \quad (7)$$

$$\gamma_L = [p(1-\tau)]\omega_H + [\tau + (1-p)(1-\tau)]\omega_L; \quad (8)$$

Given that banks could, in principle, lend to all firms in the economy, we assume that the effects of the aggregate shock described above extend to all firms. In other words, with probability $p$, a proportion $\gamma_H$ of all firms in the economy succeed, and a proportion $1-\gamma_H$ fail. With probability $1-p$, a proportion $\gamma_L$ of all firms in the economy succeed, and a proportion $1-\gamma_L$ fail.

All agents in the economy observe $\tau$, but agents (including banks) do not know apriori which firms are exposed to the aggregate shock. An individual bank makes its financing and loan decisions based on its probability of success, $p$. As we discuss later, the regulator, however, internalizes the aggregate risk of the economy that is represented by the proportion $\tau$ of firms who are exposed to the aggregate shock.

### 3.4 Investors

Equity holders are risk-neutral and invest their capital in a portfolio of the safe asset, bank equity and firm equity. The bank equity market is characterized by a set $\{\tilde{R}_m^E\}; m \in [0,1]$ of equity contracts, where $\tilde{R}_m^E$ is the return on the equity contract offered by bank $m$. $\tilde{R}_m^E$ is, in general, a random variable whose realization depends on the return on the bank’s assets. Equity holders and banks take the equity returns, $\{\tilde{R}_m^E\}; m \in [0,1]$, offered by all banks as given in making their
equity capital supply and demand decisions, respectively. Because equity holders are risk-neutral, an equity holder invests all his capital in the asset—the safe asset, bank equity or firm equity—that provides the highest expected return. If multiple banks and/or firms offer the same (maximum) expected equity return, then equity holders are indifferent between them. In this case, each equity holder randomly selects a bank or firm to invest in with the choice being drawn from the uniform distribution over the set of banks and/or firms that offer the same expected equity returns.\footnote{The convexity of the payoff of an equity contract implies that it is optimal for an equity holder to invest her capital in a single bank or firm.} In equilibrium, all banks and firms offer the same expected equity returns that exceed the safe asset return so an equity holder randomly selects a single bank or firm to invest in.

Depositors have a strictly increasing, strictly concave and twice differentiable utility function \( u(.) \). Each depositor invests her capital in a portfolio comprising of the safe asset and bank deposits. The deposit market is characterized by a deposit rate, \( R_D \), which is the return (per unit of capital invested) from bank deposits \textit{provided} the bank is able to meet its deposit liabilities. Banks are protected by limited liability. If a bank is unable to fully meet its liabilities, its assets are allocated among its depositors in a pro rated manner. To accommodate the possibility of bank default, we formally characterize the deposit market by a set \( \{ \tilde{R}_D^m \} ; m \in [0, 1] \) of deposit contracts, where \( \tilde{R}_D^m \) is the return on the deposit contract offered by bank \( m \). Here, \( \tilde{R}_D^m \) is a random variable that equals the deposit rate, \( R_D \), if bank \( m \) does not default on its liabilities, and is strictly less than \( R_D \) if the bank defaults. Analogous to the equity market, depositors and banks take the deposit returns, \( \{ \tilde{R}_D^m \} ; m \in [0, 1] \), offered by all banks as given in making their deposit capital supply and demand decisions, respectively. Each depositor invests a portion of her capital in the safe asset and the remaining portion in deposits of a \textit{single} bank that provides her with the maximum expected utility. If multiple banks offer the same (maximum) expected utility to the depositor, she randomly selects a single bank to invest in with her choice being drawn from the uniform distribution over the set of banks that offer the maximum expected utility. We implicitly assume here that there are transaction costs (e.g., search costs, differences in geographic proximity, etc) that prevent a depositor from allocating her capital across multiple banks.

Each depositor chooses the proportions \( \beta \) and \( 1 - \beta \) of her capital to invest in bank deposits and the safe asset, respectively, to maximize his expected utility, that is, each depositor’s allocation
choice satisfies
\[ d(\tilde{R}_D^m) = \arg \max_{0 \leq \beta \leq 1} E \left[ u((1 - \beta) + \beta\tilde{R}_D^m) \right]. \] (9)

We make the following standing assumptions on depositors’ utility function \( u \).

**Assumption 1** (i) \( \lim_{x \to \infty} xu'(x) = \infty \). (ii) \( d(\tilde{R}_D^m) \) is increasing in \( \tilde{R}_D^m \) where \( \tilde{R}_D^m \) is viewed as a vector describing the returns on deposits contingent on the bank’s realized asset payoff.

The first assumption ensures that depositors invest all their capital in banks if the deposit rate goes to infinity. The second assumption formalizes the intuition that the amount of capital allocated to deposits increases with the deposit return.

### 3.5 Realized Returns on Deposits and Equity

Consider a bank \( m \in [0, 1] \) with deposit capital \( D_m \), equity capital \( E_m \), loan capital \( L_m \), and safe asset holdings \( S_m \). The variables must satisfy the constraint (5). The end-of-period gross payoff (before payments to depositors) is

\[
\text{Gross Payoff} = \tilde{\rho}_m \begin{cases} 
\omega_H L_m R_L + S_m & \text{if loan pool succeeds} \\
\omega_L L_m R_L + S_m & \text{if loan pool fails}
\end{cases} \] (10)

If the deposit rate is \( R_D \) (the return on deposits unless the bank defaults), the total realized payoff of all the depositors of the bank \( m \) is as follows.

\[
\text{Depositors’ Payoff} = \tilde{\xi}_m \begin{cases} 
\min(\omega_H L_m R_L + S_m, D_m R_D) & \text{if the bank’s loan pool succeeds} \\
\min(\omega_L L_m R_L + S_m, D_m R_D) & \text{if the bank’s loan pool fails}
\end{cases} \] (11)

Since the bank’s equity holders are residual claimants, their total payoff is

\[
\text{Equityholders’ Payoff} = \tilde{\rho}_m - \tilde{\xi}_m. \] (12)

The realized returns on deposits and equity are, therefore, given by

\[
\tilde{R}_D^m = \frac{\tilde{\xi}_m}{D_m}; \tilde{R}_E^m = \frac{\tilde{\rho}_m - \tilde{\xi}_m}{E_m}. \] (13)
Note that the realized return, $\tilde{R}_m^D$ on deposits is, in general, a random variable that differs from the deposit rate, $R_D$, if the bank is unable to fully meet its deposit liabilities. If the bank is able to fully pay off depositors in all states, then $\tilde{R}_m^D \equiv R_D$.

### 3.6 Equilibrium Conditions

An equilibrium of the unregulated economy is characterized by (i) a bank deposit rate, $R^*_D$; (ii) a set, $\{\tilde{R}_m^D\}; m \in [0,1]$ of deposit contracts where $\tilde{R}_m^D \leq R^*_D$ and $\tilde{R}_m^D < R^*_D$ iff the bank $m$ defaults; (iii) a set, $\{\tilde{R}_m^E\}; m \in [0,1]$ of equity contracts; and (iv) a loan rate, $R^*_L$ such that the following conditions are satisfied:

1. **Depositor Decisions:** Each depositor chooses her allocation of capital to the safe asset and bank deposits to maximize her expected utility taking the set, $\{\tilde{R}_m^D\}; m \in [0,1]$, of deposit contracts as given. If multiple banks offer the same maximum expected utility to a depositor, she randomly selects a single bank to invest in with uniform probability over the set of banks offering the maximum expected utility. The promised deposit returns, $\{\tilde{R}_m^D\}; m \in [0,1]$, coincide with the realized deposit returns defined by (11) and (13).

2. **Equity holder Decisions:** Each equity holder chooses whether to invest in the safe asset, a bank or a firm taking the returns of the set of bank equity contracts, $\{\tilde{R}_m^E\}; m \in [0,1]$, and the loan rate, $R^*_L$, as given. If multiple banks and/or firms offer the same maximum expected returns, the equity holder randomly selects a bank or firm to invest in with uniform probability over the set of banks and/or firms offering the maximum expected return. The promised bank equity returns, $\{\tilde{R}_m^E\}; m \in [0,1]$, coincide with the realized equity returns defined by (12) and (13).

3. **Bank Decisions:** Each bank chooses how much capital to raise via deposits and equity, and how much capital to allocate to the safe asset and firms to maximize its expected profit taking the set of deposit contracts, equity contracts, and the loan interest rate as given.

4. **Entrepreneur Decisions:** Each entrepreneur chooses how much capital to borrow from a bank or a firm taking the loan interest rate, $R^*_L$, as given.
5. **Limited Liability:** All agents are protected by limited liability so their payoffs must be non-negative.

6. **Market Clearing:** Equity markets, bank deposit markets and loan markets clear.

### 4 Autarky

As we show below, all equilibria are symmetric where banks raise the same amount of equity and deposit capital, and make identical investment decisions. Hereafter, without loss of generality, we focus on a *representative* depositor, equity holder, bank, and entrepreneur. We use the superscript ‘*aut*’ to denote autarky equilibrium variables. As we see shortly, depending on parameter values, autarkic equilibria can feature either default by banks when their loan portfolios fail, or no default because the values of banks’ assets are sufficient to pay off depositors in full.

#### 4.1 Types of Equilibria

The equilibria of the autarkic economy can take two different forms depending on parameter values. We describe each type of equilibrium, and derive the necessary and sufficient conditions under which it exists. To focus attention on the equilibria of interest to our analysis, we make the following assumption.

