Participation and unbiased pricing in CDS settlement mechanisms

Ahmad Peivandi †

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Abstract

The centralized market for the settlement of credit default swaps (CDS), which governs more than $10 trillion’s worth of outstanding CDS contracts, has been criticized for mispricing the defaulted bonds that underlie the contracts. I take a mechanism design approach to the settlement of CDS contracts. I fully characterize robust settlement mechanisms that deliver unbiased prices for the underlying assets and show that all robust settlement mechanisms are ”payoff equivalent” to a ”posted price” mechanism. I exploit my analysis to propose a modification to the existing CDS market and the settlement procedure to improve the efficiency of the mechanism. Because forced participation in the settlement mechanism is not possible, my approach requires the development of a new notion of the core of games of incomplete information. This new notion can be applied to mechanism design environments in which side trades are allowed or when joining the mechanism is a cooperative decision.

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‡Georgia State University, Robinson College of Business. Email: apeivandi@gsu.edu.
1 Introduction

A credit default swap (CDS) is an insurance contract against the risk of default.⁴ After the default of a bond issuer, the corresponding CDS contracts are settled through a centralized mechanism. This mechanism produces a price for the defaulted bond that is used to measure the amount of loss due to the default. The current mechanism in use underprices the asset in most cases and is also sensitive to the number of CDS positions, which results in efficiency losses due to underinsurance (Gupta and Sundaram (2013) and Chernove et al. (2013)). I show that all robust mechanisms that deliver unbiased prices in expectation and all robust mechanisms that are not sensitive to the agent’s number of positions, are pay-off equivalent to a posted price mechanism. Since a posted price mechanism is impossible or very difficult to implement, the results of the paper show that there is no practical auction procedure that would deliver unbiased prices. I exploit my analysis to propose a modification to the existing CDS market and the settlement procedure to improve the efficiency of the mechanism. I show the proposed mechanism delivers partial participation and unbiased pricing. In addition, I model the participation of players in this mechanism and, thereby, develop a novel notion of the core of games of incomplete information. The new notion is necessitated by the fact that the decision to join the blocking mechanism precedes participation in the mechanism. Therefore, this notion can be used to model participation to centralized markets in which side trades or side matches prior to, or concurrent with, the centralized mechanism are allowed. Examples include dark pools⁵ and some centralized job markets, such as the National Residency Matching Program (NRMP).

Each CDS contract corresponds to a reference entity’s bond. The corresponding CDS contracts can be settled via physical settlement or cash settlement. In the case of cash settlement, the protection seller pays the face value minus the value of the defaulted bond to the protection buyer. In the case of physical settlement, the protection buyer hands the defaulted bond to the protection seller and receives the face value of the bond. Although physical settlement has the advantages of not requiring a price and the protection buyer’s full insurance, it is often impossible to physically settle all contracts.

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⁴More broadly, a CDS is an insurance contract against the risk of a credit event.
⁵Dark pools are private platforms to trade securities.
First, in most cases, the number of outstanding CDS contracts is more than the number of bonds. Second, even if protection buyers could purchase the defaulted bonds for physical settlement, doing so would artificially raise the price of the defaulted bond.

For these reasons, an alternative way of settling the contracts by cash transfer has emerged. The challenge for cash settlement is to identify a value for the defaulted bond. To determine the quantity of contracts to be settled physically or by cash transfer, as well as a price for cash settlement, the International Swaps and Derivatives Association (ISDA) introduced a two-stage mechanism. In the first stage, only agents with CDS contracts participate. In this stage, the mechanism determines the number of defaulted bonds to be bought or sold in the second stage of the mechanism and a price cap or floor. In the second stage, a uniform price auction determines a price for the defaulted bond. As of 2009, all CDS contracts are pegged to the value of the defaulted bond determined by this mechanism, unless both the protection buyer and protection seller choose to opt out. In addition, agents may sell their CDS contracts to other agents prior to participation in the settlement mechanism. The mechanism used by the ISDA has been the subject of criticism. Chernove et al. (2013) have observed that the defaulted bonds in this mechanism are underpriced by an average of 6%. Due to the winner’s curse in the second stage of the current mechanism, mispricing is inevitable. This mispricing implies that the protection buyer cannot fully insure against the risk of default by the issuer of the bond and that there is uncertainty in regard to the future payoff of a defaulted bond. Also, the outcome of the mechanism is dependent form the number of bonds that are traded at the second stage of auction, Chernove et al. (2013). This creates extra uncertainty for CDS traders, since when agents trade CDSs the number of physical bonds and the total number of CDSs at the time of CDS settlement are unknown.

The goal of any design should be to settle contracts with unbiased prices. A set-

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3 As stated in Summe and Mengle (2011), at the time of Delphi Corporation’s bankruptcy, it was estimated that there were $28 billion in CDSs outstanding but only $2 billion in defaulted bonds. If short selling were facilitated in this market, in a physical settlement, the protection buyer would short sell the defaulted bond rather than hand it to the protection seller. Because defaulted bonds are traded over the counter, short selling the defaulted bonds is difficult or even impossible.

4 The ISDA argues that requiring all parties to a CDS to be bound by the results of the mechanism ensures certainty, consistency, enhanced transparency, and liquidity; see http://www.isda.org/press/press031209.html
tlement mechanism is unbiased if the cash settlement price is equal to the value of the defaulted bond or if an agent’s payoff is equal to the payoff from physical settlement of all contracts. I take a mechanism design approach and look for a settlement mechanism that is unbiased. Moreover, the mechanism must satisfy three important properties:

1. Ex-post incentive compatibility: The mechanism is incentive compatible for all possible agents’ beliefs.

2. Weak budget balance: The designer does not have to incur a cost to execute the settlement mechanism.

3. Robustness with respect to agents’ participation decisions: The settlement delivers unbiased prices regardless of the agent’s participation choice.

The revelation principle implies that, without loss of generality, one can restrict attention to direct settlement mechanisms that are incentive compatible (Myerson (1981)). The first property is a robustness property against agents’ beliefs (see Bergemann and Morris (2005)). Because there is cash transfer in the mechanism, the second property is also standard.

The third property is also important, as agents cannot be compelled to participate in the ISDA settlement mechanism. If both parties of a CDS contract agree, they can choose to settle some of their contracts outside of the settlement mechanism. Also, an agent may sell some of his contracts to another agent prior to participating in the settlement clearinghouse. I define “participation choice” to describe the agents’ decision about how they participate in the central mechanism as well as how they settle contracts outside of the mechanism. Participation choices should satisfy two important criteria. First, when a pair of agents make a decision about their participation, it should benefit both of them. Second, these participation decisions should be self-confirming. This means that, when a pair of agents engage in side settlement or CDS trade, each agent update his belief about other agent’s type. Given the updated beliefs, agents choose to exit when there is a benefit to doing so. The robustness property noted above requires the mechanism to be unbiased in all possible agent participation choices. The main theoretical innovation of this paper is to model how agents choose their level of participation in the mechanism.
A mechanism that sets the cash settlement price equal to the expected value of the defaulted bond, conditional on the designer information, and sets a constant cash settlement quantity is called a posted price mechanism. I show that all mechanisms with properties 1–3 listed above are “payoff equivalent” to a posted price mechanism. The difficulty in designing a settlement mechanism, when participation is voluntary, is that agents may manipulate the outcome of the mechanism through participation. Because the designer faces the budget constraint, he cannot pay the agents to participate in the mechanism. When a posted price mechanism is employed, agents can no longer manipulate the settlement procedure through strategic choice of participation.

As discussed, participation in this mechanism is voluntary because CDS contracts are bilateral between pairs of agents, no settlement mechanism can enforce participation. Choosing not to fully participate harms the transparency of the ISDA settlement procedure and drives liquidity away from the auction. This is important because liquidity and transparency were among the main reasons offered for a central mechanism to settle CDSs in the first place. I show that all unbiased mechanisms that induce full participation of all agents with all of their contracts are payoff equivalent to a posted price mechanism.

The current mechanism in use and the posted price mechanism are robust with respect to the network, in the sense that they treat two networks with the same number of contract in the same way. A mechanism is robust with respect to the network if the price and quantities are the same in two networks, for which agents have a net equal number of contracts. The rationale for this property is that it lowers the systemic risk and transaction costs.

I extend the characterization result in two ways. First, I consider the case in which agents cannot sell their CDS contracts to other agents prior to the settlement mechanism. However, a pair of CDS traders can choose their level of participation by settling some of their contracts outside of the mechanism. Given the same model of participation as above, I again show that the only mechanism that satisfies ex-post incentive compatibility and robustness with respect to agents’ decisions about participation is a posted price mechanism. Second, I generalize the notion of unbiasedness. I consider

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5 Some CDS contracts are between agents in different countries, which makes enforcing participation even more difficult.
settlement mechanisms for which, from an ex-ante point of view, the payoff of each 
agent is equal to his payoff from cash settlement with some known price. This prop-
erty ensures that the ex-ante payoff of agents is proportional to their net number of 
contracts. This is relevant here because CDS contracts are homogeneous. I show that 
the characterization results hold if one replaces unbiasedness with this property.

Taken together, my results show that the posed price mechanism is the only robust 
settlement mechanism that delivers a price that is not sensitive to the number of posi-
tions. Therefore, there is no auction procedure that would achieve the efficient outcome 
of unbiased pricing. I exploit my analysis to propose a modification to the existing 
CDS market and the settlement procedure to improve the efficiency of the mechanism. 
At the time of contracting, the buyer and the seller of CDS decide whether the CDS 
contract is covered or naked. The holder of a covered CDS is required to submit the 
underlying defaulted bonds to the settlement mechanism in the first round of the settle-
ment procedure. If this does not occur, the buyer does not receive the face value of the 
bond from the CDS seller. With this modification, I show all covered CDSs are settled 
physically in the clearinghouse. Covered CDS buyers buy CDSs to hedge against the 
risk of default of the underlying bond. Therefore, biased pricing of the CDSs does 
less harm to naked CDS compared to covered CDS buyers who buy the CDS contract 
to hedge against the risk of default. I show that with this modification, covered CDS 
traders fully participate in the settlement mechanism.

2 Related literature

My study contributes more broadly to the market design literature and, more specifi-
cally, to the nascent literature on the valuation and settlement of CDS contracts. Studies 
that examine the CDS contract settlement mechanism focus exclusively on the proper-
ties and modifications of the current mechanism in use. Gupta and Sundaram (2013) 
observe that there is a price bias for auctions held in the 2008–2012 period. Simi-
larly, Helwege et al. (2009) compare the mechanism price to the pre- and post-auction 
prices of the defaulted bond in a sample of ten early auctions and find no mispricing 
in their sample. Coudert and Gex (2010) study the settlement procedure for a number 
of cases. Their empirical study also reveals a price bias in the auction. Du and Zhu
(2015) develop a theoretical model to explain why the current auction misprices the defaulted bond and propose a double auction to achieve efficiency. They consider the case in which a continuum of agents could have different valuations for the defaulted bond. Therefore, in their model, allocative efficiency becomes relevant. Chernove et al. (2013) document the same price bias as do Gupta and Sundaram (2013). Taking into account multiple financial frictions in the market, they solve for equilibria of the two-stage auction, assuming that agents have no private information about the value of the defaulted bond. Assuming no private information about the value of the defaulted bond, they use a CDS auction to discover a price of the defaulted bond.

Motivated by the observation that the number of participants in CDS auctions rarely exceeds 15, I consider a framework in which there is a small number of participants with private signals about the common value of the defaulted bond. Thus, strategic behavior plays a more crucial role in my analysis. More importantly, my paper is the only one (to the best of my knowledge) to take a mechanism design approach to analyze the CDS market. I complement the existing literature by characterizing the settlement mechanisms that satisfy the key robustness properties described earlier.

I also contribute to the broader literature in game theory by developing a new notion of the core of a game of incomplete information. The notion of "unravel-proofness" under incomplete information can be interpreted as a stability condition. The set of mechanisms that satisfy the property can be interpreted as the core of the underlying game of incomplete information. The notions of core and stability have been generalized to games of incomplete information in Wilson (1978), Dutta and Vohra (2005), Myerson (2007), Serrano and Vohra (2007), Yenmez (2013), Liu et al. (2014), and Pomatto (2015). The notions differ across the above studies in the way that agents communicate their private signals. Wilson (1978) considers two extreme cases: (i) all agents in a block share their private information completely (fine core) and (ii) agents share no private information. Dutta and Vohra (2005) and Myerson (2007) consider the blocks for which the decision to join the block comes from a Bayesian Nash Equilibrium. Liu et al. (2014) study the implications of common knowledge of stability of a two-sided match when one side of the market has incomplete information about the other side.

6See http://www.creditfixings.com/CreditEventAuctions/fixings.jsp
In my notion, a block exists if the exit game has an equilibrium in which a subset of agents with a positive measure subset of types participates in the blocking mechanism. The exit game that I describe resembles the voting game in Holmström and Myerson (1983). In their setup, all agents participate in the voting game, and they examine whether a mechanism can be "Pareto improved" through reallocation by an unanimously elected alternative mechanism.

