Ownership Structure, Incentives and Asset Prices

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Abstract

We develop a dynamic equilibrium model to study the interplay among managerial incentive contracts, the ownership dynamics of large controlling shareholders and asset prices. Our unified framework synthesizes an asset pricing model with a dynamic principal-agent model by modeling large shareholders as mediators who determine incentive contracts for firm managers, while also influencing asset prices through their dynamic trading decisions. Assuming a Gaussian output process and CARA preferences for agents, we explicitly characterize the equilibrium in which long-term contracts for managers, the ownership decisions of large shareholders and asset prices are simultaneously and endogenously determined. The explicit incorporation of the separation between ownership and control results in a stochastically evolving stock price in sharp contrast with the benchmark owner-manager case in which the stock price is deterministic. Agency conflicts between large shareholders and managers lead to less volatile stock returns, lower expected stock returns and lower Sharpe ratios, ceteris paribus. We derive a number of empirical implications for the relations among block ownership, managerial incentives and stock returns. The sensitivity of managerial pay to stock returns is negatively related to the expected stock return and volatility. Block ownership negatively affects the expected stock return and volatility, but positively affects incentives suggesting that block ownership and explicit contracts serve as complementary mechanisms in corporate governance.
1 Introduction

Traditional theoretical research in financial economics reveals a fundamental dichotomy between its two cornerstones; asset pricing and corporate finance (Gorton, He and Huang (2014)). At a broad level, corporate finance research focuses on the separation of ownership and control in modern corporations. Agency conflicts among a firm’s various stakeholders that arise from this separation endogenously influence firms’ cash flows in various corporate finance models. The market’s valuation of the cash flows through the underlying stochastic discount factor or pricing kernel is, however, typically exogenously specified. In contrast, traditional asset pricing models usually view corporate cash flows as exogenous and focus on identifying a stochastic discount factor or pricing kernel to price the assumed cash flows. In reality, however, firms’ earnings and their valuation by market participants are simultaneously and endogenously determined.

A nascent, but growing literature aims to bridge the gap by examining the feedback between the effects of agency conflicts among firms’ stakeholders on cash flows and security prices that represent the values of various claims to the cash flows. One group of extant studies (e.g., Admati, Pfleiderer and Zechner (1994), DeMarzo and Urosevic (2006), and Gorton et al. (2014)) specifically focuses on the role that asset prices play in facilitating risk-sharing between the small and large shareholders of firms and, thereby, the incentives of large shareholders who control firms. These studies abstract away from non-market mechanisms such as incentive contracts that play a central role in mediating the tradeoff between risk-sharing and incentives. Another group of studies examines how contracts interact with asset prices (e.g., Ou-Yang (2005)), but abstract away from the distinction between large and small shareholders. The frameworks analyzed by these studies confront the fundamental conceptual problem that stems from assuming that shareholders are competitive price-takers, but are nevertheless able to coordinate with each other to enforce managerial incentive contracts.

We contribute to the literature by developing a dynamic equilibrium model to demonstrate the interplay among managerial incentive contracts, which are determined by large shareholders, asset prices, and the ownership dynamics of large shareholders. Our perspective is motivated by empirical evidence that the vast majority of firms around the world are controlled by large shareholders (e.g., Holderness (2009)) as well as theoretical research that highlights the importance of large shareholders in influencing corporate governance (e.g., Shleifer and Vishny (1986)).
unified framework integrates an asset pricing model with a principal-agent model by viewing large
shareholders as mediators who determine incentive contracts for managers, while also influencing
asset prices through their trading decisions.

Using our framework, we provide a full-fledged and dynamic equilibrium description of the inter-
actions among the ownership dynamics of large shareholders, managerial contracts and stock
prices. In the benchmark owner-manager case, which corresponds to the scenario studied by De-
Marzo and Urosevic (2006; hereafter D-U), one obtains the counterfactual implication that stock
prices are deterministic functions of time. In contrast, the incorporation of the separation between
large shareholders and the manager results in stochastically evolving stock prices stemming from
the provision of incentives to managers. Relative to the benchmark owner-manager scenario, agency
conflicts between large shareholders and managers lead to less volatile stock returns, lower expected
stock returns and lower Sharpe ratios, ceteris paribus. Our results could partly explain the higher
equity risk premium in emerging markets in which owner-managed (or family controlled) firms
are more prevalent than in developed markets. Interestingly, the separation between ownership
and control, and the facilitation of risk-sharing through optimal contracts, effectively insulate the
ownership stakes of large shareholders from firm-specific parameters such as the mean and volatil-
ity of earnings, and are largely determined by the risk aversions of large and small shareholders.
We also obtain testable implications for the equilibrium relations among block ownership stakes,
expected excess returns and volatilities of stock prices, and managerial incentives. In particular,
we highlight the importance of examining the relations among these endogenous variables in an
equilibrium framework.