**Assumption 2**

\[ q\Lambda'(\frac{E}{F}) > 1, \quad q\Lambda'(\frac{E + D}{F}) < 1. \quad (14) \]

The first inequality in (14) states that, if all the equity capital in the economy is invested in firms, but no deposit capital is invested, the expected marginal return from production is greater than one. In this scenario, because there is a mass, \( F \), of firms, each firm obtains capital, \( \frac{E}{F} \). It then follows from (2) that the loan rate is \( \Lambda'(\frac{E}{F}) \) so that the expected loan return is \( q\Lambda'(\frac{E}{F}) \). The second inequality states that, if all the available capital in the economy—deposit and equity capital—is invested in firms, then the expected marginal return from production is less than 1. Indeed, in this scenario, each firm obtains capital, \( \frac{E + D}{F} \). It then follows from (2) that the loan rate is \( \Lambda'(\frac{E + D}{F}) \) so that the expected loan return is \( q\Lambda'(\frac{E + D}{F}) \). The two conditions together ensure the existence of “interior” equilibria where the representative bank raises deposit and equity financing; equity holders are indifferent between investing in banks and firms; and firms are indifferent between
obtaining capital from banks and equity holders.

We begin our analysis of the equilibria of the autarkic economy with the following lemma.

**Lemma 1** In any autarkic equilibrium where banks default with a positive probability, the expected returns on bank equity, deposits, and loans are equal and greater than the return on the safe asset technology (that we normalized to 1). Moreover, banks do not invest in the safe asset.

The intuition for the lemma is as follows. If depositors do not supply any capital to banks, then depositors’ capital is not invested in firms (i.e., the real economy), but only in the safe asset. Hence, the total investment in the real economy is just the total equity capital, $E$ so each firm receives an investment of $\frac{E}{F}$. In this case, the first inequality in (14) implies that the expected return on firm loans is strictly greater than one. For depositors to not supply any capital to the bank, the expected return on deposits should be at most 1. Because the expected loan return exceeds one, while the expected return on deposits is at most one, the bank can make unbounded profit by raising deposits and investing the proceeds in firms. Hence, depositors must provide nonzero capital to the bank in equilibrium.

Next, if the expected return on bank equity or deposits is less than the expected return on bank loans, the bank can make an unbounded profit by raising either deposits or equity and investing it in firms. If the expected return on bank equity is more than the expected return on firm loans, then it is optimal for equity holders to invest all of their capital in banks. However, it is unprofitable for banks to raise any equity so the equity market would not clear. If the expected return on bank deposits is greater than the expected bank loan return, it is unprofitable for banks to raise any deposit capital. Because, as argued above, depositors would like to supply nonzero capital to all banks in any equilibrium, the deposit market does not clear. Hence, the expected returns on bank deposits, equity and loans must be equal. Further, because the expected deposit return exceeds the safe asset return, 1, the expected return on bank loans also exceeds the safe asset return. Therefore, banks do not invest in the safe asset in equilibrium.

The following lemma shows that all equilibria featuring risky bank deposits are symmetric.

**Lemma 2** All autarkic equilibria in which bank’s default with positive probability are symmetric.
If the expected return on bank equity is not equal across banks, then equity holders only invest in banks with the highest expected equity return. Thus, some banks do not receive any equity capital. Lemma 1 shows that the expected deposit return is equal to the expected equity return for each bank. Hence, banks that do not raise any equity have lower expected deposit returns as well. Since these banks offer deposit contracts that are riskier and have lower expected returns, depositors do not supply capital to these banks. This contradicts the fact that depositors would like to supply capital to all banks in any equilibrium. Therefore, in all autarkic equilibria, all banks have the same expected deposit, equity and loan returns. Since the expected loan return is the same across all banks, all banks raise the same amount of total capital. Further, because the expected deposit and equity returns are identical across banks, banks have the same capital structures, that is, they raise the same amounts of equity and deposit capital.

With the same reasoning as in Lemma 1, we can show the following.

**Lemma 3** In any autarkic equilibrium where banks do not default, the expected returns on bank equity, deposits, and loans are equal to the return on the safe asset technology (that we normalized to 1). Moreover, bank deposits are safe.

If the expected return on deposits is 1, but deposits are risky, then depositors provide no capital to banks. As argued in Lemma 1, this is impossible in equilibrium since \( q \Lambda'(\frac{E}{F}) > 1 \).

### 4.2 Equity and Risky Deposits Equilibrium

In the first type of equilibrium, banks are financed with equity and risky deposits. When the representative bank’s pool of loans fails, the value of its assets is insufficient to pay off depositors in full so bank deposits are risky. Let \( \hat{R}^{aut}_{D} \equiv (R^{s,aut}_D, R^{f,aut}_D) \) denote the equilibrium random return on the representative bank’s deposits, where \( R^{s,aut}_D \) is the return when the representative bank’s assets succeed and \( R^{f,aut}_D \) is the return when the assets fail and the bank is unable to pay off depositors in full. Similarly, let \( \hat{R}^{aut}_E \equiv (R^{s,aut}_E, R^{f,aut}_E) \) be the equilibrium random return on the representative bank’s equity.

**Theorem 1 (Equity and Risky Deposits Equilibrium)** The Equity and Risky Deposits equilibrium is determined by the following conditions:
\[ N\left(\frac{X^{\text{aut}}}{F}\right) = R_L^{\text{aut}}; \quad X^{\text{aut}} = \mathcal{E} + D^{\text{aut}} \]  

(15)

\[ L^{\text{aut}} = E^{\text{aut}} + D^{\text{aut}}; \quad S^{\text{aut}} = 0 \]  

(16)

\[ E[\tilde{R}_E^{\text{aut}}] = E[\tilde{R}_D^{\text{aut}}] = q\tilde{R}_L > 1 \]  

(17)

\[ D^{\text{aut}} = Dd(\tilde{R}_D^{\text{aut}}) = Dd(R_D^{\text{aut}}, R_{D}^{l,\text{aut}}) \]  

(18)

\[ R_{D}^{l,\text{aut}} D^{\text{aut}} = \omega_L R_L^{\text{aut}} L^{\text{aut}}; \quad R_{D}^{l,\text{aut}} < R_{D}^{s,\text{aut}} \]  

(19)

\[ R_{E}^{s,\text{aut}} = \frac{\omega_H R_L^{\text{aut}} L^{\text{aut}} - R_{D}^{\text{aut}} D^{\text{aut}}}{E^{\text{aut}}}; \quad R_{E}^{l,\text{aut}} = 0 \]  

(20)

In equilibrium, the total capital, \(\mathcal{E}\), held by equity holders is supplied to banks and firms. If the total capital supplied by depositors to banks is \(D^{\text{aut}}\), it follows from the fact that banks invest all their capital in firms that the total capital invested in firms either by banks or by equity holders is \(X^{\text{aut}} = \mathcal{E} + D^{\text{aut}}\). Because each firm demands the same amount of capital for a given loan rate by (2), the total capital invested in firms is equally allocated among the mass \(F\) of firms so that each firm obtains capital \(\frac{X^{\text{aut}}}{F}\). It then follows from (2) that the equilibrium loan rate, \(R_L^{\text{aut}}\), is given by the first equation in (15). Conditions (16) represent the representative bank’s budget balance condition, and the fact that the representative bank does not invest in the safe asset in equilibrium. Equation (17) states that the expected returns on bank deposits, equity and loans are equal in equilibrium. The equalities, (18), imply that the total deposit capital supplied to banks, \(D^{\text{aut}}\), must be given by the total capital, \(D\), held by depositors multiplied by the optimal supply of capital by the representative depositor to a bank given the deposit return, \(\tilde{R}_D^{\text{aut}}\), that is given by (9). Conditions (19) and (20) express the fact that the assets of defaulted banks are distributed to their depositors, and that equity holders are residual claimants to a bank’s payoff upon success (that is, after payments to depositors).

The following proposition establishes that there is a continuum of autarkic equilibria, where each equilibrium is determined by the total capital that depositors supply to banks.

**Proposition 1** There exists an interval \([D_{\min}, D_{\max}] \subset (0, D)\), such that an equilibrium with risky deposits exists for all \(D^{\text{aut}} \in [D_{\min}, D_{\max}]\), where \(D^{\text{aut}}\) is the total capital supplied by depositors to banks. The size of the representative bank, and the total production in the economy, increases with
\(D^{aut}\), while the expected return on bank deposits, equity and loans decreases with \(D^{aut}\). Moreover, when \(D^{aut} = D_{\text{min}}\), \(E^{aut} = 0\), and when \(D^{aut} = D_{\text{max}}\), \(E^{aut} = \mathcal{E}\).

As we discussed above, the total equity capital in the economy is invested in firms either directly by equity holders or via banks. Consequently, the total capital invested in production is determined by the total capital, \(D^{aut}\), that depositors supply to banks. As we argued earlier in the discussion of the intuition for Lemma 1, depositors supply nonzero capital to banks in any equilibrium. By Assumption 2, there is also no equilibrium in which depositors supply all their capital to banks so that \(D^{aut} = D\). Indeed, in this scenario, if the capital, \(\mathcal{E} + D\), were invested in firms, the expected loan return would be less than one so that it would be optimal for banks to invest some capital in the safe asset, which contradicts Lemma 1.