My notion of "unravel-proofness" differs in two important ways from notions of the core introduced in the literature. First, in the prior notions of the core, the decision to join the blocking mechanism comes after the realization of the grand mechanism’s allocation. Therefore, the blocking designer or members of the blocking coalition take the allocation of the grand mechanism as exogenous. Because agents have quasilinear preferences and heterogeneous beliefs about a common value in my framework, the no-trade theorem implies that no subset of agents should agree to a reallocation once the contracts are settled (see Milgrom and Stokey (1982)). Therefore, if one applies the notion of stability in the literature to the environment considered in this paper, all settlements would be stable and durable, as defined in Holmström and Myerson (1983). In my notion of the block, agents simultaneously choose whether they want to participate in the blocking mechanism. The second main difference between my notion of the core and the notions in prior literature is that, in my setup, the blocking mechanism and the settlement mechanism can coexist. This is because an agent may choose to partially participate in the settlement mechanism. None of the models of stability in the literature accommodates this possibility.

My paper is also related to a body of literature that studies the incentives of agents to participate in some centralized clearinghouses. Ashlagi and Roth (2014) study the incentives of hospitals to partially enroll their patient-donor pairs in the kidney exchange program. Ekmekci and Yenmez (2015) study the incentives of schools to participate in the centralized school choice clearinghouse. Sönmez and Ünver (2015) propose an incentive scheme in the kidney exchange program. There are two key differences between my framework and the frameworks analyzed by the above studies. First, unlike kidney exchange, monetary transfer is possible in the CDS settlement clearinghouse, and, second, participation in the CDS settlement mechanism is the decision of (at least) two agents.
3 Leading Example

I illustrate how the CDS contracts work and the main theoretical contribution of the paper with the following example. The reader may omit reading this example and start from Section 4.

**Leading Example:** There are three agents, 1, 2, and 3. There is a bond with a face value of 100. Assume that the issuer of the bond has defaulted and that the value of the defaulted bond is $v(s)$, where $s = (s_1, s_2, s_3)$ is the agents’ signal profile. Agent 1 is a protection seller, and Agents 2 and 3 are protection buyers. Agents 2 and 3 may each have 10 CDS contracts with the protection seller. These homogeneous CDS contracts are on the bond. There are three possible cases (see Figure 1):

1. Agents 2 and 3 have 10 CDS contracts with Agent 1.
2. Agent 2 has 10 CDS contracts with Agent 1, and Agent 3 has no CDS contracts.
3. Agent 3 has 10 CDS contracts with Agent 1, and Agent 2 has no CDS contracts.

Denote the number of CDS contracts that agent $i$ has in case $j$ by $n^j_i$. Assume $n^j_i > 0$ if agent $i$ is a protection buyer in case $j$, $n^j_i < 0$ if he is a protection seller, and $n^j_i = 0$ if he does not have any CDS contracts. For example, $n^1_1 = -20$ and $n^1_2 = n^1_3 = 10$.

These contracts are settled by either physical settlement or cash settlement. In the case of physical settlement, the protection buyer hands the defaulted bond to the
protection seller and, in return, receives 100. Therefore, the protection buyer’s payoff from the physical settlement of one contract is \(100 - v(s)\), and the protection seller’s payoff from the physical settlement is \(-(100 - v(s))\). In the case of cash settlement, the protection seller pays the loss to the protection buyer(s) in the form of monetary transfer. Therefore, if \(p\) is a price for the defaulted bond, the protection seller pays \(100 - p\) to the protection buyer to settle one CDS contract. If \(q^j_i\) is the number of agent \(i\)'s contracts that are settled through cash settlement, and \(p^j_i\) is the cash settlement price, agent \(i\)'s payoff is as follows:

\[
u_i((n^j_i - q^j_i)(100 - v(s)) + q^j_i(100 - p^j_i)).\]

where \(u_i : \mathbb{R} \rightarrow \mathbb{R}\) is agent \(i\)'s utility function. Agents’ signals about the value of the defaulted bond is either 0 or 1, \(s_i \in \{0, 1\}\) for \(i \in \{1, 2, 3\}\). Signals are independently distributed, and \(s_i = 1\) with probability \(\frac{1}{2}\). The value of the defaulted bond conditional on signals is as follows:

\[v(s) = 21(2s_1 + s_2 + s_3).\]

Agents 1 and 2 each possess nine defaulted bonds. Therefore, some of the contracts must be settled through cash settlement.

I describe a direct settlement mechanism. A description of a mechanism is a price and a quantity function for each agent in each network. The quantity is the number of CDS contracts that are settled by cash settlement, and the price is the cash settlement price. Let \(q^j_i\) and \(p^j_i\) denote the quantity of cash settlement and cash settlement price, respectively, for agent \(i\) in network \(j\). Consider the following settlement mechanism:
\[ q_1(s) = -6 + 4s_1 - s_2 - s_3, \]
\[ p_1(s) = 28 - 28s_1 + 8s_2 + 8s_3 + 20s_1s_2 + 20s_1s_3 - \frac{13}{4}s_2s_3 - \frac{27}{4}s_1s_2s_3, \]
\[ q_2(s) = 3 - 2s_1 + s_2, \]
\[ p_2(s) = 28 - 28s_1 + \frac{7}{4}s_2 + \frac{133}{4}s_1s_2 + 21s_3, \]
\[ q_3(s) = 3 - 2s_1 + s_3, \]
\[ p_3(s) = 28 - 28s_1 - \frac{7}{4}s_3 + \frac{133}{4}s_1s_3 + 21s_2, \]
\[ q_4(s) = -4, \]
\[ q_5(s) = -3.5, \]
\[ q_6(s) = 3.5, \]
\[ q_7(s) = 42, \]
\[ q_8(s) = 42. \]

These prices and quantities guarantee ex-post incentive compatibility. Moreover, the following holds:

\[ \forall s \in \{0, 1\}^3 \quad \forall j \in \{1, 2, 3\} : \sum_{i=1}^{3} q_j(s) = 0, \]  \hfill (1)

\[ \forall s \in \{0, 1\}^3 \quad \forall j \in \{1, 2, 3\} : \sum_{j=1}^{3} q_j(s)(100 - p_j(s)) = 0. \]  \hfill (2)

Equation (1) is a market-clearing condition. Note that \( q_j(100 - p_j) \) is the cash transfer that agent \( i \) receives in network \( j \); therefore, equation (2) is the budget-balanced condition. In addition to these properties, if \( U_j(s) \) is agent \( i \)'s payoff from the settlement mechanism in case \( j \), the following holds:

\[ E_s[q_j'(s)(100 - p_j'(s))] = E_s[u_j'(100 - v(s))]. \]

This condition is called unbiased pricing. This means that, from an ex-ante point of view, that all contracts are settled by physical settlement or by cash settlement, with the price equal to the value of the defaulted bond.

I study agents’ incentives to participate in the settlement mechanism when an arbitrary group of agents can form coalitions and settle some of their contracts with an arbitrary blocking mechanism. As an illustration, consider the settlement mechanism that I described above. I consider a block by Agents 1 and 3 (see Figure 2). In this
blocking mechanism, seven contracts are settled by physical settlement, and three contracts are settled by cash settlement. The cash settlement prices for Agents 1 and 3 are 30 and 38.5, respectively. The following inequalities hold (see the appendix for the calculations.):

\[
E_{s_2}[U_1^1(0, s_2, 0)] \leq E_{s_2}[U_1^e(0, s_2, 0)], \quad E_{s_2}[U_1^1(1, s_2, 0)] \geq E_{s_3}[U_1^e(1, s_2, 0)],
\]

\[
E_{s_2}[U_3^1(0, s_2, 0)] \leq E_{s_2}[U_3^e(0, s_2, 0)], \quad E_{s_2}[U_3^1(0, s_2, 1)] \geq E_{s_2}[U_3^e(0, s_2, 1)].
\]  

(3)

Therefore, there exists a Bayesian Nash Equilibrium in which Agents 1 and 3 choose the exit option when their signals are 0. In this blocking, when Agents 1 and 3 visit the blocking mechanism, i.e., when \((s_1, s_3) = (0, 0)\), the blocking designer’s payoff is \(3(38.5 - 30)\), which is positive.

Given this model of agents’ participation, in this paper, I answer the following two questions. First, which settlement mechanism ensures that all agents will participate with all of their contracts and is unbiased and budget balanced? Second, if I allow agents to settle a number of their contracts with some blocking mechanisms and take into account agents’ payoff from blocking mechanisms, which settlement mechanism is unbiased and budget balanced? As I will show, the answer to both questions is a mechanism whereby the designer sets constant price and quantity.
4 Model

Without loss of generality, I assume the face value of the defaulted bond is 100. Each CDS contract has a protection buyer and a protection seller. In the case of a default, the protection buyer should be compensated for the loss on the reference asset (bond) by the protection seller. These CDS contracts are homogeneous, and each corresponds to one bond. I assume that the default has happened, and I consider the contract settlement problem. Let $K$ be the set of all agents. These agents may have CDS contracts on the bond between each other. A contract matrix specifies the number of contracts of a pair of agents. In a contract matrix $N = [n_{i,j}]$, agents $i, j \in K$ have net $n_{i,j}$ contracts. Assume $n_{i,j} > 0$ if $j$ is a protection seller and $i$ is a protection buyer, $n_{i,j} = 0$ if they do not have any CDS contracts, and $n_{i,j} < 0$ if $i$ is the protection seller. Throughout this paper, I use the words network and contract matrix interchangeably. Set $n_i$ to be the net number of contracts that agent $j$ has, $n_i = \sum_{j \in K} n_{i,j}$. Each agent has a number of defaulted bonds; assume agent $i$ has $b_i \geq 0$ defaulted bonds. Each agent has a private signal for the value of the defaulted bond. Agent $i$’s signal is drawn from $S_i = [0, 1]$. Given $s \in [0, 1]^k$, a profile of agents’ signals, the expected value of the defaulted bond is $v(s)$. I assume $v(s)$ is non-decreasing and continuous in agents’ signals. If $A \subseteq K$ is a subset of agents, set $S_A = \prod_{i \in A} S_i$. Given $B \subseteq A \subseteq K$ and $s \in S_A$, let $\pi_B(s) \in S_B$ be the projection of $s$ on its $B$ elements. For economy of exposition, I use the notation $s_{-i}$ and $s_{-i,j}$ for $\pi_{K\setminus\{i\}}(s)$ and $\pi_{K\setminus\{i,j\}}(s)$ respectively.

CDS contracts are settled by either physical settlement or cash settlement. In the case of physical settlement, the protection buyer hands in the defaulted bond to the protection seller and, in return, receives 100. Therefore, the protection buyer’s payoff from the physical settlement of one contract is $100 - v(s)$, and the protection seller’s payoff from the physical settlement is $-(100 - v(s))$. In the case of cash settlement, the protection seller pays the loss to the protection buyers. Therefore, if $p_i$ is the price of the defaulted bond, and $q_i$ is the number of agent $i$’s contracts that are settled through

\footnote{Note that $n_{i,j} + n_{j,i} = 0$.}
cash settlement, the agent’s payoff at signal profile \( s \in S_K \) is as follows\(^8\):

\[
u_i(b_i v(s) + (n_i - q_i)(100 - v(s)) + q_i(100 - p_i)).
\]

where \( u_i : \mathbb{R} \to \mathbb{R} \) is agent \( i \)'s utility function. The utility function, \( u_i(\cdot) \), is strictly increasing and is normalized such that \( u_i(0) = 0 \). Note that one can rewrite the payoff of agent \( i \) as follows:

\[
u_i(b_i v(s) + n_i(100 - v(s)) + q_i(v(s) - p_i)).
\]

where the second term inside \( u_i(\cdot) \) is his payoff if all of the contracts are physically settled or if the price is equal to the value of the defaulted bond. The third term can be thought of as the bias. In general, a settlement mechanism is a reallocation of the defaulted bonds and monetary transfer. Note that combinations of physical settlement and cash settlement can generate any allocation of the defaulted bonds and monetary transfer.

Let \( K(N) \) be the set of agents who have some CDS contracts in network \( N \), formally,

\[K(N) = \{ i \in K | n_{i,j} \neq 0 \text{ for some } j \in K \}.
\]

In this environment, a direct \textbf{settlement mechanism} takes the network and the profile of reported signals as inputs and returns a cash settlement quantity and a cash settlement price for each agent. A direct settlement mechanism consists of functions \( q_i^N : S_K \to \mathbb{R} \) and \( p_i^N : S_K \to \mathbb{R} \) for all agents \( i \in K \) and network \( N \). The cash settlement quantity is \( q_i^N \), and \( p_i^N \) is the cash settlement price for agent \( i \) in network \( N \). Let \( p^N = (p_i^N)_{i \in K} \) and \( q^N = (q_i^N)_{i \in K} \) be the profile of price and quantity functions when the network is \( N \) and \( P = (p^N) \) and \( Q = (q^N) \) be the price and quantity profiles. Note that I allow agents to have different cash settlement prices; in other words, I am not restricting the case to \( p_i^N = p_j^N \) for all \( i, j \in K \). Therefore, any reallocation of money and the defaulted bonds can be generated by cash settlement and physical settlement. The defaulted bonds that are used for physical settlement must be cleared, formally, for

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\(^8\) Agent \( i \) gives/receives \( n_i - q_i \) of his defaulted bonds and receives/pays the face value of the bond; in addition, he receives/pays his loss for the rest of the contracts.
all networks $N$ and $s \in S_K$:

$$\sum_{i \in K} (n_i - q_i^N(s)) = 0.$$  

This is equivalent to $\sum_{i \in K} q_i^N(s) = 0$. This mechanism is ex-post incentive compatible if, for all networks $N$, $i \in K$, and $s = (s_i, s_{-i})$ & $s' = (s'_i, s_{-i}) \in S_K$, the following holds:

$$u_i(b_i v(s) + (n_i - q_i^N(s))(100 - v(s)) + q_i^N(s)(100 - p_i^N(s))) \geq$$

$$u_i(b_i v(s) + (n_i - q_i^N(s'))(100 - v(s)) + q_i^N(s')(100 - p_i^N(s')))$$  

(4)

This means that agent $i$ with private information $s_i$ should not find it profitable to mis-report his signal as $s'_i$, when all other agents’ signal profiles are $s_{-i}$. Because the utility function is increasing, inequality (4) is equivalent to:

$$(n_i - q_i^N(s))(100 - v(s)) + q_i^N(s)(100 - p_i^N(s)) \geq$$

$$(n_i - q_i^N(s'))(100 - v(s)) + q_i^N(s')(100 - p_i^N(s')).$$

I use the notation $(P, Q, U)$ for a settlement mechanism with price, quantity, and payoff functions $p_i^N(\cdot), q_i^N(\cdot)$, and $U_i^N(\cdot)$ for all networks $N$ and agents $i \in K$. Agent $i$’s payoff in network $N$, when all agents are reporting their signals truthfully, is as follows:

$$U_i^N(s) = u_i(b_i v(s) + n_i(100 - v(s)) + q_i^N(s)(v(s) - p_i^N(s))).$$

For economy of exposition, I define $\Lambda_i^N(s) = n_i(100 - v(s)) + q_i^N(s)(v(s) - p_i^N(s))$ to be the **risk-neutral payoff** of agent $i$ from settling the CDS contracts when the network is $N$; therefore, $U_i^N(s) = u_i(b_i v(s) + \Lambda_i^N(s))$.\footnote{In general $U_i^N(s) = u_i(\mathcal{X}_i(s) + n_i(100 - v(s)) + q_i^N(s)(v(s) - p_i^N(s)))$ where $\mathcal{X}_i(s)$ is agent $i$’s payoff from his all other assets. This generalization would not change the subsequent results in the paper.} Note that the cash settlement part of the risk-neutral payoff, $q_i^N(s)(v(s) - p_i^N(s))$, is a monetary transfer to the agent. The mechanism is ex-post **budget balanced** if, for all networks, $N$ and $s \in S_K$, the equality $\sum_{i \in K} q_i^N(s)(v(s) - p_i^N(s)) = 0$ holds for all signal profiles $s$. Because $n_i$’s sum to zero, this is equivalent to $\sum_{i \in K} \Lambda_i^N(s) = 0$ for all $s$. It is ex-post weakly budget balanced if, for all networks, $N$ and $s \in S_K$, the inequality $\sum_{i \in K} \Lambda_i^N(s) \leq 0$ holds. It is ex-ante budget balanced if $E_p[\sum_{i \in K} \Lambda_i^N(s)] = 0$ for all networks $N$ and signal profiles $s$. I define

\footnote{I am not restricting attention to risk-neutral agents.}
ex-ante weakly budget-balanced mechanisms similarly. I restrict attention to ex-post incentive compatible and ex-ante weakly budget-balanced settlement mechanisms. A mechanism has no short sell if \( q_i(s) \leq n_i - b_i \) for all \( i \in K \) and \( s \in S_K \).

Agents’ prior about the signals is denoted by \( \mu \); the probability of observing signal profile \( s \) is \( \mu(s) \). The designer does not know \( \mu \), but she holds a prior \( \rho \) about \( \mu \). Let \( \kappa \) be the support of the designer’s belief. Let \( \rho \) be the designer’s prior about the agents’ prior, that is, \( \rho \) is a probability distribution over agents’ priors, \( \mu \). I use \( E_\rho \) to refer to the expected value symbol, given the designer’s information. I maintain the following assumption throughout the paper.

**Assumption 4.1. Full Rank Belief:** If function \( x : S \rightarrow \mathbb{R} \) satisfies \( E_\mu[x(s)] = 0 \) for almost all \( \mu \) in support of the designer’s belief, then \( x(s) = 0 \) for all \( s \in S \).

This assumption means that the designer is sufficiently unaware of the agents’ prior. This assumption is violated if, for example, the designer knew the expected value of the agents’ signals. This assumption would be satisfied, however, if the designer believes that the agents’ prior is a small perturbation of some distribution.

An agent is short selling if he ends up with a net negative number of defaulted bonds after the settlement, that is, \( b_i < n_i - q_i \). The problem is trivial if the market facilitates short selling; in this event, the agent should simply set \( q_i^N(s) = 0 \) and settle all contracts with physical settlement. However, short selling in this market is generally difficult or impossible. Therefore, any settlement should satisfy the no short sell constraint, which is \( q_i \geq n_i - b_i \) for all \( i \in K \).

## 5 Desired Properties

### 5.1 Participation

As of 2009, all CDS contracts are pegged to the result of the centralized CDS settlement mechanism. ISDA has argued that such policy ensures certainty, consistency, enhanced transparency, and liquidity. Even though CDS contracts are hardwired to the outcome of the settlement mechanism, if both parties of a CDS contract agree, they are allowed
to side settle their contracts by a concerted settlement procedure. Hence, I develop a model to understand how agents participate in the settlement mechanism.\(^{11}\)

Lack of participation harms the transparency settlement procedure since it drives liquidity away from the auction. Moreover, agents may manipulate the result of auction through lack of participation in the clearinghouse. This is important because liquidity and transparency were among the main reasons offered for a central mechanism to settle CDSs in the first place.

An agent who does not have any CDS contracts is not obligated to participate in the settlement mechanism; however, he can choose to participate if there is a positive payoff. This motivates the following definition: A mechanism is **ex-post individually rational** for agents without contracts if an agent without a contract leaves the auction with a non-negative payoff. In other words, for all networks \(N\), all signal profiles \(s \in S_K\), all agents \(i \in K\) that satisfy \(n_{i,j} = 0\) \(\forall j \in K\), the inequality \(\Lambda_i^N(s) \geq 0\) holds. It is **interim individually rational** if all agents without CDS contract leave the clearinghouse with a non-negative expected utility; in other words, the inequality \(E_{s,i}[U_i^N(s)] \geq E_{s,i}[u_i^N(h,\nu(s))]\) holds for all agents \(i \in K\) that satisfy \(n_{i,j} = 0\) \(\forall j \in K\).

I formally model the participation decision of agents who have CDS contracts. In standard mechanism design, agents can choose whether to participate in the mechanism. They participate when they have a non-negative payoff from participating in the mechanism. Participation in this environment is different for an important reason. Agents with CDS contracts are required to participate by default; however, if both parties of a CDS contract agree, they can settle some of their contracts through another mechanism. In this environment, agents’ outside options are no longer exogenous; rather, they depend on their signals as well as other agents’ signals. In other words, if an agent agrees to settle a CDS contract through another mechanism, it reveals information about his own private signal. I do not assume the number of contracts that a pair of agents has is private information; rather, agents are legally allowed to not bring a number of their contracts to the settlement mechanism if all parties of these contracts agree.\(^{12}\)

\(^{11}\)See http://www.isda.org/companies/auctionhardwiring/auctionhardwiring.html

\(^{12}\)Since contracts are hardwired to a centralized clearinghouse, the designer of the mechanism knows that network of contracts.
All of the results of the paper are proven when I only consider side settlements by a pair of agents. To provide a more general model of participation, however, I consider side settlements by multiple pairs of agents. Due to bilateral decisions that groups of agents may make prior to participating in the mechanism about the number of contracts, the designer may face contract matrices that are different from the original network of contracts. When the contract matrix is $N$, if a group of agents choose to settle some of their contracts outside of the settlement mechanism, the designer faces a new contract matrix, namely $M$. In this case, $M$ is a reduction of $N$. Formally, $M = [m_{i,j}]$ is a reduction of $N = [n_{i,j}]$ if, for all $i, j \in K$, the inequality $|m_{i,j}| \leq |n_{i,j}|$ holds. I use the notation $M \prec N$, if $M$ is a reduction of $N$. Let $A$ be the set of all agents who choose to settle some of their contracts outside of the settlement mechanism. Note that $A = K(M - N)$, where $M - N$ is a contract matrix in which agents $i$ and $j$ have $m_{i,j} - n_{i,j}$ contracts.

A blocking mechanism can be viewed as a settlement mechanism when the set of agents is $A$ and the network of contracts is $N - M$. The main differences are that it does not have to be budget balanced and does not have to clear the number of defaulted bonds used. Nevertheless, the blocking mechanism must provide the designer with an expected positive surplus. A blocking mechanism has an important role: It settles all contracts that were not brought to the settlement mechanism. I use the notation $(P', Q', U')$ for the blocking mechanism. Let $U^e_i$ be the payoff of an agent from joining the blocking mechanism, i.e.:

$$U^e_i(s) = u_i[b_{i\cdot v_i(s)} + \Lambda'_i(s) + \Lambda''_i(s)]$$  

where $\Lambda'_i(s) = (n_i - m_i - q'_i(s))v(s) + q'_i(s)(100 - p'_i(s))$. I present two models for the blocking: (i) complete information case and (ii) incomplete information case.

5.1.1 Complete Information Case

Agents in $A$, for a subset of their types, block the settlement mechanism and reduce the network from $N$ to $M$ if there exists a blocking mechanism $(P', Q', U')$ and a prescribed non-zero measure subset of types $S'_i \subseteq S_i$ for agents in $A$, such that the following holds:
1. For all $i \in A$ and $s_A \in \prod_{i \in A} S_i$, the following inequality holds:

$$E_{s \setminus A} [U_i^N(s_A, s_{-A})] \leq E_{s \setminus A} [U_i^e(s_A, s_{-A})]$$

(6)

where $U_i^e$ is agent $i$’s payoff from joining the blocking mechanism; see equation (5). Agents in $A$ join the coalition when their types are in the prescribed subset of types. Inequality (6) means that, if all signals of agents in $A$ are in the prescribed sets, then the expected payoff of all agents $i \in A$ from the settlement mechanism with network $N$ is not larger than an agent’s total payoff from the blocking mechanism and the payoff from the settlement mechanism with network $M$. This gives agent $i$ an incentive to join the coalition when all blocking agents’ signals are in the prescribed sets.

2. For all $s_A \in S_K$, such that $\pi_{A \setminus \{i\}}(s_A) \in \prod_{j \in A \setminus \{i\}} S'_j$ and $\pi_{\{i\}}(s) \in S_i \setminus S'_i$, the following inequality holds:

$$E_{s \setminus A} [U_i^N(s_A, s_{-A})] \geq E_{s \setminus A} [U_i^e(s_A, s_{-A})].$$

(7)

Inequality (7) means that if agent $i$’s signal is not in $S'_i$, and the signal of all other agents in $A$ are in the prescribed sets, then agent $i$’s expected payoff from the settlement mechanism with network $N$ is not smaller than the agent’s total payoff from the blocking mechanism and his payoff from the settlement mechanism with network $M$.

3. The blocking designer has an expected positive payoff. Formally, the following inequality must hold:

$$E[- \sum_{i \in A} \Lambda'_i(s) | \pi_A(s) \in S_A] > 0.$$
the strategy of exiting only if the type is in the prescribed set, then these inequalities guarantee that agent $i \in A$, upon learning the types of all agents in $A \setminus \{i\}$, would not regret his participation decision. The mechanism is **ex-post unraveled** if blocking exists.

To understand inequality (8), think of the blocking designer as an agent. Note that, in general, the blocking mechanism does not have to balance the budget or clear the number of defaulted bonds that are used for physical settlement. Because the blocking mechanism designer may buy defaulted bonds from agents in $A$. Inequality (8) means that the blocking designer’s expected payoff, conditional on the event that the block is formed, must be positive. The first term that appears in the summation is the blocking designer’s payoff from defaulted bonds, and the second term is his payoff from the monetary transfer. The strict inequality (8) guarantees that no mechanism is blocked by the null mechanism.\(^{13}\)

A settlement mechanism $(P, Q, U)$ is **ex-post unravel-proof** if, for any pair of contract matrices $M$ and $N$ and subset of agents $A \subseteq K$, where agents in $A$ reduced $N$ to $M$, agents in $A$ cannot form a block for a subset of their types. A settlement mechanism is **weakly ex-post unravel-proof** if there is no pair of agents who could form a block. One can strengthen the notion of unravel-proof to consider blocks by more than two agents. The results of the paper, however, are correct in either case.

The following proposition provides sufficient conditions for unravel proofness.

**Proposition 5.1.** If agents are risk neutral and the following inequality holds, then there is no ex-post unraveling in which agents in $A$ reduce the network from $N$ to $M$.

$$
\sum_{i \in A} E_{s\rightarrow A} A^N_i (s_A, s_{\neg A}) \geq \sum_{i \in A} E_{s\rightarrow A} A^M_i (s_A, s_{\neg A}) \text{ for almost all } s_A \in S_A \tag{9}
$$

**Proof.** See the Appendix for a proof. \(\Box\)

Note that inequality (9) means that the sum of agents’ risk-neutral expected payoffs that are in $A$ is greater when the network is $N$ compared to that of network $M$. Since sum of agents payoff in the side settlement is negative, the inequality implies

\(^{13}\)A mechanism where all the quantities are equal to zero.
that total payoff of agents in $A$ does not improve if they choose to side settle. Hence, side settlement is not beneficial for all agents in $A$.