If \(D^{aut}\) is the total deposit capital in any candidate equilibrium, then (15) uniquely determines the expected bank loan return, which equals the expected return on bank deposits by Lemma 1. The equilibrium condition (18) then uniquely determines the random deposit return, \(\tilde{R}_D^{aut} \equiv (R_{D,s}^{aut}, R_{D,f}^{aut})\), so the total capital raised by banks, \(L^{aut}\), is determined by (19). The candidate equilibrium is, indeed, an equilibrium iff the total capital raised by banks is (i) greater than or equal to the total deposit capital supplied to banks, but (ii) less than the total deposit capital supplied to banks plus the total available equity capital. In other words,

\[
D^{aut} + \mathcal{E} \geq L^{aut} \geq D^{aut}. \tag{21}
\]

As we show in the proof of the proposition, there exists an interval of values of the total deposit capital supplied to banks for which the above inequalities are satisfied and, therefore, correspond to an equilibrium. As the total deposit capital in equilibrium, \(D^{aut}\), increases, the total capital invested in firms, \(X^{aut} = \mathcal{E} + D^{aut}\), increases. This, in turn, lowers the expected loan return by (15) and, therefore, the expected return on bank deposits. In other words, an increase in deposit capital is associated with a lower risk premium on bank deposits so that deposits become less risky. For banks to receive greater deposit capital that is also less risky, however, they must also raise more equity capital to serve as a buffer to pay back depositors when their assets yield low returns. Hence, an increase in the equilibrium deposit capital is associated with an increase in equity capital and, therefore, a larger bank size. The equilibrium corresponding to the maximum
possible deposit capital, $D_{\text{max}}$, is the one for which the economy invests the maximum amount of capital in production. As we show in the proof of the proposition, banks raise all available equity capital in the economy in this equilibrium. That is, this equilibrium corresponds to a “full intermediation” equilibrium in which firms (the real economy) are financed entirely by banks.

We emphasize here that Proposition 1 does not imply that banks’ capital structure is irrelevant as in the Modigliani-Miller theorem. As we discussed above, bank size varies across the different equilibria, and this stems from the link between banks’ financing and investment decisions. The proposition only implies that there are multiple autarkic equilibria that feature different bank capital structures. As we show in Section 6, regulatory intervention breaks the equilibrium multiplicity. For a given set of values of regulatory parameters, the equilibrium of the regulated economy is unique.

### 4.3 Equity and Safe Deposits Equilibria

In the second type of equilibria, deposits are risk-free. In these equilibria, depositors are indifferent between the safe asset and bank deposits.

**Theorem 2 (Equity and Safe Deposits Equilibrium)** The Equity and Safe Deposits Equilibria are determined by the following conditions:

\[
\Lambda'(\frac{X^{\text{aut}}}{F}) = R^{\text{aut}}_L; \tag{22}
\]

\[
S^{\text{aut}} = E + D - X^{\text{aut}}; \quad L^{\text{aut}} = E^{\text{aut}} + D^{\text{aut}} - S^{\text{aut}} \tag{23}
\]

\[
R^{s,\text{aut}}_D = R^{f,\text{aut}}_D = E[\hat{R}^{\text{aut}}_E] = qR^{\text{aut}}_L = 1 \tag{24}
\]

\[
D^{\text{aut}} = Dd(R^{s,\text{aut}}_D, R^{s,\text{aut}}_D) = D \tag{25}
\]

\[
R^{s,\text{aut}}_E = \frac{\omega H R^{\text{aut}}_L L^{\text{aut}} - R^{s,\text{aut}}_D D^{\text{aut}}}{E^{\text{aut}}}; \quad R^{f,\text{aut}}_E = \frac{\omega L R^{\text{aut}}_L L^{\text{aut}} - R^{f,\text{aut}}_D D^{\text{aut}}}{E^{\text{aut}}} \tag{26}
\]

In these type of equilibria, depositors and equity holders are indifferent between investing in the safe asset, banks and firms. Equations (22) and (24) determine the loan rate, deposit rate and expected equity return uniquely. Note that if $\omega L R^{\text{aut}}_L L^{\text{aut}} - R^{f,\text{aut}}_D D^{\text{aut}} \geq 0$, conditions (16) and (26) become equivalent.
Proposition 2 The equity and safe deposits equilibrium exists if and only if the following condition is met:

\[ E \geq (1 - \frac{\omega}{q})X^{\text{aut}}. \]  

Here, \( X^{\text{aut}} \) satisfies \( \Lambda'(\frac{X^{\text{aut}}}{F}) = \frac{1}{q} \). Moreover, any level of total equity capital, \( E^{\text{aut}} \), held by banks in the interval, \( [(\frac{q}{\omega_L} - 1)(X^{\text{aut}} - E), E] \) corresponds to an equilibrium with safe deposits.

For the deposits of the representative bank to be risk-free, it should not default in the bad state where its loan pool fails. In this state, the total payoff from its loan pool and investment in the safe asset is \( \omega_L R_L^{\text{aut}} L^{\text{aut}} + S^{\text{aut}} = \omega_L R_L^{\text{aut}} (X^{\text{aut}} - E + E^{\text{aut}}) + D - X^{\text{aut}} + E \). The total payment to depositors in the bad state is \( D \). Hence, for deposits to be risk-free, we must have:

\[ \omega_L R_L^{\text{aut}} (X^{\text{aut}} - E + E^{\text{aut}}) + D - X^{\text{aut}} + E \geq D. \]

Because \( E^{\text{aut}} \leq E \), condition (27) ensures that there exists a bank equity capital level, which ensures that deposits are risk-free. Further, any level of total equity capital, \( [(\frac{q}{\omega_L} - 1)(X^{\text{aut}} - E), E] \) corresponds to an equilibrium that features risk-free deposits.

To focus attention on the more interesting and relevant equilibrium with risky deposits, we hereafter assume that condition (27) is violated so the only autarkic equilibria feature risky deposits as we discussed in Section 4.2.

5 Efficient Allocations

We now derive the efficient allocations in the economy by considering the problem of a hypothetical social planner. There are three inefficiencies in the autarkic economy. First, there is limited market participation as depositors cannot directly invest in productive firms, but only indirectly via banks. Second, each depositor is undiversified and invests in a single bank. Depositors are, therefore, exposed to bank-specific risk. Third, markets are incomplete because there is no traded security contingent on the aggregate shock. As a result, depositors, equity holders and banks cannot internalize the effects of the aggregate shock in creating correlation among banks’ loan portfolios when they make their investment decisions. The combination of the above inefficiencies leads to incomplete sharing of bank-specific risks among depositors as well as imperfect sharing of aggregate
risk among depositors and equity holders. Further, limited market participation by depositors as well as the incomplete internalization of aggregate risk could lead to under- or over-investment of capital in production depending on the level of aggregate risk in the economy.

The social planner invests households’ capital in entrepreneurs/firms and the safe asset. The planner then distributes the payoffs from investments among depositors, equity holders and entrepreneurs. (Banks are risk-neutral financial intermediaries and, therefore, receive no payoff in an efficient allocation.) Given that depositors are risk-averse, while all other agents are risk-neutral, we assume that the social planner’s objective is to maximize the depositors’ expected utility subject to keeping the sum of the payoffs of the other agents—the equity holders and the entrepreneurs—no less than the sum of their payoffs in autarky. As we showed in Section 4.2, there is a continuum of autarkic equilibria determined by an interval of possible values of the deposit capital raised by the representative bank. We derive the set of efficient allocations corresponding to the set of autarkic equilibria. In other words, for each autarkic equilibrium, we characterize the efficient allocation in which the expected utility of depositors is maximized subject to the sum of the payoffs of equity holders and entrepreneurs being no less than the sum of their payoffs in the autarkic equilibrium.

Accordingly, consider any autarkic equilibrium in which the representative bank has equity, debt and total capital, \((e^{aut}, d^{aut}, l^{aut})\), and the expected return on bank deposits, equity and loans is \(E[\tilde{R}^{aut}]\). (Recall that we assume that the condition (27) in Proposition 2 is violated so that the only autarkic equilibria feature risky deposits.) The total expected payoff of entrepreneurs and equity holders in this autarkic equilibrium is:

\[
\Delta = \underbrace{E[\tilde{R}^{aut}]\mathcal{E}}_{\text{expected payoff of equity holders}} + \underbrace{qF\Lambda\left(\frac{X^{aut}}{F}\right) - E[\tilde{R}^{aut}]X^{aut}}_{\text{expected payoff of entrepreneurs}}.
\]

Let us denote the set of aggregate states of the economy as \(\Omega = \{H, L\}\). As described in Section 3.3, the state \(H\) is the “positive” aggregate state where a proportion \(\gamma_H\) of all firms succeed, and \(L\) refers to the “negative” aggregate state where a proportion \(\gamma_L\) of all firms succeed, where \(\gamma_H, \gamma_L\) are defined in (7) and (8), respectively. Suppose that the social planner invests \(X_S\) and \(X_F\) in the safe asset and entrepreneurs’ projects, and distributes \(P_B^D\) to the deposit holders and \(P_E^D\) to the
equity holders and entrepreneurs in the state \( \omega \in S \). The planner’s objective is then

\[
\max\; X_F, X_S, P_D^L, P_D^H, P_E^L, P_E^H \geq 0 \quad E[u(P_D^\omega)] \text{ subject to}
\]

\[
X_S + X_F \leq D + \mathcal{E}
\]

Equation (28) is the resource constraint, which means that the total capital that the social planner invests does not exceed the total capital that is initially available. Equations (30) and (29) mean that the total capital allocated to all agents does not exceed the total available capital in the low and high states, respectively. Because a proportion \( \gamma_H \) of firms succeeds in the high state, the total revenue of all firms in the high state is \( \gamma_H \mathcal{F} \Lambda(X_F \mathcal{F}) \). Similarly, the total revenue of all firms in the low state is \( \gamma_L \mathcal{F} \Lambda(X_F \mathcal{F}) \). Equation (31) means that the total allocated capital to equity holders and entrepreneurs is no less than the sum of their payoffs in the autarkic equilibrium. Since the utility function is increasing, the social planner does not dispose off any capital so that (30) and (29) must be binding at the optimum. Therefore,

\[
P_D^H = \gamma_H \mathcal{F} \Lambda(X_F \mathcal{F}) + X_S - P_D^H
\]

\[
P_E^L = \gamma_L \mathcal{F} \Lambda(X_F \mathcal{F}) + X_S - P_D^L
\]

\[
X_S = D + \mathcal{E} - X_F
\]

Because depositors are risk-averse, it is optimal for the social planner to allocate the same level of capital to all depositors. Hence, depositors are not exposed to idiosyncratic risk. In other words, the allocation to agents only depends on the aggregate state. The planning problem reduces to the
We now fully characterize the efficient allocations.