The contribution of this paper is that I consider the possibility that the outside option of an agent depends on the set of other agents who exercise their outside option. This is because the agents who choose to exit may decide to band together and settle some of their contracts among themselves through a different mechanism. Such a possibility was first considered in the literature on cooperative games and culminated in the notion of the core. The notion of unraveling presented in this paper is related to the block in matching theory and the block in cooperative game theory. The difference between blocking in matching theory and the notion of unraveling is that, in my setup, the network is predetermined and only price and quantity of cash settlement are chosen through a mechanism. Unravel-proofness is a property of a mechanism that is defined over networks, but stability is defined over a possible match. Unlike similar concepts in corporate game theory, the notion of unravel-proofness can be naturally extended to environments with incomplete information. A generalization of the unravel-proofness notion to environments with incomplete information is presented in the following section.

5.1.2 Incomplete Information Case

I extend the blocking mechanism definition to environments in which agents who participate in a block do not know each other’s signals but share a prior. When agents make decisions about whether to join the blocking mechanism, they update their belief upon observing other agents’ decisions. An agent’s choice to participate in a blocking mechanism reveals information about his private signal. Other agents take this into account when making their decisions. The notion of interim blocking is defined as follows:

For all $i \in A$, subsets $\emptyset \neq S_i' \subseteq S_i$ are called the prescribed sets. Let event $E^i$ be defined as follows:

$$E^i = \{s \in S_k | \pi_{A\setminus\{i\}}(s) \in \prod_{j \in A \setminus \{i\}} S_j', \pi_{\{i\}}(s) \in S_i \setminus S_i'\}.$$
Define event $E$ as 

\[ E = \{ s \in S_K | \pi_A(s) \in \prod_{j \in A} S'_j \}. \]

Note that $E^i$ is the event that the private signals of all agents in $A$, except for agent $i$, are in the prescribed sets. Event $E$ is the event that the private signals of all agents in $A$ are in the prescribed sets. The inequalities in the blocking mechanism definition, inequalities (6), (7), and (8), change to the following inequalities:

\[ E_{s-i}[U^N_i(s)|E] \leq E_{s-i}[U^f_i(s)|E], \quad (10) \]

\[ E_{s-i}[U^N_i(s)|E^i] \geq E_{s-i}[u^o_i(s)|E^i]. \quad (11) \]

\[ E[\sum_{j \in A} \Lambda'_j(s)|E] > 0. \quad (12) \]

To interpret inequalities (10) and (11), imagine a game whose players are agents in $A$. These agents, after observing their private signals, choose whether to participate in the blocking mechanism. If all of these agents decide to participate in the blocking mechanism, their payoff is that of the blocking mechanism plus that of the settlement mechanism when the network is $M$. If some decide not to participate in the blocking mechanism, their payoff is only that of the settlement mechanism when the network is $N$. The mechanism is unraveled if this game has a Bayesian Nash Equilibrium in which agents in $A$, for a subset of their types, choose the blocking mechanism.

With the new definition of a block, unravel-proofness is naturally redefined. A mechanism is **interim unravel-proofness** if there is no pair of agents who could form an interim block. One can strengthen the notion of unravel-proofness to consider blocks by more than two agents. The results of the paper, however, are correct in either case.

**Proposition 5.2.** A settlement mechanism $(P, Q, U)$ that settles all contracts with cash settlement with a constant price, i.e., $p_i^N = p_i$ and $q_i^N = n_i$, is interim unravel-proof. Moreover, if agents are risk neutral and the following inequality holds, then there is no
interim unraveling in which agents in $A$ reduce the network from $N$ to $M$:

$$\sum_{i \in A} E_{s-A}[\Lambda_i^N(s_A, s_{-A})] \geq \sum_{i \in A} E_{s-A}[\Lambda_i^M(s_A, s_{-A})] \text{ for almost all } s_A \in S_A$$

(13)

Proof. The proof is an adaptation of the proof of proposition 5.1 and, hence, it is omitted. □

5.2 Unbiasedness

As I stated in Section 2, several authors have criticized the current settlement mechanism for underpricing the underlying bond. The current mechanism sets a biased price, which results in a difference between physical settlement and cash settlement. Because it is not known at the time of contracting a CDS whether the CDS contract will be settled by cash settlement or physical settlement, the biased pricing results in uncertainty and, hence, efficiency loss. I look for mechanisms that overcome this issue. Due to the information rent, based on agents’ private information about the value of the defaulted bond, as discussed in Section 1, ex-post correct pricing is not possible unless all contracts are physically settled. I define unbiasedness in two ways, as seen below.

5.2.1 Weakly Unbiased

A weakly-unbiased mechanism is one that does not misprice the defaulted bond in expectation. Formally, mechanism $(P, Q, U)$ is weakly unbiased if, for all networks $N$ and agents $i \in K$:

$$E[\Lambda_i^N(s)] = [n_i(100 - v(s))]$$

(14)

Equation (14) means that, from an ex-ante perspective, the agents’ risk-neutral payoff from the settlement mechanism is the same as their payoff from physical settlement of all contracts or cash settlement with the price equal to the value of the defaulted bond. Note that, because both price and quantity may depend on the signal profile, this condition is not equivalent to $E[p_i^N(s)] = E[v]$.

Observation 5.1. If $E[\Lambda_i^N(s)] = n_i(100 - E[v])$, then $E_p[\sum_{i \in K} \Lambda_i^N(s)] = (\sum_{i \in K} n_i)(100 -$
Therefore, a mechanism is weakly ex-ante budget balanced if it is weakly unbiased.

5.2.2 Unbiased

If the mechanism is not strategy-proof, some agents may settle some of their contracts outside of the settlement mechanism. Therefore, agents’ total payoff is not only the payoff from the settlement mechanism; it also should include the payoff from the blocking mechanisms. A mechanism is unbiased if the agents’ total payoff, including the payoff from the settlement mechanism and the blocking mechanisms, from an ax-ante perspective, is equal to the agents’ payoff from physical settlement of all contracts or cash settlement with the correct price.

To formally define unbiased mechanisms, I first define the notion of participation-choice. For some networks, a pair of agents may find it profitable to settle some of their contracts with a blocking mechanism. Using the participation model introduced in Section 5.1, I allow pairs of agents to take these actions; a mechanism is unbiased if it is weakly unbiased, regardless of these actions. The results of the paper are valid if one considers only coalitions by pair of agents in the definition of unbiasedness. To provide a richer model, however, I consider groups of agents.

Consider an ex-post incentive-compatible settlement mechanism, namely \((P, Q, U)\), which may not be unravel-proof. Let \(\Omega\) be the set of all possible networks. I allow for several coalitions to coexist; let \(P_N\) be the set of blocking coalitions when the true network of contract is \(N\). Hence, a participation-choice is a collection of sets \((P_N)_{N \in \Omega}\), whereby elements of \(P_N\) capture the sub-networks that join a coalition if the true network of the contract is \(N\). For all networks \(N\), each element of \(c \in P_N\) has a network \(c_N \prec N\), a subset of agents \(c_A \subseteq K(c_N)\), a set of type profiles \(S^c_i \subseteq S_i\) for all \(i \in c_A\), and a blocking mechanism for the \(c_N\) network, \((\hat{P}^c, \hat{Q}^c, \hat{U}^c)\). Let \(c_i = \prod_{i \in c_A} S^c_i\) for all \(i \in c_A\). This participation-choice should satisfy three conditions:

1. Let \(A_N(s)\) be the set of all coalitions that are formed when the signal profile is \(s \in S^K\). Formally, \[A_N(s) = \{c \in A_N | \pi_{c_A}(s) \in c_i\}.\]

It must be that \(\bar{N}(s) = \sum_{c \in A_N(s)} c_N \prec N\). This means that the network that is left
after coalitions are formed is a reduction of $N$.

2. Agent $i$’s payoff from this participation-choice when the network is $N$ is $u_i(\Lambda_i^{PN}(s))$, where:

$$\Lambda_i^{PN}(s) = \sum_{c \in A_N} \hat{\Lambda}_i^c(s) + \Lambda_i^{N-N}(s).$$

Joining the coalitions for the prescribed types must be a Bayesian Nash equilibrium. Formally, for all networks $N$ and $c \in A_N$, let events $E_c$ and $E_i^c$ be defined as:

$$E_c = \{ s \in S_K | \pi_{A_c}(s) \in \prod_{j \in K(c_N)} S_j \},$$

$$E_i^c = \{ s \in S_K | \pi_{A_c}(s) \in \prod_{j \in K(c_N) \setminus \{i\}} S_j, \pi_{\{i\}}(s) \in S_i \setminus S_i^c \}.$$

For all $i \in c_A$, the following inequalities should hold:

$$E_{s_i}[u_i^{PN}(s)|E_c] \geq E_{s_i}[u_i^{PN \setminus c}(s)|E_c],$$

$$E_{s_i}[u_i^{PN}(s)|E_i^c] \leq E_{s_i}[u_i^{PN \setminus c}(s)|E_i^c].$$

3. Given a coalition $c \in A_N$, when the signal profile is $s \in S_K$, agent $i \in K(c_N)$ enters the coalition $c$ if $\pi_{c_A}(s) \in S_i^{c_j}$. The blocking designer’s expected payoff from the blocking mechanism should be positive, conditional on the event that all agents in $c_A$ join the blocking mechanism. This is similar to inequality (12), which ensures that the blocking mechanisms are self-sustaining.

Consider a participation-choice for each agents’ prior $\mu$. The mechanism is unbiased if, for all networks $N$, all agents $i \in K$, and all participation-choices, the following holds:

$$E_\rho[\Lambda_i^{PN}(s)] = E_\rho[n_i(100 - v(s))]$$

This condition indicates that, from an ex-ante perspective, agent $i$’s total payoff from the blocking mechanism and the settlement mechanism for all possible participation-choices is equal to the agent’s payoff from physical settlement of all contracts. Note that, because I allow some contracts to be settled outside of the settlement mechanism,
the notion of budget balancedness must be modified. The mechanism is weakly budget balanced regardless of agents’ participation-choice if, for all networks $N$ and all participation-choices, the following holds:

$$E_p[\sum_{i\in K(N)} \Lambda_i^{N-N(s)}(s)] \leq 0.$$ 

5.3 Robustness With Respect to Network

The current mechanism in use takes only the net number of contracts, and not the details of the network of contracts, as an input. To elaborate, consider the following example: Agent 1 has sold 500 CDSs to Agent 2, Agent 2 has sold 400 CDSs to Agent 3 and Agent 3 has sold 300 CDSs to Agent 1. The clearinghouse offsets 300 contracts in this loop, hence this network is treated as the case where agent 2 has 200 long CDS positions (CDS buyer) and agent 1 and 3 each have 100 short positions (CDS sellers). This motivates the property of robust with respect to network. Here is the formal definition: A mechanism is robust with respect to the network if, for all pairs of networks $M = [m_{i,j}]$ and $N = [n_{i,j}]$ that satisfy $\sum_{j\in K} m_{i,j} = \sum_{j\in K} n_{i,j}$ for all $i \in K$, the following equalities hold: $\forall i \in K$, $q_i^N(s) = q_i^M(s)$ and $p_i^N(s) = p_i^M(s)$. Since netting cancel out some of the positions, this property lowers the transactions costs and contemporary risk.

6 Results

I provided a model of participation when groups of agents can join the blocking mechanism. Nevertheless, all of the results of the paper are proven when only a pair of agents engage in side settlement.

I look for mechanisms that satisfy a combination of properties that I have introduced in the previous section. Before presenting the characterization results, I introduce a class of mechanisms called posted-price mechanisms.

A posted price mechanism sets prices and quantities that do not depend on agents signals. More over there exists $q^n$ for all $n \in \mathbb{Z}$, satisfying $q^n + q^{-n} = 0$ for all $n \in \mathbb{Z}$,
and price $p$, such that the following holds for all network $N$ and $i \in K$,

$$q^N_i(s) = \sum_{j \in K} q^{N,j}_i,$$

$$p^N_i(s) = p.$$

A posted-price mechanism has a fair price if $p = E[v]$.

**Proposition 6.1.** Any posted price settlement mechanism with a fair price is weakly unbiased and ex-post individually rational for agents without contracts. Moreover, if agents are risk neutral, the settlement mechanism is ex-post and interim unravel-proof.

**Proof.** The only non-trivial property is unravel-proofness when agents are risk neutral. A posted price mechanism satisfies the sufficient conditions in propositions 5.1 and 5.2. □

A posted price mechanism satisfies the robustness with respect to network if for all $m, n \in \mathbb{Z}$, $q^m + q^n = q^{m+n}$.

### 6.1 Characterizations

**Theorem 6.2.** If $(P, Q, U)$ is a settlement mechanism that is full information unravel-proof, weakly unbiased, robust with respect to network, and ex-post individually rational for agents without contracts, then almost surely it is a posted-price mechanism with a fair price.

**Proof.** A sketch of the proof is presented; for the complete proof, see the appendix. Proof is by induction. To prove the inductive step, I use the following lemma.

**Lemma 6.3.** Consider the assumptions and the setup in proposition 5.1, if the settlement mechanism is a posted-price when the network is $M$, then almost surely for all $s \in S_K$:

$$\sum_{i \in A} \Lambda^N_i(s) = \sum_{i \in A} \Lambda^M_i(s).$$
This lemma connects the mechanism in a network to its reductions. Because I consider side settlements only by two agents, I use lemma 6.3 for coalitions of size two, $|A| = 2$. To explain how this lemma is applied in the proof, I provide an example.

Consider the case of three agents $\{i, j, k\}$, where $k$ has one contract with $i$, and $i$ has one contract with $j$ (see Figure 3); call this network of contracts $N$. Let $M_1$ be a network where agent $k$ has one contract with agent $i$ and $M_2$ be a network where agent $i$ has one contract with agent $j$ (see Figure 3). Because the result is true for two agents, I can apply the lemma 6.3 for two pairs of networks $N \& M_1$ and $N \& M_2$. In addition, the mechanism is budget balanced when the network is $N$; hence, I have:

$$
\Lambda_i^N(s) + \Lambda_j^N(s) = \Lambda_i^{M_1}(s), \quad \Lambda_j^N(s) + \Lambda_k^N(s) = \Lambda_i^{M_2}(s) \quad \text{and} \quad \Lambda_i^N(s) + \Lambda_j^N(s) + \Lambda_k^N(s) = 0
$$

(16)

Therefore,

$$
\Lambda_i^N(s) = \Lambda_i^{M_1}(s) + \Lambda_i^{M_2}(s), \quad \Lambda_j^N(s) = -\Lambda_i^{M_2}(s) \quad \text{and} \quad \Lambda_k^N(s) = -\Lambda_i^{M_1}(s).
$$

This proves the inductive step for this case.

I provide a sketch of the proof of lemma 6.3. Note that $\sum_{i \in A} n_i = \sum_{i \in A} m_i$, weakly-unbiasedness implies:

$$
\sum_{i \in A} E_p[\Lambda_i^N(s)] = \sum_{i \in A} E_p[\Lambda_i^M(s)]
$$

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If the lemma does not hold, then for some \( s^* \in S \),
\[
\sum_{i \in A} \Lambda_i^N(s^*) < \sum_{i \in A} \Lambda_i^M(s^*). \tag{17}
\]
Inequality (17) implies that there exists \( T_i \in \mathbb{R} \), such that
\[
\sum_{i \in A} T_i < 0 \quad \text{and for all } i \in A \Lambda_i^N(s^*) = \Lambda_i^M(s^*) + T_i. \tag{18}
\]
I construct a blocking for \( A \). Because the mechanism is a posted price when the network is \( M \), \( \Lambda_i^N(s^*) = \Lambda_i^M(s^*) + T_i \) is linear. Convexity of \( \Lambda_i^N(s_i, s^*_{-i}) \) follows from incentive compatibility of the mechanism. Because (18) holds, \( \Lambda_i^N(s_i, s^*_i) \) lies above \( \Lambda_i^M(s_i, s^*_{-i}) + T_i \) (see Figure 4). In the candidate blocking, I want agent \( i \) to join the blocking mechanism only when his signal is \( s^*_i \). (Here, the set of type is a measure zero set; in the proof, I provide a blocking with a positive measure of types). Note that the candidate blocking mechanism must be ex-post incentive compatible. Therefore, all I need to find is a blocking mechanism with payoff \( \Lambda_i \) such that \( \Lambda_i^M(\cdot, s^*_i) + \Lambda_i(\cdot, s^*_{-i}) \) is convex and lies weakly below \( \Lambda_i^N(\cdot, s^*_i) \) and equal to \( \Lambda_i^N(\cdot, s^*_{-i}) \) at \( s^*_i \). Intuitively, this can be done by considering a convex function that lies between \( \Lambda_i^N(\cdot, s^*_{-i}) \) and \( \Lambda_i^M(\cdot, s^*_{-i}) + T_i \). The fact that \( \sum_{i \in A} T_i < 0 \) guarantees that the blocking mechanism satisfies equation (18). \( \Box \)

The unravel-proofness property in Theorem 1 is the full information notion. The
same result holds with the imperfect information model.

**Theorem 6.4.** Under assumption 4.1 if \((P, Q, U)\) is a settlement mechanism that is interim unravel-proof, robust with respect to network, weakly-unbiased, and interim individually rational for agents without contracts, then almost surely it is a posted-price mechanism with a fair price.

*Proof.* See the appendix for the proof.  

The motivation for unravel-proofness is that participation in the mechanism ensures liquidity and transparency of the settlement procedure. This property, however, can be replaced with a stronger version of the weak-unbiasedness property, namely, the unbiasedness property.

**Theorem 6.5.** Under assumption 4.1 if \((P, Q, U)\) is a settlement mechanism that is robust with respect to the network, unbiased, weakly budget balanced regardless of agents’ participation choice, and interim individually rational for agents without contracts, then almost surely it is a posted-price mechanism with a fair price.

*Proof.* See the appendix for the proof.  

7 Discussion

7.1 Selling CDS Contracts

Agents are legally allowed to sell some of their CDS contracts prior to participating in the settlement mechanism. In this section, I explore the possibility that agents trade their contracts before the settlement mechanism but after they learn their signals. This motivates a change in the definition of unravel-proofness to ensure that no pair of agents has an incentive to take the following two actions: (i) settling some of their contracts with another mechanism and (ii) selling some of their contracts to another agent. I change the definition of network reduction as follows:

Formally, \(M\) is a reduction of \(N\) if there exists a sequence of networks \((M^t = [m^t_{i,j}])_{i,j \in K}^{t=\tau}\) such that \(M^n = N, M^1, M^2, ..., M^\tau = M\) and given \(M^t\) for \(0 \leq t \leq \tau - 1\), \(M^{t+1}\) satisfies one of the following two cases:
1. Contract matrix $M^{t+1} \neq M^t$ is such that, for all $i, j \in K$, $|m_{i,j}^t| \geq |m_{i,j}^{t+1}|$ and if $m_{i,j}^t \neq 0$, then $m_{i,j}^t$ and $m_{i,j}^{t+1}$ have the same sign. In this case, the set of agents who took this action is all agents $i \in K$ such that $m_{i,j}^t \neq m_{i,j}^{t+1}$ for some agent $j \in K$.

2. Contract matrix $M^{t+1}$ is constructed from $M^t$ by removing some contracts that are between two agents $i \in K$ and $j_1 \in K$ and adding these contracts to contracts between $i$ and another agent, $j_2 \in K$. It must be that

$$m_{i,j_1}^{t+1} + m_{i,j_2}^{t+1} = m_{i,j_1}^t + m_{i,j_2}^t.$$ 

No other contract is removed or added. This is the case where agent $j_1$ buys some of the contracts that are between $j_2$ and $i$ from agent $j_2$. In this case, agents $i, j_1$ and $j_2$ took an action.

The definition of unraveling is similar to that of the previous case. Let $A$ be the set of agents who (potentially) take these actions. These agents block and reduce the network from $N$ to $M$ if there exists a blocking mechanism that satisfies the inequalities in the definition of blocking.\(^{14}\) The blocking mechanism has two roles: (i) it settles all contracts that were not brought to the settlement mechanism, and (ii) it compensates agents for selling their CDS contracts. With this modification, unbiasedness and unravel-proofness are naturally redefined.

**Theorem 7.1.** Given the new definitions, the results of Theorems 6.2, 6.4 and 6.5 remain true if one drops the property of robustness, with respect to the network, from the list of properties.

*Proof.* See the appendix for the proof. \(\square\)

7.2 Ex-ante Uniform Price

One of the criticisms of the current settlement procedure in use is that the outcome of the mechanism is sensitive to the number of positions. Therefore, at the time of contracting the pay-off of agents from the clearinghouse is not predictable. I propose

\(^{14}\)For the case of complete information, these inequalities are inequality (6), (7), and (8), and, for the case of incomplete information, they are (10), (11), and (12).
a generalization for the unbiased and weakly-unbiased properties. A mechanism is weakly ex-ante uniform price if, from an ex-ante perspective, CDS contracts have the same payoff.

Here is a formal definition: A mechanism satisfies weak ex-ante uniform price property if, for some price function $p : S \rightarrow \mathbb{R}$, all networks $N$, and all agents $i \in K$:

$$E_{\rho}\[\Lambda_i^N(s)\] = E_{\rho}[n_i(100 - p(s))].$$

Ex-ante uniform price property is defined naturally. Given a participation-choice for agents’ prior, agent $i$’s ex-ante payoff is defined as $E[u_i^N(s)]$ for all $s \in S_K$. The mechanism is unbiased if, for all agents $i \in K$, all networks $N$ and all participation plans and some price function $p : S \rightarrow \mathbb{R}$, the following holds:

$$E_{\rho}[\Lambda_i^{P}(s_{K(N)})] = E_{\rho}[n_i(100 - p(s))].$$

(19)

**Proposition 7.2.** Properties of unbiasedness and weakly-unbiasedness can be replaced with ex-ante uniform price and weakly ex-ante uniform price, respectively, in Theorems 6.2, 6.4, 6.5, and 7.1.

**Proof.** See the appendix for the proof. □

### 7.3 Simple Blocking Mechanisms

**Proposition 7.3.** Theorems 6.2, 6.4, 6.5, and 7.1 hold true if one restricts attention to blocking mechanisms that have a constant price and quantity.

The proof follows from the proof of theorems 6.2, 6.4, 6.5, and 7.1

In general a blocking mechanism might be a complex mechanism. However, proposition (7.3) shows that the results of the paper hold if one restricts attention to simple blocking mechanisms.
8 Modifying the Current Settlement Mechanism

The results of the paper show that the only robust settlement mechanism that delivers unbiased prices or gives agents ex-ante uniform payoff is a posted price mechanism. However, a posted price mechanism cannot be practically used, because it gives too much discretion to the designer of the mechanism. I propose a modification to the existing settlement mechanism to improve the efficiency of the settlement procedure.

8.1 Current Settlement Mechanism

I explain how the existing settlement procedure works. The current settlement procedure is comprised of two stages. Prior to the auction, the designer sets two values called maximum spread and quotation size.

Stage one: Only agents who have net non-zero CDS positions can participate in this stage of the mechanism. Agents submit requests to buy or sell the defaulted bonds at the final price of the mechanism. Net CDS sellers may submit request to buy defaulted bonds and net CDS buyers may submit requests to sell defaulted bonds. The size of the requests may not exceed the net number of positions. Moreover, in this stage, agents submit bids and offers for the quotation size. The difference between bids and offers may not exceed the predetermined maximum spread. The initial market midpoint (IMM) is set as follows: Crossing or touching bids and offers are removed. The IMM is the average of the best half bid and the best half offer. The net open interest (NOI) is calculated as follows: the buy physical orders are netted against the sell physical orders. The NOI is to sell if there are more buy physical orders and it is to sell if the opposite happens. Penalties are levied on agents that are on the wrong-side of the market. If the NOI is to sell, those who bid higher than the IMM, pay an amount equal to

\[ 0.01(bid - IMM) \times Q \]

\(^{15}\)For a more detailed explanation of CDS settlement procedure with numerical examples see Haworth (2011).

\(^{16}\)A bid \( b \) is crossing or touching with an offer \( o \), if \( b \geq o \).

\(^{17}\)If \( n \) bids and offers remain, the best halves are the \( \frac{n}{2} \) highest bids and \( \frac{n}{2} \) lowest offers. If \( n \) is odd, the best \( \frac{n+1}{2} \) bids and offers are used.)
Here $Q$ is the predetermined quotation size. If the NOI is to buy, those who offer less than the IMM, pay a penalty equal to $0.01(\text{IMM} - \text{offer}) \times Q$.

**Stage two:** There is no participation constraint in this stage of the mechanism. A uniform price auction is used to fill out the NOI. Therefore, if the NOI is to buy the goal of the auction is to buy defaulted bonds and if it is to sell the auction sells defaulted bonds. Moreover, if the NOI is to sell, then the final price may not exceed the IMM plus a predetermined cap amount; if the NOI is to buy, the final price may not be less than the IMM minus the cap amount. If the auction is to sell and there are not enough orders to fill the NOI, the final price is set equal to zero. If the auction is to buy and there is not enough order to fill the NOI, the final price is set equal to the par, which in our model is 100.

All CDS contracts are settled by cash settlement at the final price of the auction. However, the design of the auction allows agents to replicate the physical settlement by trading the defaulted bond. To elaborate, assume the final price of the auction is $p$ and the value of the defaulted bond is $v$, the payoff from cash settlement of a long CDS position is $100 - p$. The payoff from selling a defaulted bond with value $v$ is $p - v$. Therefore, cash settlement and selling the defaulted bond to the clearinghouse give a total payoff of $100 - p + p - v = 100 - v$, which is the physical settlement payoff.

### 8.2 Modification

I propose a modification to the current CDS settlement mechanism and prove the properties. At the time of contracting, agents must decide if a CDS contract is covered or not. The mechanism gives incentives to covered CDS buyers to have the underlying bond at the time of settlement. I add stage zero as follows:

**Stage zero:** All net Covered CDS buyers hand in defaulted bonds for each net covered CDS position to the clearinghouse and receive a monetary transfer equal to the defaulted bond’s face value from the net covered CDS sellers for each defaulted bond they submit. If a covered CDS holder does not submit the underlying bond for a covered CDS contract, he does not get the monetary transfer. Net Covered CDS sellers submit orders to receive defaulted bonds. The orders may not exceed the net number of Covered CDS positions. Net covered CDS sellers receive the final price of
the defaulted bond for defaulted bonds that they did not choose to get in stage zero. Orders submitted in stage zero are netted and added to the NOI in stage one.

   Stage one and Stage two proceed as before.

   I call this settlement mechanism **modified settlement mechanism (MSM)**.

**Proposition 8.1.** MSM has the following properties:

1. Robustness with respect to network.
2. All net covered CDS buyers receive full insurance payoff for their covered CDSs.
3. No pair of risk neutral agents have an incentive to side settle covered CDSs.

**Proof.** Properties 1 and 2 are direct implications of the MSM design. Assume for a contradiction that a block exists. Let $N$ be the original network of contracts and $M$ be the left over contracts. Note that both agents, if any of them choose not to join the block, can secure the payoff from network $M$ plus the payoff from physical settlement of $q$ contracts. Since side settlement is zero sum, it cannot be the case that both agents benefit from side settlement.