**Theorem 3 (Efficient Allocation)** The efficient allocation is as follows.

1. **Full Insurance:** If the aggregate risk, $\tau$, is below a threshold, $\bar{\tau}$, then the depositors receive the same allocation in the low and high states, that is, they receive full insurance against aggregate risk. In this scenario the payout to depositors is given by

   \[
P_L,_{\text{eff}} D = P_H,_{\text{eff}} D = A_1
   \]

   The total capital invested in firms is determined by the following:

   \[
   \Lambda'(\frac{X_{\text{eff}} F}{F}) = \frac{1}{q},
   \]

   The expected return on firm’s investment, denoted by $E[\bar{R}_{\text{eff}}]$, is the same as the safe asset return.

   \[
   E[\bar{R}_{\text{eff}}] = [p\gamma_H + (1 - p)\gamma_L] \Lambda'(\frac{X_{\text{eff}} F}{F}) = 1.
   \] 

2. **Incomplete Insurance:** If the aggregate risk is above the threshold, $\bar{\tau}$, then depositors bear aggregate risk. In this case, the whole payout goes to depositors in the low state, since they are risk averse and other agents are risk neutral. Hence, The total investment, $X_{\text{eff}} F$, in entrepreneurial
firms, is the solutions to the following optimization program:

$$\max_{X_F} E[u(P_D^\omega)] = \max_{X_F} pu(P_D^H) + (1 - p)u(P_D^L)$$

which is determined by the following first order condition:

$$\frac{\partial}{\partial X_F}[pu((\gamma_H F \Lambda(\frac{X_F}{F}) + D + \mathcal{E} - X_F - \frac{\Delta}{p})) + (1 - p)u(\gamma_L F \Lambda(\frac{X_F}{F}) + D + \mathcal{E} - X_F)] = 0 \quad (36)$$

The expected return on firm’s investment is greater than the safe asset return.

$$E[\tilde{R}_{eff}] = [p\gamma_H + (1 - p)\gamma_L] \Lambda(\frac{X_{eff}}{F}) > 1. \quad (37)$$

The threshold, \(\bar{\tau}\), is such that the right hand-side of inequalities (32) and (33) are equal, which is equivalent to the following:

$$\bar{\tau} = \frac{\Delta}{p\mathcal{F}\Lambda(\frac{X_{eff}}{F})(\omega_H - \omega_L)}. \quad (38)$$

The social planner faces a tradeoff between providing full insurance to risk-averse depositors against aggregate shocks, and investing in production by firms. If aggregate risk is below a threshold, the wedge between the total output in the high and low aggregate states is low enough that the social planner can fully insure depositors, while providing equity holders and entrepreneurs with their autarkic payoffs. In this scenario, the social planner chooses the investment level to maximize the expected payoff from production, and invests the remaining capital in the safe asset to ensure that depositors are fully insured.

If aggregate risk is above a threshold, however, the difference between the total outputs in the low and high states is large. Consequently, it is socially costly to fully insure depositors so they must bear some aggregate risk. As equity holders and entrepreneurs are risk neutral, it is optimal for them to bear the maximum possible risk in the efficient allocation. Hence, depositors receive all available capital in the low aggregate state, while entrepreneurs and equity holders receive nothing. This allocation of capital minimizes the risk faced by depositors, which in turn maximizes their
expected utility. In this situation, total production is below the level in the full insurance scenario because investing too much capital in production imposes excessive risk on depositors.

We now show how the efficient level of investment in entrepreneurial firms, $X_{F}^{eff}$, is affected by the aggregate risk, $\tau$.

**Proposition 3 (Aggregate Risk and Production)** The efficient level of investment, $X_{F}^{eff}$, does not vary with $\tau$ for $\tau \leq \bar{\tau}$, but decreases with $\tau$ for $\tau > \bar{\tau}$.

When the aggregate risk is below the threshold, $\bar{\tau}$, the social planner simply maximizes expected production so that the investment decision is independent of the aggregate risk parameter $\tau$. If the aggregate risk is above the threshold, an increase in the aggregate risk causes the social planner to lower investment in production and invest in the safe asset to offset the effects of higher aggregate risk on depositors’ expected utility. The following corollary shows that, depending on the level of aggregate risk, the autarkic economy could feature either underinvestment or overinvestment relative to the efficient allocation.

**Corollary 1** There exists a threshold, $\tilde{\tau} \geq \bar{\tau}$ such that the autarkic economy features underinvestment relative to the efficient allocation if $\tau < \tilde{\tau}$ and overinvestment if $\tau > \tilde{\tau}$.

When aggregate risk is low, risk-averse depositors invest too much capital in the safe asset in the autarkic equilibrium relative to the efficient allocation where risk is optimally shared between depositors and the risk-neutral agents in the economy (equity holders and entrepreneurs). As a result, greater capital is invested in risky production in the efficient allocation. When aggregate risk is above a threshold, risk-neutral banks in the autarkic economy collectively invest too much capital in risky production because banks do not internalize the effects of aggregate risk in creating correlation among banks’ asset portfolios when they make their investment decisions. In this scenario, it is optimal for the social planner to invest greater capital in the safe asset relative to the autarkic economy, which implies that there is overinvestment in production in the autarkic economy.
6 Regulation

We now introduce a regulator and examine whether it can implement the efficient allocations characterized in the previous section through appropriate regulatory policies. We take a more holistic perspective in which the regulator can embody the roles of the Government through fiscal policy, and banking regulators through policies such as deposit insurance, capital and liquidity requirements.

6.1 Regulatory tools

We begin by describing the possible tools that the regulator can use that mimic tools used in the regulation of banks.

6.1.1 Taxation and deposit insurance

The regulator can tax or subsidize agents in the model: depositors, equity holders, banks and entrepreneurs. Taxes or subsidies may be contingent on the realizations of the aggregate shocks or the idiosyncratic shocks of a firm or bank. In this general perspective, deposit insurance can be viewed as a subsidy to depositors. We require the regulator to maintain a balanced budget in all states.

Without loss of generality, we consider lump-sum taxation where the regulator can collect or distribute (in the case of a subsidy) a “lump sum” amount to investors (depositors and equity holders), banks or entrepreneurs. The state of a randomly chosen bank’s loan portfolio is either success (s) or failure (f), and the aggregate state of the economy is either high (H) or (L). Therefore, the state of an arbitrary bank is (i, j) ∈ {s, f} × {H, L}. We let $\pi^{i,j}$ denote the payment per unit of deposits to depositors if the state of the bank is (i, j). If $\pi^{i,j} < 0$, it represents a payment by the depositor to the regulator; if $\pi^{i,j} > 0$, it represents a payment from the regulator to the depositor.

Similarly, a lump-sum tax (subsidy) $T_k^{i,j}$ is levied on (paid to) agent $k$—a depositor (D), equity holder (E), bank (B) or entrepreneur (EN)—if the state of the corresponding bank is (i, j).
6.1.2 Liquidity Requirement

The regulator can require banks to hold a minimum proportion of capital in the safe and liquid asset. The liquidity requirement for a representative bank takes the following form:

\[ s \geq \beta(e + d), \]

for some \( 1 \geq \beta > 0 \). Since we have the accounting identity, \( l + s = e + d \) by (5), the above inequality is equivalent to:

\[ s \geq \frac{\beta l}{1 - \beta}. \]

Under this policy banks are required to invest \( \frac{\beta l}{1 - \beta} > 0 \) in the safe asset for every unit of investment in risky firms.