□

A pair of risk averse agents may benefit from risk sharing by side settling covered CDS contracts, however, they cannot manipulate the MSM to change the outcome in their favor.

**Proposition 8.2.** If a covered CDS buyer prefers physical settlement to any other settlement with the same expected payoff and the CDS seller is risk neutral, then this pair of agents settle all of their contracts in the settlement mechanism.

The proof follows from the proof of proposition (8.1). If covered CDS buyers bought the CDS to hedge against the risk of default, they would prefer physical settlement to any other settlement with the same expected payoff. Note that net covered CDS buyers can secure the physical settlement payoff; therefore, they engage in side settlement only if the CDS buyer receives a premium from the CDS seller. Since the seller can also secure the physical settlement payoff, the covered CDS seller agrees to side settlement only if the CDS buyer provides risk sharing for the CDS seller. This
does not happen if the CDS seller is risk neutral. Net CDS sellers are typically large financial institutions and covered CDS buyers are typically pension funds and insurance companies that buy CDS to hedge against the risk of default.

9 Conclusion

I took a mechanism design approach to address the design of a CDS contract settlement problem. The design would have been trivial if short selling defaulted bonds was possible because; one could settle all contracts physically. Physical settlement of all contracts would result in an ex-post unbiased-settlement procedure. Inability to short sell makes the design problem non-trivial. An important issue considered in this paper, neglected by other authors, is participation. Any settlement mechanism should take into account this issue when making predictions regarding the settlement price and agents’ payoffs. I show that any settlement mechanism that is robust with respect to the network and, from a designer’s ex-ante standpoint, sets an unbiased price is a posted-price mechanism. This mechanism sets a price equal to the expected value of the defaulted bond, conditional on designer’s information. Moreover, this mechanism is almost surely the unique mechanism that satisfies enumerated properties in the introduction. The main result of the paper should be interpreted as an impossibility result, which means that there is no non-trivial settlement mechanism that can render unbiased pricing or non-sensitive pricing to the number of CDS positions in the CDS settlement procedure. I exploit the result to suggest that the market should make a distinction between covered CDSs and naked CDSs and require all covered CDSs to be settled physically at the settlement clearinghouse.

The tool that is developed in this paper is a new approach to extending the notion of the core to the case of incomplete information. I considered the “exit game” before joining the blocking mechanism. This model can be applied to mechanism design problems in which (i) agents are allowed to get together and use another mechanism for their purpose and (ii) the decision to leave the mechanism occurs before formation of the grand mechanism. An example of such environments is dark markets in the stock exchange.
10 Appendix

10.1 Proof of Inequalities in the Leading Example:

One can rewrite agent $i$’s payoff that he receives when $q_i$ contracts are settled physically
with price $p_i$ as $u_i(n_i(100 - v(s)) + q_i(v(s) - p_i))$. I have:

$$
E[U_1^1(0, s_2, 0)] = \frac{1}{2}(E[U_1(0, 0, 0) + E[U_1(0, 1, 0)]) = -20(100 - E[v|s_1 = 0, s_3 = 0]) + \\
\frac{1}{2}(168 + 105) = -20(100 - E[v|s_1 = 0, s_3 = 0]) + 136.5
$$

$$
E[U_1^1(1, s_2, 0)] = \frac{1}{2}(E[u_1(1, 0, 0) + u_1(1, 1, 0)] = -20(100 - E[v|s_1 = 1, s_3 = 0]) + \\
\frac{1}{2}(-84 - 105) = -20(100 - E[v|s_1 = 1, s_3 = 0]) - 94.5
$$

$$
E[U_2^0(0, s_2, 0)] = -20(100 - E[v|s_1 = 0, s_3 = 0]) - 3.5(10.5 - 42) - 3(10.5 - 30) = \\
- 20(100 - E[v|s_1 = 0, s_3 = 0]) + 168.75
$$

$$
E[U_2^0(1, s_2, 0)] = -20(100 - E[v|s_1 = 1, s_3 = 0]) - 3.5(52.5 - 42) - 3(52.5 - 30) = \\
- 20(100 - E[v|s_1 = 1, s_3 = 0]) - 104.25
$$

$$
E[U_3^1(0, s_2, 0)] = 10(100 - E[v|s_1 = 0, s_3 = 0]) - 84
$$

$$
E[U_3^1(0, s_2, 1)] = 10(100 - E[v|s_1 = 0, s_3 = 1]) - 21
$$

$$
E[U_3^2(0, s_2, 0)] = 10(100 - E[v|s_1 = 0, s_3 = 0]) - 84
$$

$$
E[U_3^2(0, s_2, 1)] = 10(100 - E[v|s_1 = 0, s_3 = 1]) - 21
$$

This proves the inequalities.

10.2 Proof of Proposition 5.1

I prove the second part of the proposition first. Assume that there is a blocking mechanism ($P', Q', U'$). Note that, when agents are risk neutral, inequality (6) is equivalent
\[ \forall i \in A \text{ and } s_A \in \prod_{i \in A} S_i', \ E_{s,A}[\Lambda_i^N(s_A)] \leq E_{s,A}[\Lambda_i^M(s_A) + \Lambda_i'(s_A)]. \quad (20) \]

If one adds this for all \( i \in A \) and takes expectations over all \( s_A \in \prod S_i' \), then the following holds:

\[ \sum_{i \in A} E[\Lambda_i^N(s)|\pi_A(s) \in \prod_{i \in A} S_i'] \leq E[\Lambda_i^M(s) + \Lambda_i'(s)|\pi_A(s) \in \prod_{i \in A} S_i'] \quad (21) \]

Inequalities \((21)\) and \((8)\) imply the following inequality:

\[ \sum_{i \in A} E[\Lambda_i^N(s)|\pi_A(s) \in \prod_{i \in A} S_i'] < E[\Lambda_i^M(s)|\pi_A(s) \in \prod_{i \in A} S_i'] \quad (22) \]

Therefore, for some \( s \in S_K \), \( \sum_{i \in A} \Lambda_i^N(s) < \sum_{i \in A} \Lambda_i^M(s) \). This contradicts the assumption in the proposition.

I prove the first part of the proposition.

### 10.3 Proof of Theorem 6.2

I prove the following proposition:

**Proposition 10.1.** Let \( M \) and \( N \) be a pair of contract matrices, where \( M \) is a reduction of \( N \). Let \( A \) be defined as in the definition of blocking and \((P, Q, U)\) be a settlement mechanism with no complete information unraveling, for which \( N \) is reduced to \( M \) by agents in \( A \). If for all \( i \in A \), \( p_i^M \) and \( q_i^M \) do not depend on \( s_i \), then the following inequality must hold:

\[ \sum_{i \in A} E_{s,A}[\Lambda_i^N(s_A)] \geq \sum_{i \in A} E_{s,A}[\Lambda_i^M(s_A)] \text{ for all } s_A \in S_A \quad (23) \]

The inequality means that sum of agents’ risk-neutral expected payoffs that are in \( A \) is greater when the network is \( N \) compared to that of network \( M \).

**Proof.** I prove that the following holds: For all intervals of types \( S_i' = [s_i, \overline{s}_i] \subseteq S_i \)
$i \in A$ such that $\prod_{i \in A} S'_i$ has a positive measure,

$$
\sum_{i \in A} E[\Lambda_i^N(s) | \pi_A(s) \in \prod_{i \in A} S'_i] \geq \sum_{i \in A} E[\Lambda_i^M(s_A, s_{-A}) | \pi_A(s) \in \prod_{i \in A} S'_i]
$$

(24)

This is equivalent to (23). Let $(P, Q, U)$ be a settlement mechanism that does not satisfy (24), i.e., for some interval of types $S'_i = [s_{\underline{i}}, s_{\bar{i}}] \subseteq S_i$ $i \in A$ such that $\prod_{i \in A} S'_i$ has a positive measure, the opposite of (24) holds. I construct a blocking mechanism $(P', Q', U')$ in which agent $i \in A$ joins the blocking mechanism if $s_i \in S'_i$. For all $i \in A$ and $s \in S_K$, set the quantity as follows: $q_i(s) = q_i^N(s) - q_i^M(s)$

$$
q_i'(s) = \begin{cases} 
q_i^N(s) - q_i^M(s) & \text{if } s_{\underline{i}} \leq s_i \leq s_{\bar{i}}, \\
q_i^N(s_{\underline{i}}, s_{-i}) - q_i^M(s_{\underline{i}}, s_{-i}) & \text{if } s_i > s_{\bar{i}}, \\
q_i^N(s_{\bar{i}}, s_{-i}) - q_i^M(s_{\bar{i}}, s_{-i}) & \text{if } s_i < s_{\underline{i}}
\end{cases}
$$

I set the price for the blocking mechanism as follows:

$$
p_i'(s) = \begin{cases} 
\hat{p}_i(s) & \text{if } s_{\underline{i}} \leq s_i \leq s_{\bar{i}}, \\
\hat{p}_i(s_{\underline{i}}, s_{-i}) & \text{if } s_i > s_{\bar{i}}, \\
\hat{p}_i(s_{\bar{i}}, s_{-i}) & \text{if } s_i < s_{\underline{i}}
\end{cases}
$$

where $\hat{p}_i(s)$ is the unique solution to the following equation:

$$
\Lambda_i^N(s) = \Lambda_i^M(s) + (n_i - m_i - q_i(s))(100 - v(s)) + q_i'(s)(100 - \hat{p}_i(s)).
$$

This equation has a unique solution, as the right-hand side is linear in $\hat{p}_i(s)$. Note that, by construction, if $s_{\underline{i}} \leq s_i \leq s_{\bar{i}}$ then $\Lambda_i'(s) = \Lambda_i^N(s) - \Lambda_i^M(s)$.

I prove that the mechanism $(P', Q', U')$ is ex-post incentive compatible. Because the settlement mechanism is incentive compatible when the network is $N$ and $q^M(\cdot, s_{-i})$ does not depend on $s_i$, for all $i \in A$ and $(s'_i, s_{-i}), (s_i, s_{-i}) \in S_K$,

$$
\Lambda_i^M(s'_i, s_{-i}) - \Lambda_i^M(s_i, s_{-i}) = q_i^M(s_i, s_{-i})(E[v | s'_i, s_{-i}] - E[v | s_i, s_{-i}])
$$

$$
\Lambda_i^N(s'_i, s_{-i}) - \Lambda_i^N(s_i, s_{-i}) \geq q_i^N(s_i, s_{-i})(E[v | s'_i, s_{-i}] - E[v | s_i, s_{-i}])
$$

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Therefore, if \( s_i \leq s_i, s'_i \leq \overline{s}_i \),

\[
\Lambda_i'(s'_i, s_{-i}) - \Lambda_i'(s_i, s_{-i}) \geq q_i'(s_i, s_{-i})(E[v|s'_i, s_{-i}] - E[v|s_i, s_{-i}])
\]

This proves that the blocking mechanism is incentive compatible in this case. Note that the settlement mechanism sets the same price and quantity if \( s_i, s'_i > \overline{s}_i \) or if \( s_i, s'_i < s_i \). Therefore, the incentive compatibility is trivial for those cases. If \( s'_i > \overline{s}_i \) and \( s_i \in [s_i, \overline{s}_i] \), then, by construction and incentive compatibility of the blocking mechanism in \([s_i, \overline{s}_i]\), the following holds.\(^{18}\)

\[
\Lambda_i'(s'_i, s_{-i}) = \Lambda'(\overline{s}_i, s_{-i}) + q_i'(\overline{s}_i, s_{-i})(E[v|s'_i, s_{-i}] - E[v|\overline{s}_i, s_{-i}]) \geq
\]

\[
\Lambda'(s_i, s_{-i}) + q_i'(s_i, s_{-i})(E[v|s'_i, s_{-i}] - E[v|s_i, s_{-i}]) + q_i'(s_i, s_{-i})(E[v|\overline{s}_i, s_{-i}] - E[v|s_i, s_{-i}])
\]

\[
\geq \Lambda'(s_i, s_{-i}) + q_i'(s_i, s_{-i})(E[v|s'_i, s_{-i}] - E[v|s_i, s_{-i}])
\]

This proves the ex-post inventive compatibility for this case. All other cases admit a similar proof. Inequality (6) is satisfied by construction. To check inequality (7), let \( i \in A, (s_i, s_{-i}) \in \prod_{j \in A} S'_j \) and \( s'_i > s_i \). For economy of exposition, let \( s = (s_i, s_{-i}), s' = (s'_i, s_{-i}) \) and \( \overline{s} = (\overline{s_i}, s_{-i}) \). Incentive compatibility of the settlement mechanism when the network is \( N \), construction of the blocking mechanism, and that prices and quantities do not depend on the agent’s signal when the signal is \( M \) imply the following:

\[
\Lambda_i^N(s') \geq \Lambda_i^N(\overline{s}) + q_i^N(\overline{s})(E[v|s'] - E[v|\overline{s}]) = \Lambda_i^M(\overline{s}) + \Lambda_i'(\overline{s}) + q_i^N(\overline{s})(E[v|s'] - E[v|\overline{s}])
\]

\[
= \Lambda_i^M(s') + \Lambda_i'(s') + (q_i^N(\overline{s}) - q_i^M(\overline{s}) - q_i'(\overline{s}))(E[v|s'] - E[v|\overline{s}]) = \Lambda_i^M(s') + \Lambda_i'(s')
\]

If \( s'_i < s_i \), then:

\[
\Lambda_i^N(s') \geq \Lambda_i^N(s) + q_i^N(s)(E[v|s'] - E[v|s]) = \Lambda_i^M(s) + \Lambda_i'(s) + q_i^N(s)(E[v|s'] - E[v|s])
\]

\[
= \Lambda_i^M(s') + \Lambda_i'(s') + (q_i^N(s) - q_i^M(s) - q_i'(s))(E[v|s'] - E[v|s]) = \Lambda_i^M(s') + \Lambda_i'(s')
\]

These prove inequality (7). Because the opposite of (24) holds, the blocking mechanism designer has a positive expected payoff. \( \Box \)

\(^{18}\)Because the mechanism is ex-post incentive compatible in \( S'_i, q_i'(\overline{s}_i, s_{-i}) \geq q_i'(s_i, s_{-i}) \)
Lemma 10.2. Consider an agent $i \in K$ and two ex-post incentive compatible settlement mechanisms with price, quantity, and risk-neutral payoff functions $p_i, q_i, \Lambda_i$ and $p'_i, q'_i, \Lambda'_i$. If $q'_i(\cdot)$ and $p'_i(\cdot)$ do not depend on $s_i$ for all $s_i \in (0, 1)$ and almost surely for all $s \in S_K$, then almost surely $q_i(s) = q'_i(s)$ and $p_i(s) = p'_i(s)$.

Proof. Let $E$ be that set of signal profiles $s_{-i}$ such that, for almost all $s_i \in [0, 1]$, $\Lambda_i(s_i, s_{-i}) = \Lambda'_i(s_i, s_{-i})$. Note that, because $\Lambda_i(s) = \Lambda'_i(s)$ holds almost surely, the complement of $E$ is measure zero. Given $s_{-i} \in E$, let $s_i, s'_i, s''_i$ be such that $\Lambda_i(s_i, s_{-i}) = \Lambda'_i(s_i, s_{-i})$, $\Lambda_i(s'_i, s_{-i}) = \Lambda'_i(s'_i, s_{-i})$, $\Lambda_i(s''_i, s_{-i}) = \Lambda'_i(s''_i, s_{-i})$ and $s'_i < s_i < s''_i$. Ex-post incentive compatibility implies that:

$$\Lambda_i(s'_i, s_{-i}) - \Lambda_i(s_i, s_{-i}) \geq q_i(s_i, s_{-i})(E[v|s'_i, s_{-i}] - E[v|s_i, s_{-i}]).$$

Note that

$$\Lambda_i(s'_i, s_{-i}) - \Lambda_i(s_i, s_{-i}) = \Lambda'_i(s'_i, s_{-i}) - \Lambda'_i(s_i, s_{-i}) = q'_i(s_i, s_{-i})(E[v|s_i, s_{-i}] - E[v|s'_i, s_{-i}])$$

$$+ \left[ \sum_{i \in A} E[\Lambda_i'N(s)] \right] \geq \sum_{i \in A} E[\Lambda_i'M(s)] \text{ for all } s \in S.$$

Because the mechanism is weakly-unbiased, the following holds:

$$\sum_{i \in A} E[p_i[\Lambda_i'N(s)] = \sum_{i \in A} E[p_i[\Lambda_i'M(s)].$$

Equality (39) and Proposition [10.1] imply that, almost surely for all $s \in S_K$, the following holds:

$$\sum_{i \in A} \Lambda_i'N(s) = \sum_{i \in A} \Lambda_i'M(s).$$

\[\square\]
Therefore, \( q'_i(s_i, s_{-i}) \geq q_i(s_i, s_{-i}) \). Considering ex-post incentive compatibility for types \( s_i \) and \( s'_i \) implies that \( q'_i(s_i, s_{-i}) \leq q_i(s_i, s_{-i}) \). Therefore, \( q'_i(s_i, s_{-i}) = q_i(s_i, s_{-i}) \) and \( p'_i(s_i, s_{-i}) = p_i(s_i, s_{-i}) \). One can choose \( s'_i \) close enough to 0 and \( s''_i \) close enough to 1. Therefore, almost surely for all \( s_i \in [0, 1] \), \( q'_i(s_i, s_{-i}) = q_i(s_i, s_{-i}) \) and \( p'_i(s_i, s_{-i}) = p_i(s_i, s_{-i}) \).

**Lemma 10.3.** Assume \( N \) is a network where only two pairs of agents have contracts. If the mechanism \((P, Q, U)\) is ex-post individually rational, full information unravel-proof, ex-ante weakly budget balanced, and weakly-unbiased, then almost surely (i) \( q^N_i(s) = 0 \) for all signal profiles \( s \in [0, 1]^{|K|} \) and agents \( i \in K \) who do not have any CDS contracts in network \( N \) and (ii) the mechanism is almost surely ex-post budget balanced.

**Proof.** Consider the contract matrix in which no agent has a CDS contract, namely, the 0 network. Because the mechanism is ex-post individually rational, \( U_0^i(s) \geq 0 \) for all \( i \in K \); therefore, \( \Lambda_0^i(s) \geq 0 \) for all \( i \in K \). Ex-ante weakly budget balanced condition implies that \( \sum_{i \in K} E_{\rho}[\Lambda_0^i(s)] \leq 0 \). Hence, almost surely, \( \Lambda_0^i(s) = 0 \) for all \( i \in K \). In network \( N \), consider a block by the two agents who have CDS contracts. Lemma 6.3 implies that, for all \( s \in S_K \), the following holds almost surely:

\[
\sum_{i \in K(N)} \Lambda^N_i(s) = 0. \tag{28}
\]

Ex-post budget balancedness implies that, for all \( i \in K \setminus K(N) \) and \( s \in S_K \):

\[
\Lambda^N_i(s) \geq 0. \tag{29}
\]

Because the mechanism is ex-ante weakly budget balanced, the following inequality holds:

\[
E_{\rho}[\sum_{i \in K} \Lambda^N_i(s)] \leq 0. \tag{30}
\]

Inequalities (28), (29), and (30) imply that, almost surely for all \( i \in K \setminus K(N) \) and \( s \in S_K \), \( \Lambda^N_i(s) = 0 \) and \( \sum_{i \in K(N)} \Lambda^N_i(s) = 0 \). Therefore, \( \sum_{i \in K} \Lambda^N_i(s) = 0 \) for all \( i \in K \). Moreover, lemma 10.2 implies that for all \( i \in K \setminus K(N) \) and almost all \( s \in S_K \), \( q^N_i(s) = 0 \). \( \square \)
I establish a generalization of the induction base as a lemma.

**Lemma 10.4.** If in network $N$, agents $i$ and $j$ have CDS contracts only with each other and for almost all $s \in K$ $q_i^N(s) + q_j^N(s) = 0$, then, almost surely, $q_i^N(s)$, $p_i^N(s)$, $q_j^N(s)$ and $p_j^N(s)$ do not depend on $s_i$ and $s_j$ and $p^N(s) = p^N(s)$. 

**Proof.** Consider a block where agents $i$ and $j$ choose to side settle all of their contracts. Lemma 6.3 implies that $\Lambda_i^N(s) + \Lambda_j^N(s) = 0$ for almost all $s \in S_K$. Set

$$A = \{s_{-i,j} \in [0,1]|^{[k-2]} q_i^N(s_i, s_{-i}) + q_j^N(s_i, s_{-i}) = 0 \text{ for almost all } (s_i, s_j) \in [0,1]^2\}$$

$$A_{s_{-i,j}} = \{(s_i, s_j) \in (0,1)^2|q_i^N(s_i, s_{-i}) + q_j^N(s_i, s_{-i}) = 0\}$$

The assumption in the lemma implies that $A$ has probability 1. Given $s_{-i,j} \in A$ and $(s_i, s_j) \in A_{s_{-i,j}}$, let $(s_i', s_j) \in A_{s_{-i,j}}$ and $(s_i, s_j') \in A_{s_{-i,j}}$ be such that $E[v|s_i', s_j] = E[v|s_i, s_j']$. Note that, because $v(s)$ is increasing and continuous, by definition of $A$, I can always find $s_i', s_j'$ such that $s_i' > s_j$. Ex-post incentive compatibility implies that:

$$q_i^N(s_i, s_j, s_{-i,j})(E[v|s_i', s_j, s_{-i,j}] - E[v|s_i, s_j, s_{-i,j}]) \leq \Lambda_i^N(s_i', s_j, s_{-i,j}) - \Lambda_i^N(s_i, s_j, s_{-i,j})$$

$$q_j^N(s_i', s_j', s_{-i,j})(E[v|s_i', s_j', s_{-i,j}] - E[v|s_i', s_j', s_{-i,j}]) \leq \Lambda_j^N(s_i', s_j', s_{-i,j}) - \Lambda_j^N(s_i', s_j', s_{-i,j})$$

$$q_i^N(s_i', s_j', s_{-i,j})(E[v|s_i, s_j', s_{-i,j}] - E[v|s_i', s_j', s_{-i,j}]) \leq \Lambda_i^N(s_i, s_j', s_{-i,j}) - \Lambda_i^N(s_i', s_j', s_{-i,j})$$

$$q_j^N(s_i, s_j, s_{-i,j})(E[v|s_i, s_j, s_{-i,j}] - E[v|s_i, s_j, s_{-i,j}]) \leq \Lambda_j^N(s_i, s_j, s_{-i,j}) - \Lambda_j^N(s_i, s_j, s_{-i,j})$$

(31)

Note that the left- and right-hand side of all these sum to zero; therefore, all of these inequalities must hold with equality. Equality of inequality (31) means that agent $i$ is indifferent about reporting $s_i$ and $s_i'$ when his signal is, in fact, $s_i'$. Let $s_i''$ be a signal for agent $i$ such that $s_i < s_i'' < s_i'$, ex-post incentive compatibility implies that:

$$q_i^N(s_i'', s_j, s_{-i,j})(E[v|s_i', s_j, s_{-i,j}] - E[v|s_i'', s_j, s_{-i,j}]) \leq \Lambda_i^N(s_i', s_j, s_{-i,j}) - \Lambda_i^N(s_i'', s_j, s_{-i,j})$$

(32)

$$q_i^N(s_i, s_j, s_{-i,j})(E[v|s_i'', s_j, s_{-i,j}] - E[v|s_i, s_j, s_{-i,j}]) \leq \Lambda_i^N(s_i', s_j, s_{-i,j}) - \Lambda_i^N(s_i, s_j, s_{-i,j})$$

(33)

$$q_i^N(s_i, s_j, s_{-i,j}) \leq q_i^N(s_i'', s_j, s_{-i,j})$$

(34)

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Adding inequalities (32) and (33) and applying inequality (34) implies inequality (31). Because inequality (31) holds with equality, inequalities (32), (33) and (34) hold with equality as well. This implies that, for all $s_i < s_i' < s_i'':$

$$q_i^N(s_i, s_j, s_{-i,j}) = q_i^N(s_i'', s_j, s_{-i,j})$$

I can choose $s_i' < s_i$ as well. Therefore, I conclude that, in a neighborhood around $s_i$, the function $q_i(., s_i, s_{-i,j})$ is constant. Therefore, $q_i^N(., s_j, s_{-i,j})$ almost surely does not depend on $s_i$. A similar argument holds for agent $j$. This proves the lemma. □

If only two agents, $i$ and $j$, have CDS contracts, then lemma 10.3 implies that, almost surely, the mechanism sets a quantity of zero for all other agents; therefore, the assumption of lemma 10.4 is valid. Lemma 10.4 implies that, when only two agents have CDS contracts with each other, then the price and the quantity do not depend on agents’ signals. If agents $i$ and $j$ have $n$ contracts, let $p^n_{i,j}(s_{-i,j}) = p^n_{j,i}(s_{-i,j})$ and $q^n_{i,j}(s_{-i,j}) = -q^n_{j,i}(s_{-i,j})$ be the price and quantity. Consider a network where three agents, $i$, $j$ and $k$, have CDS contracts as follows: $i$ has $n$ CDS contracts with $j$, and $j$ has $n$ CDS contracts with $k$. Assume $i$ is a CDS buyer and $k$ is a CDS seller. Call this network $M$. Apply lemma 6.3 for $A = \{i, j\}$ and conclude:

$$\Lambda^M_i(s) = n(100 - v(s)) + q^n_{j,k}(s_{-j,k})(v(s) - p^n_{j,k}(s_{-j,k}))$$

(35)

Note that, in network $M$, agent $j$ has net zero CDS contracts. Robustness with respect to network implies that

$$\Lambda^M_i(s) = n(100 - v(s)) + q^n_{i,k}(s_{-i,k})(v(s) - p^n_{i,k}(s_{-i,k}))$$

(36)

Equations (35) and (36) imply that

$$q^n_{j,k}(s_{-j,k})(v(s) - p^n_{j,k}(s_{-j,k})) = q^n_{i,k}(s_{-i,k})(v(s) - p^n_{i,k}(s_{-i,k}))$$

(37)

Note that the only term that depends on $s_i$ in both sides of equation (37) is $v(s)$. Therefore, equation (37) implies that $q^n_{j,k}(s_{-j,k}) = q^n_{i,k}(s_{-i,k})$ and $q^n_{j,k}(s_{-j,k}) = q^n_{i,k}(s_{-i,k})$. There-
fore, it must be that $q_{i,j}(s_{-i,j}) = q^n$ and $p_{i,j}(s_{i,j}) = p^n$ for all $i, j \in K$. Because the mechanism is unbiased, $p^n = E_{\rho}[v]$. This proves the induction base.