6.1.3 Equity Constraint or Capital Requirement

The regulator can impose a minimum capital requirement on banks, which restricts the total amount of debt that banks can raise. Specifically, the total amount of deposit capital less the amount invested in the safe asset (cash) is required to be no greater than a proportion, \( \theta \), of capital invested in risky firms, that is,

\[ d \leq \theta l + s. \]

Combined with (5), the constraint corresponding to the minimum capital requirement for a representative bank is

\[ e \geq (1 - \theta)l. \]

6.2 Optimal Regulation

We now discuss the implementability of the efficient allocations characterized in Section 5 by a combination of the policies discussed in Section 6.1. Recall from Section 5 that there is a set of efficient allocations corresponding to the set of possible autarkic equilibria. For generality, we remain agnostic about the specific efficient allocation that the regulator wishes to implement. Our results below hold for any choice of efficient allocation.
Accordingly, consider an arbitrary efficient allocation

$$\Gamma \equiv \left\{ X_{F}^{\text{eff}}, X_{S}^{\text{eff}}, P_{D}^{L,\text{eff}}, P_{D}^{H,\text{eff}}, E[\tilde{R}^{\text{eff}}], \Delta \right\} ,$$

(43)

where $X_{F}^{\text{eff}}$ is the efficient level of investment in entrepreneurial firms; $X_{S}^{\text{eff}}$ is the efficient level of investment in the safe asset; $P_{D}^{L,\text{eff}}, P_{D}^{H,\text{eff}}$, are the total payoffs to depositors in the low and high aggregate states, respectively; $\Delta$ is the total expected payoff to equity holders and entrepreneurs; and $E[\tilde{R}^{\text{eff}}]$ is the expected marginal return on total investment in entrepreneurial firms. The following assumption ensures that the available equity capital in the economy alone is insufficient to produce at the efficient level. This ensures a nontrivial role for bank regulation in the implementation of the efficient allocation by requiring that banks must raise deposits.

**Assumption 3** The total available equity capital is insufficient to produce at the efficient level, that is,

$$\mathcal{E} \leq X_{F}^{\text{eff}}.$$  

(44)

To derive the equilibria of the regulated economy, and to examine the implementability of the efficient allocation, we proceed as in Section 4. Relative to the derivation of the autarkic equilibria, the regulatory requirements (40) and (42) appear as constraints on the decisions of the various agents: depositors, equity holders, banks and entrepreneurs.

### 6.2.1 Optimal Regulatory Policy

We now characterize the optimal regulatory policy using the set of policy tools described in the previous section: lump-sum taxation, deposit insurance, a liquidity requirement defined by (40), and a capital requirement defined by (42). We begin with the following useful lemma, which pins down the relative proportions of the representative bank’s investments in the safe and risky assets, respectively, in any regulated equilibrium.

**Lemma 4** In any regulated equilibrium, the marginal expected return on the representative bank’s assets is equal to the marginal expected cost of the representative bank’s capital, and the expected return on bank equity is equal to the expected return on loans. In other words, the following must
\[ \beta + (1 - \beta)(qR_{L}^{reg}) = \alpha E[\tilde{R}_{E}^{reg}] + (1 - \alpha)E[\tilde{R}_{D}^{reg}], \]  
(45)

\[ qR_{L}^{reg} = E[\tilde{R}_{E}^{reg}], \]  
(46)

where \( R_{L}^{reg} \) is the loan rate, \( \tilde{R}_{E}^{reg} \) is the return on bank and firm equity, \( \tilde{R}_{D}^{reg} \) is the return on bank deposits, \( \beta \) is given by equation (39) and \( \alpha = (1 - \beta)(1 - \theta) \) is the proportion of equity capital in the bank’s total capital in the regulated equilibrium.

Moreover the expected total payoff from deposits is equal to the expected total payoff from investing deposits in entrepreneurial firms less the opportunity cost of investment in the safe asset. In other words,

\[ E[\tilde{R}_{D}^{reg}]d^{reg} = qR_{L}^{reg}d^{reg} + (1 - qR_{L}^{reg})s^{reg}, \]  
(47)

where \( d^{reg} \) and \( s^{reg} \) are, respectively, the representative bank’s deposit capital and investment in the safe asset in the regulated equilibrium.

If the liquidity requirement, (40), is not binding, then the representative bank voluntarily invests nonzero capital in the safe asset. In this case, the expected return on loans to firms must equal the safe asset return, which is one. Therefore, the expected returns on bank deposits and equity must be one as well. In this case, the right-hand side and left-hand side of (45) and (46) are equal to one. If the expected loan return is greater than one, banks do not voluntarily invest in the safe asset. In this case, the liquidity constraint, (40) is binding. The L.H.S. of (45) is the expected return on the representative bank’s portfolio when it is required to invest a portion \( \beta \) in the safe asset. The R.H.S. of (45) is the weighted average of the expected deposit return and the expected equity return, which is the expected marginal cost of capital for the representative bank.

Note that, because the equity constraint, (42), may be binding in a regulated equilibrium, the expected deposit return and the expected equity return may not be equal. The intuition for equalities (45) and (46) is similar to the intuition for Lemma 1. In all regulated equilibria, equity holders should be indifferent between investment in representative banks’ equity and firms, and firms are indifferent between obtaining financing from banks and directly from equity holders. Hence, the expected returns on bank equity, firm equity and bank loans are equal. Equality (47)
then follows from the fact that the expected total payoff to the representative bank’s depositors and equity holders must equal the expected total payoff from its assets, which comprise the safe asset and loans to firms.

We now describe the implementation of the efficient allocation via regulation.

**Theorem 4 (Implementation of Efficient Allocation)** (i) There exists a continuum of policy tools all of which implement the efficient allocation. The policy tools are determined by an interval of possible values of the parameter $\beta$ that define the liquidity requirement, (40). For each $\beta$ in the interval, there exist a corresponding capital requirement, deposit insurance policy, and taxation policy that implement the efficient allocation. The policy tools are as follows:

\[ \beta^{\text{reg}} \in [\beta_0^{\text{reg}}, X_S^{\text{eff}} D], \]  

\[ \varrho^{\text{reg}} = \frac{\beta^{\text{reg}}(D - X_S^{\text{eff}})}{X_S^{\text{eff}}(1 - \beta^{\text{reg}})} \]  

For $j \in \{H, L\}$ and $i \in \{f, s\}$:

\[ \pi^{f,j} = -\frac{\omega_L}{q} E[\tilde{R}^{\text{eff}}] + \frac{X_S^{\text{eff}}}{D} \left( \frac{\omega_L}{q} E[\tilde{R}^{\text{eff}}] - 1 \right) - \frac{\omega_L E[\tilde{R}^{\text{eff}}]}{qD} \left( \frac{X_S^{\text{eff}} - \beta^{\text{reg}} D}{\beta^{\text{reg}}} \right) + \frac{P^{\text{eff}}_{i,j} D}{D}, \]  

\[ \pi^{s,j} = -\frac{\omega_H}{q} E[\tilde{R}^{\text{eff}}] + \frac{X_S^{\text{eff}}}{D} \left( \frac{\omega_H}{q} E[\tilde{R}^{\text{eff}}] - 1 \right) - \frac{\omega_H E[\tilde{R}^{\text{eff}}]}{qD} \left( \frac{X_S^{\text{reg}} - \beta^{\text{reg}} D}{\beta^{\text{reg}}} \right) + \frac{P^{\text{eff}}_{i,j} D}{D}, \]  

\[ T_E^{i,j} + T_F^{i,j} + T_B^{i,j} = -\pi^{i,j}_{D} D. \]  

(ii) For a given set of policy tools, the equilibrium of the regulated economy is unique. The returns to depositors and equity holders, the loan rate, and the total amounts of equity capital, deposit capital,
loan capital as well as investments in the safe asset in the regulated equilibrium are as follows:

\[
R_{S,reg}^D = \omega_H E[\tilde{R}_{eff}] - \frac{X_S^{eff}}{D}(\omega_H E[\tilde{R}_{eff}] - 1) + \left(\frac{\omega_H E[\tilde{R}_{eff}]}{qD} - \frac{E[\tilde{R}_{eff}]}{pD}\right)\left(\frac{X_S^{eff}}{\beta_{reg}} - \frac{\beta_{reg} D}{\beta_{reg}}\right),
\]

\[
R_{D,reg}^L = \omega_L E[\tilde{R}_{eff}] - \frac{X_S^{eff}}{D}(\omega_L E[\tilde{R}_{eff}] - 1) + \frac{\omega_L E[\tilde{R}_{eff}]}{qD}\left(\frac{X_S^{eff} - \beta_{reg} D}{\beta_{reg}}\right),
\]

\[
E[\tilde{R}_{D}] = E[\tilde{R}_{eff}] - \frac{X_S^{eff}}{D}(E[\tilde{R}_{eff}] - 1), \quad E[\tilde{R}_{E}] = qR_{L}^{reg} = E[\tilde{R}_{eff}],
\]

\[
E_{reg}^{\gamma} = \frac{X_S^{eff} - \beta_{reg} D}{\beta_{reg}}, \quad D_{reg}^{\gamma} = D, \quad S_{reg}^{\gamma} = X_S^{eff}.
\]

Here \(\beta_{0}^{reg} = \max\{\frac{X_S^{eff}}{\beta_{reg}}, \tilde{\beta}_{0}^{reg}\}\) where \(\tilde{\beta}_{0}^{reg} = \max\{\beta \leq \frac{X_S^{eff}}{D} \mid \frac{p(\omega_H - \omega_L)}{q}L_{reg} \geq E_{reg}\}\).

**Corollary 2 (Relations Among Regulatory Tools)** The policy tools described in theorem (4) satisfy the following:

1. The expected bank deposit, bank equity and firm equity returns are independent of \(\beta_{reg}\).

2. The capital requirement, \(\theta_{reg}\), the deposit insurance payment when the representative bank’s pool of loan fails, and total lump-sum taxation on equity holders, firms and banks in the high aggregate state are all increasing in \(\beta_{reg}\).

3. Total lump-sum taxation on equity holders, firms and banks in the low aggregate state, and the deposit insurance payment when the representative bank’s pool of loans succeeds are all decreasing in \(\beta_{reg}\).

As we showed in Theorem 3, an efficient allocation is determined by the total amounts of capital invested in entrepreneurial firms and the safe asset, respectively. The efficient allocation can be implemented by having depositors invest their entire capital in banks and imposing a liquidity requirement on banks, which ensures that the total investments in the safe asset and firms are consistent with the efficient allocation. Because the total capital held by equity holders is eventually invested in firms either directly or indirectly via banks, the efficient allocation can be implemented by multiple regulated equilibria that correspond to different levels of equity capital raised by banks.

By (40), the liquidity parameter, \(\beta\), determines the proportion of a bank’s total capital that must be invested in the safe asset. The right endpoint of the interval of possible values of \(\beta\) in (48)
corresponds to a regulated equilibrium in which banks are financed solely via deposits. The left endpoint corresponds to a regulated equilibrium in which banks raise all available equity capital, \( E \) or deposits are safe. Bank size, therefore, declines across the interval of regulated equilibria. The liquidity parameter, \( \beta \), ensures that banks invest the efficient amount of capital in the safe asset. Further, given a value of \( \beta \), the capital requirement, (49), ensures that banks raise the right amount of equity capital, while the deposit insurance and lump sum taxation policies guarantee that the payoffs of depositors, equity holders and entrepreneurs correspond to the efficient allocation.

As part 1 of Corollary 2 states, however, the expected bank deposit, bank equity and firm equity returns are the same in these equilibria. The equilibria only differ in the relative proportions of capital that equity holders provide to banks vis-a-vis firms. Part 2 of the corollary implies that a stricter liquidity requirement (higher \( \beta \)) is associated with a looser capital requirement, that is, a higher \( \theta \), which corresponds to a lower minimum capital requirement (see (42)). A stricter liquidity requirement implies that banks must invest a greater proportion of their total capital in the safe asset. To ensure that the efficient amount of total capital in the economy is invested in entrepreneurial firms, banks must, therefore, raise less equity. In other words, a greater proportion of the capital held by equity holders goes to entrepreneurial firms rather than banks. Because banks are smaller as the liquidity requirement becomes stricter, this also means that entrepreneurial firms are financed through a greater proportion of outside equity capital relative to bank loans.

If \( \beta \) increases, the representative bank’s portfolio becomes less risky as a greater proportion of its capital is invested in the safe asset. Consequently, less deposit insurance is required, that is, the payment to depositors when the representative bank’s pool of loan fails (succeeds) decreases (increases). Because equity holders, banks and firms share the total surplus net of payments to depositors, their total payoff decreases in the high aggregate state, and increases in the low aggregate state as \( \beta \) increases. Because the total surplus does not vary with \( \beta \) in each state (the equilibria corresponding to different \( \beta \) all implement the same efficient allocation), it follows that total lump sum taxes on equity holders, banks and firms decrease (increase) in the high (low) aggregate state.

At a broad level, Theorem 4 provides some support for both proponents and opponents of strict bank regulation. There is, in fact, a range of regulatory policies all of which implement the same efficient allocation. The regulated equilibria generated by the range of policies differ in the tightness of the liquidity constraint and the resulting size of banks. Importantly, however, the
different regulatory tools must be tuned to each other to ensure that the regulatory policy actually implements the efficient allocation. In particular, a stricter liquidity requirement is associated with a looser capital requirement and less deposit insurance.

### 6.2.2 Effects of Aggregate Risk

We now show how aggregate risk affects the optimal regulatory tools and the regulated equilibria.

**Corollary 3** If the aggregate risk, $\tau > \bar{\tau}$, where $\bar{\tau}$ is as defined in (38), then the expected return on bank equity is greater than the expected return on deposits. If $\tau \leq \bar{\tau}$, the expected returns on bank equity and deposits are equal.

If $\tau > \bar{\tau}$, then the expected marginal return on investment in firms is greater than one (see (37)). Hence, banks do not voluntarily invest in the safe asset. The liquidity requirement, however, forces banks to invest in the safe asset. It then follows from Lemma 4 that the expected return on the representative bank’s assets is less than the expected return on loans. However, the expected return on firm loans must equal the expected return on firm and bank equity. To ensure that the representative bank’s profits are nonnegative, it must then be the case that the expected cost of bank deposits (that equals the expected return on bank deposits) must be less than the expected cost of bank equity (the expected return on bank equity).

**Proposition 4** If $\tau > \bar{\tau}$, then the relationships among the policy tools introduced in Theorem 4 is unique. In other words, any set of policies that achieve the first best must be of the form described in theorem (4).

Lemma 4 implies that in any efficient regulated equilibrium the return on the representative bank’s equity is equal to the expected marginal return on production. As depositors receive a return of at least 1 in both aggregate states in the efficient allocation, depositors supply all of their capital to banks. Also, equation (47) uniquely identifies the expected deposit return. In an efficient regulated equilibrium, only banks would invest in the safe asset. Hence, a liquidity buffer policy uniquely identifies the representative bank’s equity capital. Since the total investment in production is efficient, the representative bank’s investment in firms is determined by the bank’s equity capital level. Given the representative bank’s capital and expected deposit return, the deposit rate is...
Theorem 5 If $\tau \leq \bar{\tau}$, the optimal regulatory policy is independent of $\tau$. If $\tau > \bar{\tau}$, as $\tau$ increases, the interval of possible values of the liquidity parameter, $\beta$, described in Theorem 4 shifts to right. Further, the expected bank deposit return, bank equity return and firm equity return all increase with $\tau$. Also, for each $\beta$, the corresponding capital requirement becomes tighter as $\tau$ increases. However, the aggregate risk, $\tau$, has ambiguous effects on the deposit insurance and lump sum taxation policies.

By Theorem 3, the efficient allocation does not vary with $\tau$ for $\tau \leq \bar{\tau}$. Hence, the optimal regulatory policy is unaffected by aggregate risk in this region. When $\tau > \bar{\tau}$, however, the efficient level of investment in the safe asset increases. Consequently, for a given bank capital level, the liquidity requirement is stricter. Further, for a given value of $\beta$ that can implement the efficient allocation, the total amount of capital invested in the safe asset must increase as $\tau$ increases. As a result, banks must raise more equity capital so the capital requirement becomes stricter. Because the efficient level of investment in entrepreneurial firms decreases with $\tau$, the concavity of firms’ production technology implies that the expected marginal return on investment in firms increases. As a result, the expected return on bank loans, firm equity, bank equity and bank deposits all increase.

6.3 Subset of Regulatory Tools

Theorem 4 shows that the efficient allocation can be uniquely implemented with all four regulatory policy tools; a liquidity requirement, a capital requirement, deposit insurance and lump sum taxation. It is, however, interesting to examine whether, and under what conditions, the efficient allocation can be implemented with a subset of these tools. This investigation is of interest from a practical standpoint because it may be difficult to coordinate the tools typically employed in bank regulation (capital requirements, liquidity requirements and deposit insurance) with fiscal policy through lump sum taxation.

Proposition 5 (Implementation in Full Insurance Region) If $\tau \leq \bar{\tau}$, then the efficient allocation is implementable via deposit insurance and lump sum taxation. However, a liquidity require-
ment and capital requirement ensure that the regulated equilibrium uniquely implements the efficient allocation. If \( \tau > \bar{\tau} \), all four regulatory tools are necessary to implement the efficient allocation.

If the aggregate risk, \( \tau \leq \bar{\tau} \) such that the efficient allocation fully insures depositors (see Theorem 3) then \( E[\tilde{R}^{eff}] = 1 \). Therefore, the expected return on deposits, bank equity, firm equity and bank loans are all equal to one. The deposit insurance policy ensures that depositors deposit all of their capital in banks. Banks and equity holders invest in firms such that the expected return on investment in firms is equal to one. Hence, the capital and liquidity requirements are not required to ensure that the total amount of capital invested in entrepreneurial firms is efficient. However, in the absence of capital and liquidity requirements, there may be other equilibria of the economy that are inefficient. In other words, we need capital and liquidity requirements to ensure that the equilibrium of the regulated economy is unique so that the regulatory policy uniquely implements the efficient allocation.

On the other hand if the aggregate risk is above the threshold—\( \tau > \bar{\tau} \)—the efficient allocation varies with aggregate risk. In particular, the efficient levels of investment in the safe asset and production vary. In this scenario, as we showed in proposition 4, a liquidity requirement is necessary to ensure that banks invest the right proportion of capital in the safe asset, and the corresponding capital requirement guarantees that banks raise enough equity capital so that the total amount of capital invested in the safe asset and production are efficient.

Proposition 5 shows that the efficient allocation is not implementable without the full set of regulatory policies when \( \tau > \bar{\tau} \). How much can regulation achieve with a subset of regulatory tools? In conformity with our analysis in Section 5, we derive the optimal restricted regulatory policy by maximizing the expected utility of depositors over the set of policies that only employ a subset of regulatory tools, while ensuring that the other agents receive at least their payoffs in autarky.

**Proposition 6 (Ineffectiveness of Liquidity Requirement without Capital Requirement)**

Suppose that \( \tau > \bar{\tau} \). If there is no capital requirement, then the liquidity requirement is ineffective. In other words, the optimal restricted regulatory policy in the absence of a capital requirement does not require a liquidity requirement either.

To understand the proposition, first consider a regulated equilibrium where the expected marginal return on production exceeds one. Since the representative bank invests in the safe asset, the ex-
pected loan return exceeds the expected return on the representative bank’s equity. Because the expected loan return equals the expected return on firm equity in equilibrium, equity holders strictly prefer to invest directly in firm equity rather than bank equity. The agents’ payoffs in the candidate equilibrium can be replicated by their payoffs in a modified equilibrium in which there is no liquidity requirement on banks, but depositors’ investment in the safe asset equals the total aggregate investment in the safe asset by depositors and the representative bank in the original equilibrium.