Given network $N$, let $f(N)$ be the number of pairs of agents who have contracts with each other and $g(N)$ be the number of pairs of agents who have contracts with each other and do not have contracts with other agents. I prove the result by induction on $h(N) = 2f(N) + g(N)$. So far, I have established the result for $h(N) \leq 3$. Given network $N$, where $h(N) > 3$ and $i \in K(N)$, I show that $\Lambda_i^N$ has the form described in the theorem. There are four cases:

- **Case 1:** For some $j$ and $k \in K$, $i$ has contracts with $j$, $j$ has some contracts with $k$, and $k$ has contract with $i$.

**Proof:** construct contract matrix $N^i, N^j$ and $N^k$ as follows: $N^i$ is constructed from $N$ by removing the contracts between $j$ and $k$, $N^j$ is constructed by removing the contracts between $i$ and $k$, and $N^k$ is constructed from $N$ by removing the contracts between $i$ and $j$. Lemma [6.3] implies:

\[
\Lambda_i^N(s) + \Lambda_j^N(s) = \Lambda_i^{N^k}(s) + \Lambda_j^{N^k}(s), \tag{38}
\]

\[
\Lambda_i^N(s) + \Lambda_k^N(s) = \Lambda_i^{N^j}(s) + \Lambda_k^{N^j}(s), \tag{39}
\]

\[
\Lambda_j^N(s) + \Lambda_k^N(s) = \Lambda_j^{N^i}(s) + \Lambda_k^{N^i}(s). \tag{40}
\]

Equalities \[(38), (39)\] and \[(40)\] imply that

\[
2\Lambda_i^N(s) = \Lambda_i^{N^k}(s) + \Lambda_j^{N^k}(s) + \Lambda_i^{N^j}(s) + \Lambda_k^{N^j}(s) - \Lambda_j^{N^i}(s) - \Lambda_k^{N^i}(s) \tag{41}
\]
Case 2: For some \( j \) and \( k \) the induction hypothesis implies that

\[
2 \Lambda_i^N(s) = \sum_{l \in K \setminus \{j\}} n_{i,l}(100 - v(s)) + q_{i,l}(v(s) - p) + \sum_{l \in K \setminus \{i\}} n_{j,l}(100 - v(s)) + q_{j,l}(v(s) - p) \tag{42}
\]

\[
+ \sum_{l \in K \setminus \{k\}} n_{k,l}(100 - v(s)) + q_{k,l}(v(s) - p) \tag{43}
\]

\[
- \sum_{l \in K \setminus \{j\}} n_{j,l}(100 - v(s)) + q_{j,l}(v(s) - p) - \sum_{l \in K \setminus \{i\}} n_{k,l}(100 - v(s)) + q_{k,l}(v(s) - p) \tag{44}
\]

\[
= 2 \sum_{l \in K} n_{i,l}(100 - v(s)) + q_{i,l}(v(s) - p) - n_{i,j}(100 - v(s)) - q_{i,j}(v(s) - p) - n_{j,k}(100 - v(s)) - q_{j,k}(v(s) - p) + n_{j,k}(100 - v(s)) - q_{j,k}(v(s) - p)
\]

\[
+ n_{k,j}(100 - v(s)) - q_{k,j}(v(s) - p) \tag{45}
\]

Note that for all \( r, s \in K \), \( n_{r,s} + n_{s,r} = 0 \) and \( q^n + q^{-n} = 0 \) for all \( n \in \mathbb{Z} \), hence, equality (45) implies that

\[
\Lambda_i^N(s) = \sum_{l \in K} n_{i,l}(100 - v(s)) + q_{i,l}(v(s) - p)
\]

- Case 2: For some \( j \) and \( k \) in \( K \), \( i \) has contracts with \( j \), \( j \) has some contracts with \( k \), and \( k \) and \( i \) do not have contracts with each other.

Proof. Remove the contracts between \( i \) and \( j \) and call the network \( N^{i,j} \). Also, remove the contracts between \( j \) and \( k \) and call the network \( N^{i,j} \). Lemma 6.3 implies that for all \( s \in S \):

\[
\Lambda_i^N(s) + \Lambda_j^N(s) = \Lambda_k^N(s) + \Lambda_j^N(s) \tag{46}
\]
I construct network $M$ as follows:

$$m_{r,s} = \begin{cases} 
n_{i,j} & \text{if } (r, s) = (i, k) 
n_{j,i} + n_{j,k} & \text{if } (r, s) = (j, k) 
0 & \text{if } (r, s) = (i, j) 
-m_{s,r} & \text{if } (r, s) \in \{(k, i), (k, j), (j, i)\} 
n_{r,s} & \text{otherwise}
\end{cases}$$

(47)

Note that $h(M) = h(N)$. Robustness with respect to network implies $\Lambda^M_r(s) = \Lambda^N_r(s)$ for all $r \in K$ and $s \in S$. In network $M$, remove the contracts between $j$ and $k$ and call the network $M'$. Note that $h(M') < h(N)$. Lemma 6.3 implies that for all $s \in S$:

$$\Lambda^N_j(s) + \Lambda^N_k(s) = \Lambda^M_j(s) + \Lambda^M_k(s) = \Lambda^{M'}_j(s) + \Lambda^{M'}_k(s).$$

(48)

Note that the inductive hypothesis applies to $M^i, N^i, N^j$ and $N^k$. Equations (47), (48) and similar argument as in the previous case proves the result for this case.

□

- Case 3: agent $i$ has contracts with two agents $j$ and $k$. This case admits a similar proof to previous cases.

- Case 4: agent $i$ only has contracts with agent $j \in K$ and $j$ has no contract with other agents. Since $f(N) \geq 3$, there exists an agent $k \in K$ who has contracts with other agents. Construct network $M$ from $N$ as follows: remove the contracts between $i$ and $j$, add contracts between $i & k$ and $j & k$, such that $m_{i,k} = n_{i,j}$ and $m_{j,k} = n_{j,i}$. Note that all agents have the same net number of contracts in $M$ and $N$, therefore, robustness with respect to network implies $\Lambda^N_i(s) = \Lambda^M_i(s)$. The rest of the proof is similar to case 2.
10.4 Proof of Theorem 6.4

I establish the result of lemma 6.3 for the case of incomplete information unravel-proof mechanisms. Let networks \( M, N \) satisfy \( M \prec N \) and assume \( p_i^M \) and \( q_i^M \) do not depend on the reported messages. If, for some non-zero measure set of signals \( \Pi_{i \in A} S'_i \), the inequality

\[
E[\sum_{i \in A} \Lambda^N_i(s) | \pi_A(s) \in \Pi_{i \in A} S'_i] < E[\sum_{i \in A} \Lambda^M_i(s) | \pi_A(s) \in \Pi_{i \in A} S'_i]
\]

holds, then one can design a blocking mechanism similar to proposition 10.1. This implies that, for all non-zero measure \( \Pi_{i \in A} S'_i \), the following holds:

\[
E[\sum_{i \in A} \Lambda^N_i(s) | \pi_A(s) \in \Pi_{i \in A} S'_i] \geq E[\sum_{i \in A} \Lambda^M_i(s) | \pi_A(s) \in \Pi_{i \in A} S'_i].
\]

Equality (49) and (50) and assumption 4.1 imply that, almost surely, for all \( s \in S_K \):

\[
\sum_{i \in A} \Lambda^N_i(s) = \sum_{i \in A} \Lambda^M_i(s).
\]

A similar argument in the proof of lemma 6.3 implies the following: For almost all \( i \in K \setminus K(N) \): \( E_\mu[\Lambda^N_i(s)] = 0 \) holds. Assumption 4.1 implies \( \Lambda^N_i(s) = 0 \) for all networks \( N \) and agents \( i \in K \setminus K(N) \). This proves the result of lemma 6.3 in this case. Proof of lemma 10.3 is followed from lemma 6.3. The rest of the proof is identical to the proof of Theorem 6.2.

10.5 Proof of Theorem 6.5

Lemma 10.5. Let networks \( M, N \) satisfy \( M \prec N \), and \( A \) is the set of agents who reduced the network from \( N \) to \( M \). If the mechanism is payoff equivalent to a posted price
mechanism when the network is M, then the following holds:

\[ \sum_{i \in A} \Lambda_i^N(s) = \sum_{i \in A} \Lambda_i^M(s). \]

**Proof:** Assume for some positive measure set of types \( \Pi_{i \in A} S_i' \), the following inequality holds:

\[ E[\sum_{i \in A} \Lambda_i^N(s) | \pi_A(s) \in \Pi_{i \in A} S_i'] < E[\sum_{i \in A} \Lambda_i^M(s) | \pi_A(s) \in \Pi_{i \in A} S_i']. \] (52)

Consider a participation-choice in which agents in \( A \) block the mechanism and reduce the network from \( N \) to \( M \) (see proposition 5.2). Because the mechanism is unbiased, for all \( i \in K \), the following holds:

\[ E[\rho_i(\Lambda_i^M(s) I_{\{\pi_A(s) \in \Pi_{i \in A} S_i' \}} + \Lambda_i^N(s) I_{\{\pi_A(s) \in \Pi_{i \in A} S_i' \}}] = E[\rho_i(n_i(100 - v(s))] \] (53)

I sum these equalities for all \( i \in K(N - M) \) and apply equation (52) to conclude:

\[ E[\rho_i(\sum_{i \in A} \Lambda_i^M(s))] > E[\rho_i(\sum_{i \in A} n_i(100 - v(s)))] = E[\rho_i(\sum_{i \in A} m_i(100 - v(s)))] \] (54)

Consider a participation-choice in which the \( M \) network does not unravel; unbiasedness implies that (54) must hold with equality. This contradiction implies that (52) does not hold. I apply the unbiasedness for the participation choice with no unraveling to conclude: For all positive measure \( \pi_A(s) \in \Pi_{i \in A} S_i' \)

\[ E[\sum_{i \in A} \Lambda_i^N(s) | \pi_A(s) \in \Pi_{i \in A} S_i'] = E[\sum_{i \in A} \Lambda_i^M(s) | \pi_A(s) \in \Pi_{i \in A} S_i']. \] (55)

Assumption 4.1 implies that, almost surely,

\[ \sum_{i \in A} \Lambda_i^N(s) = \sum_{i \in A} \Lambda_i^M(s). \] (56)

The rest of the proof is the replicate of the proof of Theorem 6.2.
10.6 Proof of Theorem 7.1

Lemma 6.3 can be extended to this case. Because the new notions of unbiasedness and unravel-proofness are stronger, and cases 2, 3 and 4 of the proof of theorem 6.2 are the only parts that use the property of robustness with respect to the network of contracts, all I need to do is to replace these steps. Here is the replacement:

- Case 2: For some \( j \) and \( k \in K \), \( i \) has contracts with \( j \), \( j \) has some contracts with \( k \), and \( k \) and \( i \) do not have contracts with each other.

Proof. Construct network \( N_i \) by removing the contracts between \( j \) and \( k \), and \( N_k \) by removing the contracts between \( i \) and \( j \). If \( i \) sells all his contracts with \( j \) to agent \( k \), network \( M \) is constructed. Lemma 6.3 implies:

\[
\begin{align*}
\Lambda_i^N(s) + \Lambda_j^N(s) &= \Lambda_i^{N_i}(s) + \Lambda_j^{N_i}(s) \\
\Lambda_j^N(s) + \Lambda_k^N(s) &= \Lambda_j^{N_j}(s) + \Lambda_k^{N_j}(s) \\
\Lambda_i^N(s) + \Lambda_j^N(s) + \Lambda_k^N(s) &= \Lambda_i^M(s) + \Lambda_j^M(s) + \Lambda_k^M(s)
\end{align*}
\]

Similar argument as in Case 2 of theorem 6.2 shows that equations (57) prove the result. \( \square \)

- Case 3: agent \( i \) has contracts with two agents \( j \) and \( k \).

Proof. The proof of this case is similar to the previous case. \( \square \)

- Case 4: agent \( i \) only has contracts with agent \( j \in K \) and \( j \) has no contract with other agents.

Proof. Let \( k_1, k_2 \in K \setminus \{i, j\} \) be a pair of agents who have contracts with each other. I construct network \( M_i \), \( M_j \) and \( M \) from \( N \) as follows:

\( M_i \) is constructed by selling \( i \)'s contracts with \( j \) to \( k \).

\( M_j \) is constructed by selling \( j \)'s contracts with \( i \) to \( k \).

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$M$ is constructed by removing the contract between $i$ and $j$. Note that $h(M) < h(N)$, $h(M_i) < h(N)$ and $h(M_j) < h(N)$, therefore, the result of the theorem holds for $M$, $M_i$ and $M_j$. Lemma 6.3 implies:

\[
\Lambda_i^N(s) + \Lambda_k^N(s) = \Lambda_i^M(s) + \Lambda_k^M(s) \\
\Lambda_j^N(s) + \Lambda_k^N(s) = \Lambda_j^M(s) + \Lambda_k^M(s) \\
\Lambda_i^N(s) + \Lambda_j^N(s) = \Lambda_i^M(s) + \Lambda_j^M(s) \tag{58}
\]

Similar argument as in Case 1 of theorem 6.2 shows that equations (58) prove the result. □

10.7 Proof of Proposition 7.2

The unbiased and weakly-unbiased properties have been used in equations (39), (50), and (53). These equations still hold if I make the replacement. However, it has been argued that equation (51) contradicts with the unbiasedness property. This equation also contradicts with the ex-ante uniform price property.

References