If the expected marginal return on production is one in the regulated equilibrium, then the representative bank would voluntarily invest in the safe asset so that the liquidity buffer policy is ineffective.

Similar arguments lead to the following proposition.

**Proposition 7 (Ineffectiveness of Capital Requirement without Liquidity Requirement)**

Suppose that $\tau > \overline{\tau}$. If there is no liquidity requirement, then the capital requirement policy is ineffective. In other words, the optimal restricted regulatory policy in the absence of a capital requirement does not require a liquidity requirement either.

If there is no liquidity requirement so that $\beta = 0$, Lemma 4 implies that the expected returns on deposits, loans and the representative bank’s equity coincide. Therefore, the capital requirement policy is not binding and hence ineffective.

Taken together, Propositions 6 and 7 show that the liquidity and capital requirement policies work in tandem to implement the optimal regulatory policy. Either of these tools is ineffective without the other.

7 Conclusions

We develop a general equilibrium model of competitive banks that are exposed to idiosyncratic and aggregate risk. There is a continuum of autarkic equilibria that vary from an equilibrium in which banks are financed purely with debt and the real economy is financed with bank debt and equity, to a “full intermediation” equilibrium in which banks raise all available equity capital and the real economy is financed entirely via bank debt. We characterize the efficient allocations and analyze how regulatory intervention tools can be used to implement them in a decentralized
economy. When aggregate risk is below a threshold, an efficient allocation features full insurance for depositors, and the level of investment in production does not vary with aggregate risk. When aggregate risk is above the threshold, however, there is imperfect insurance for depositors, and productive investment declines with aggregate risk.

For a given efficient allocation, there is a range of regulatory policies all of which implement the allocation, but the various tools in an optimal regulatory policy must be finely tuned to each other. In particular, capital and liquidity requirements move in opposing directions; an optimal regulatory policy that features a stricter capital requirement has a looser liquidity requirement. When aggregate risk is below a threshold, the efficient allocation can be implemented via deposit insurance and taxation alone. Capital and liquidity requirements are, however, necessary to ensure that the equilibrium of the regulated economy is unique so the efficient allocation is uniquely implemented via regulation. When aggregate risk is high, all four regulatory tools are essential components of an optimal regulatory policy. Taken together, our results provide qualified support for proponents and opponents of stricter banking regulation. Lower capital requirements for banks could be optimal, but they must be accompanied by stricter liquidity requirements and vice versa.

The basic framework we develop can be extended in several ways. We could extend the model to a dynamic setting that would allow us to address how bank regulation varies with fluctuations in the business cycle (e.g., see Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2012)). A second potential extension would be to introduce liquidity shocks among depositors and/or among banks’ assets, thereby creating the possibility of bank runs. A third possible extension would be to introduce Keynesian frictions, thereby creating a role for monetary policy. We leave the analyses of these and other extensions to future research.

Appendix

Proof of Theorem 1

First we show that in any equilibrium that banks go bankrupt, all the equations stated in the proposition are satisfied. Equation (18) is satisfied by the definition of the supply function \( d(R_1, R_2) \) as it determines the supply of deposit for given rates \( R_1 \) and \( R_2 \). Equations (19) and the equality \( R^i_{LF} = 0 \) are satisfied by the assumption that banks go bankrupt in the equilibrium. When the representative bank’s pool of loans fails, banks can not fulfill the promised rate \( R^i_{LF} \), hence after solvency all the salvage value which is \( \omega L^i_{LF} R^i_{LF} \) goes to depositors. By Lemma 1 we know
\[ E[\tilde{R}_D^{\text{aut}}] = E[\tilde{R}_E^{\text{aut}}] = qR_L^{\text{aut}} > 1, \] which is equation (17). Equalizing marginal cost and marginal revenue for banks implies equation (15). Equation (20) will follow since equity holders are residual claimants. Finally, by lemma 1, we know that \( S^{\text{aut}} = 0 \) which proves equation (16).

Now we show that given the prices and quantities, all markets clear and banks make zero profit. This shows that the proposed values in theorem (1) form an equilibrium. Equality \[ E[\tilde{R}_E^{\text{aut}}] = qR_L^{\text{aut}} \] implies that equity holders are indifferent between investing in firms and banks, therefore, equity holders would supply the proposed level of equity stated in the theorem. Equations (15) and (16) imply that the loan, equity and deposits market clear. Finally, we show banks make zero profit in the proposed equilibrium. By equations (19) and (20) we can see that the total revenue of the banks which is \( \omega_L R_L^{\text{aut}} L^{\text{aut}} \) in the low state and \( \omega_H R_L^{\text{aut}} L^{\text{aut}} \) in the high state, is exactly equal to the total payouts to depositors and equity holders in both states, so banks will make zero profit as well.

**Proof of Proposition 1**

Consider a given value \( D^{\text{aut}} \). We want to find conditions under which equilibrium exists for this given value. This is equivalent to the fact that the equations in Theorem 1 have a solution. Since \( \Lambda'(X^{\text{aut}}) = R_L^{\text{aut}} \) and \( \Lambda \) is concave, given \( D^{\text{aut}} \), we can find \( R_L^{\text{aut}} \) which is a decreasing function of \( D^{\text{aut}} \). The discussion following Proposition (1) shows that given a level of deposits \( D^{\text{aut}} \) that satisfies \( \Lambda'(D^{\text{aut}} + \mathcal{E}) > \frac{1}{q} \), the problem reduces to solving the following system of equations:

\[
E[\tilde{R}_D^{\text{aut}}] = pR_D^{\text{aut}} + (1 - p)R_L^{\text{aut}} = q\Lambda'(\frac{D^{\text{aut}} + \mathcal{E}}{\mathcal{F}}) > 1,
\]

\[
D^{\text{aut}} = Dd(\tilde{R}_D^{\text{aut}}) = Dd(R_D^{s,\text{aut}}, R_D^{f,\text{aut}}).
\]

We identify the bounds on \( D^{\text{aut}} \) such that the system above has a solution. Since depositors have a continuous utility function and are risk averse, when \( E[\tilde{R}_D^{\text{aut}}] \) is fixed, the supply of deposit is continuously decreasing in \( R_D^{s,\text{aut}} - R_D^{f,\text{aut}} \). \( R_D^{s,\text{aut}} - R_D^{f,\text{aut}} \) is minimized when \( E^{\text{aut}} = \mathcal{E} \) so banks can pay more in the case of failure. Set \( R_D^{s,\text{aut}}(D) \) and \( R_D^{f,\text{aut}}(D) \) to be the deposit rate when banks raise all available equity, raise \( D \) in deposits and the expected deposit rate is \( q\Lambda'(\frac{D + \mathcal{E}}{\mathcal{F}}) \). The upper bound on the deposits, \( D^{\text{aut}} \) is the largest solution to the following:

\[
D_{\text{max}} = Dd(R_D^{s,\text{aut}}(D_{\text{max}}), R_D^{f,\text{aut}}(D_{\text{max}})).
\]

For the lower bound on deposit, note that we have \( R_D^{f,\text{aut}} = \frac{\omega_L R_L^{\text{aut}} L^{\text{aut}}}{D^{\text{aut}}} \geq \omega_L R_L^{\text{aut}} \). The supply of deposits is minimized if the equality happen, which is equivalent to \( E^{\text{aut}} = 0 \) and \( R_D^{\text{aut}} = \omega_H R_L^{\text{aut}} \). To find \( D_{\text{min}} \) then it is enough to solve the equation

\[
D^{\text{aut}} = Dd(\tilde{R}_D^{\text{aut}}) = Dd(\omega_H R_L^{\text{aut}}, \omega_L R_L^{\text{aut}})
\]

where \( R_L \) is given by \( \Lambda'(\frac{X^{\text{aut}}}{\mathcal{F}}) = \Lambda'(\frac{\mathcal{E} + D^{\text{aut}}}{\mathcal{F}}) \).
Note that when $D^{aut}$ increases $R_{D}^{l, aut}$ also increases. Then equation (19) implies that $L^{aut}$ increase as well which completes the argument.

**Proof of Theorem 2**

The proof is similar to the the proof of theorem 1. Suppose there is an equilibrium in which banks do not go bankrupt. Equation (24) follows from Lemma (3). Equation (22) follows by equalizing marginal revenue and cost for the representative bank. Since deposit is similar to the safe asset, we can assume that depositors invest only in the bank deposits, therefore equation (25) is satisfied. Equation (26) will follows from the fact that equity holders are residual claimants and the proof is complete.

The proof that these quantities form an equilibrium follows the same steps as the proof of theorem 1. The difference is that since deposit is a safe asset, depositors may supply any deposits level. For convenience, we assume in the equilibrium $D^{aut} = D$; however, as long as depositors provide enough capital such that $D^{aut} + E \geq X^{aut}$, we can get an isomorphic equilibrium to the one stated in the theorem.

**Proof of Theorem 3**

Note that $A_2$ is decreasing in $\tau$ and when $\tau = 0$ we have $A_1 < A_2$.

First consider the case that $\tau$ is low enough such that $A_1 \leq A_2$. Note that inequality (32) implies inequalities (33) and (34). Therefore, the only binding constraint is that depositors expected payout is no more than $A_1$. Since depositors are risk averse and have expected payout $A_1$, their utility will be maximized when we maximize (expected payout) $A_1$ and allocate to them the same amount ($A_1$) in both low and high states. The problem reduces to maximizing $A_1$. The first order condition for $X_F$ in the equation for $A_1$ is:

$$q \Lambda'(\frac{X_F}{F}) = 1.$$ 

This implies that $\Lambda'(\frac{X_F^{eff}}{F}) = \frac{1}{q}$, as we claimed.

Now assume that $A_1 > A_2$. Since $A_1 \leq pA_3 + (1-p)A_2$, inequalities (32) and (33) are binding. In order to maximize the depositor’s expected utility, the risk averse depositors should receive all possible cash flow in the low state. This would minimize the difference of the payouts between the low and high states. Risk neutral agents will be compensated in the high state of the economy. In particular,

$$P_H^D = \gamma_H F \Lambda(\frac{X_F}{F}) + D + \varepsilon - X_F - \frac{\Delta}{p},$$

$$P_L^D = \gamma_L F \Lambda(\frac{X_F}{F}) + D + \varepsilon - X_F.$$ 

As we can see FOC for the central planner’s problem is exactly equation (36). Inequality (37)
will follow from proposition 3 which we proved below.

Finally as we can see in the proof, \( \bar{\tau} \) is obtained by equalizing the right-hand sides of the equations for \( A_1 \) and \( A_2 \), implying equation (38).

**Proof of Proposition 3**

For \( \tau < \bar{\tau} \), as stated in equation (35), \( X_{eff}^F \) is constant. For \( \tau > \bar{\tau} \), in order to prove \( X_{eff}^F \) is decreasing, it is enough to show the cross derivative of the objective function is negative. The cross derivative is:

\[
\frac{\partial^2}{\partial X_F \partial \tau} [p u(P^H_D) + (1 - p) u(P^L_D)] = \\
p(1 - p) (\omega_H - \omega_L) \Lambda'(\frac{X_F}{F})(u'(P^H_D) - u'(P^L_D)) + \\
p(1 - p) (\omega_H - \omega_L) \mathcal{F} \Lambda(\frac{X_F}{F})[(\gamma_H \Lambda'(\frac{X_F}{F}) - 1)u''(P^H_D) - (\gamma_L \Lambda'(\frac{X_F}{F}) - 1)u''(P^L_D)] < 0
\]

Concavity of the utility function \( u \) implies that the first term is negative. It is enough to show the last term is negative as well. It suffices to show the following:

\[
(\gamma_H \Lambda'(\frac{X_F}{F}) - 1)u''(P^H_D) - (\gamma_L \Lambda'(\frac{X_F}{F}) - 1)u''(P^L_D) < 0
\]

Since \( u'' < 0 \), it is enough to show

\[
A^H = \gamma_H \Lambda'(\frac{X_F}{F}) - 1 \geq 0 \\
A^L = \gamma_L \Lambda'(\frac{X_F}{F}) - 1 \leq 0
\]

The first order condition for \( X_F \) in the central planner problem is (equation (36)):

\[
pA^H u'(P^H_D) + (1 - p)A^L u'(P^L_D) = 0.
\]

Since \( u'(P^H_D), u'(P^L_D) > 0 \) , \( A^H \) and \( A^L \) must have opposite signs. Note that:

\[
A^H - A^L = (\gamma_H - \gamma_L) \Lambda'(\frac{X_F}{F}) > 0.
\]

Hence \( A^H > 0 > A^L \). This completes the argument.

**Proof of Theorem 4**

Similar to the proof of theorem 1, we show that the proposed values constitute an equilibrium. More precisely, given the prices and policy tools, we show agents choose quantities as stated in the
theorem, markets clear and banks make zero profit. In the first step we compute $R_{D}^{f,\text{reg}}$. When a bank’s pool of loan fails, total cash flow is:

$$\omega_{L}R_{L}^{\text{reg}}L + X_{S}^{\text{eff}}.$$  

By our assumption on $\beta$ we know that the cash flow in the low state is not enough to pay depositors in full, so we have:

$$\mathcal{D}R_{D}^{f,\text{reg}} = \omega_{L}R_{L}^{\text{reg}}L + X_{S}^{\text{eff}}$$

which gives us the equation for $R_{D}^{f,\text{reg}}$ as in the theorem. Now we show that the supply of deposit is $\mathcal{D}$. Since $P_{D}^{H,\text{eff}} \geq P_{D}^{L,\text{eff}} \geq \mathcal{D}$ deposit is at least as good as the safe asset in every state, hence depositors invest all of their capital in the banks. To calculate $E[\tilde{R}_{D}^{\text{reg}}]$, we have:

$$E[\tilde{R}_{D}^{\text{reg}}] = pR_{D}^{s,\text{reg}} + (1-p)R_{D}^{f,\text{reg}} = E[R_{D}^{\text{eff}}] - \frac{X^{\text{eff}}}{\mathcal{D}}(E[R_{D}^{\text{eff}}] - 1).$$

Since this is less than (proposed) value for $E[\tilde{R}_{D}^{\text{reg}}] = E[R_{D}^{\text{eff}}]$, banks prefer deposits to equity and hence they attract all the available deposit before financing with equity. This shows the deposit market clears. To prove that equity market clears, we have to demonstrate: $E[\tilde{R}_{E}^{\text{reg}}] = qR_{L}^{\text{reg}}$. This relation shows that equity holders are indifferent between investing in bank’s or firm’s equity. Note that in the low state, equity holders receive nothing and in the high state we have:

$$R_{E}^{\text{reg}}E + R_{D}^{s,\text{reg}}\mathcal{D} = \omega_{H}R_{H}^{\text{reg}}L + X_{S}^{\text{eff}}$$

where we equalize the revenue of banks with the payout to depositors and equity holders. Therefore,

$$R_{E}^{\text{reg}} = \frac{1}{E_{\text{reg}}}[\omega_{H}R_{H}^{\text{reg}}L + X_{S}^{\text{eff}} - R_{D}^{s,\text{reg}}\mathcal{D}]$$

If we set $E_{\text{reg}} = \frac{X_{S}^{\text{eff}} - \beta^{\text{reg}}\mathcal{D}}{\beta^{\text{reg}}} \text{ and } L_{\text{reg}} = (1 - \beta^{\text{reg}})(E_{\text{reg}} + \mathcal{D})$ we get $pR_{E}^{\text{reg}} = qR_{L}^{\text{reg}}$. Parameter $\theta^{\text{reg}}$ is defined using the (binding) relation (41) which is in this case

$$\mathcal{D} = \theta^{\text{reg}} L_{\text{reg}} + X_{S}^{\text{eff}}.$$  

This gives us $\theta^{\text{reg}} = \frac{\beta^{\text{reg}}(\mathcal{D} - X_{S}^{\text{eff}})}{X_{S}^{\text{eff}}(1 - \beta^{\text{reg}})}$. Since equity holders are indifferent between investing in firms and banks, we can assume they invest in the banks as much as banks demand in the equilibrium. In order to show banks demand $E_{\text{reg}} = \frac{X_{S}^{\text{eff}} - \beta^{\text{reg}}\mathcal{D}}{\beta^{\text{reg}}}$ in the equity financing, we have to verify equation (45) which is
\[
\beta^{reg} + (1 - \beta^{reg} - \alpha^{reg})qR_{L}^{reg} = (1 - \alpha^{reg})E[\tilde{R}_{D}^{reg}]
\]

where \(\alpha^{reg} = (1 - \theta^{reg})(1 - \beta^{reg})\). Substituting \(E[\tilde{R}_{D}^{reg}]\) from above we should show

\[
\beta^{reg} = (1 - \alpha^{reg})\frac{X_{S}^{eff}}{D}
\]

which is equivalent to

\[
\frac{\beta^{reg}(D - X_{S}^{eff})}{X_{S}^{eff}} = \theta^{reg}(1 - \beta^{reg})
\]

which holds by definition of \(\theta^{reg}\). Finally, firms demand exactly \(X_{F}^{eff}\) given \(R_{L}^{eff}\) because

\[
R_{L}^{eff} = \frac{E[R_{D}^{eff}q]}{q} = \lambda'(\frac{X_{F}^{eff}}{F})
\]

This completes the proof because of equation (2).

**Proof of Corollary 2**

To show \(\theta^{reg}\) is increasing in \(\beta^{reg}\), we have:

\[
\frac{\partial \theta}{\partial \beta} = \frac{D - X_{S}^{eff}}{X_{S}^{eff}}(1 - \beta)^{-2}
\]

which is positive since \(D \geq X_{S}^{eff}\) by the assumption \(E \leq X_{F}^{eff}\). For deposit insurance when bank’s pool of loan fails, we have

\[
\frac{\partial \pi_{f,j}}{\partial \beta} = C\beta^{-2}
\]

which is positive since \(C = \omega_{L}E[R_{D}^{eff}X_{S}^{eff}qD] > 0\). To show that the total lump-sum taxation in high state is increasing in \(\beta^{reg}\), it is enough to show that \(R_{D}^{reg}\) is increasing in \(\beta^{reg}\), which is equivalent to showing that

\[
\left(\frac{\omega_{H}E[\tilde{R}_{D}^{eff}]}{qD} - \frac{E[\tilde{R}_{D}^{eff}]}{pD}\right) < 0
\]
Since $p\omega_H < p\omega_H + (1 - p)\omega_L = q$, the above inequality holds. The last two statements of the corollary follows from these relations and the fact that expected payout to deposit holders is fixed in $\beta^{reg}$. 
References