We model an infinite horizon, continuous-time economy with a representative all-equity firm.
The firm’s shareholders comprise of small shareholders, who are competitive price-takers, and a
representative large shareholder who hires the firm’s manager. The manager influences the firm’s
earnings through her unobservable effort. All agents can dynamically consume over time and
also have access to a risk-free savings technology. The manager’s savings are observable so that
we can, without loss of generality, restrict consideration to “savings proof” contracts, that is,
contracts under which the manager consumes his compensation payments. As in D-U, the large
shareholder can trade the firm’s stock at discrete dates, while small shareholders can trade the
stock continuously. The large shareholder cannot commit to her trading policy. The stock price
is endogenously determined at each instant of time by the market clearing condition for the firm’s equity.

The large shareholder offers the manager a long-term incentive contract that is contingent on the firm’s earnings. The two parties can renegotiate their existing contract at each trading date, but any renegotiation must be weakly Pareto improving for both parties to be accepted so that we can restrict consideration to renegotiation-proof contracts without loss of generality. The large shareholder’s trades influence the terms of the manager’s contracts that affect the manager’s effort and the firm’s earnings. The firm’s earnings, in turn, influence the stock price and, therefore, the large investor’s trades. In this manner, the large shareholder’s trading dynamics, managerial incentives and the stock price are simultaneously and endogenously determined. For tractability, we restrict consideration to Strong Markov Perfect Public Equilibria in which the large shareholder’s contract choices in any period depend only on the state vector that comprises of her ownership at the beginning of the period, her holdings in the money market account and the manager’s promised payoff or expected continuation utility from her long-term contract.

Assuming that the output process is Gaussian and all agents have CARA preferences (e.g., Holmstrom and Milgrom (1987)), we analytically characterize the equilibrium and compare it with the equilibrium in the benchmark “owner-manager” scenario in which there is no separation between the large shareholder and the manager. As in the owner-manager scenario, which corresponds to the case studied by D-U, the large shareholder’s ownership dynamics are deterministic. Because the output process is i.i.d. over time and there are no wealth effects with CARA preferences, the large shareholder’s ownership choices are the solutions to a sequence of static mean-variance optimization problems at the trading dates so that they vary deterministically. In sharp contrast with the benchmark owner-manager case, however, the equilibrium stock price varies stochastically. The stochastic variation stems from the random evolution of the manager’s promised payoff process that reflects incentive provision to the manager via her long-term contract.

For a given ownership stake of the large shareholder, the expected excess return and volatility of the stock are lower than in the benchmark “owner-manager” scenario. The lower expected excess return stems from the impact of agency costs of risk-sharing between the large shareholder and the manager. The more surprising result that the stock return volatility is lower in the agency scenario arises due to the fact that incentive provision entails nontrivial risk-sharing between the
large shareholder and the manager. As the stock price is determined by the residual earnings—total earnings net of the managerial compensation payments—the higher volatility of managerial compensation payments effectively lowers the volatility of the stock returns. The observation that the owner-manager case involves a more volatile stock return and a higher risk premium partially explains the higher equity risk premium in emerging markets, in which owner-manager firms, that is, family-controlled public firms are more prevalent, than in developed markets (e.g., Harvey (1995)).

We also characterize the equilibrium variables in the steady state where the large shareholder’s ownership is constant through time. Interestingly, the steady state ownership level of the large shareholder is identical to the steady state level in the benchmark owner-manager scenario, and equals the ownership level in the competitive equilibrium where the large shareholder trades taking the stock price as given. In other words, agency conflicts do not affect the steady state ownership level itself, but only the convergence to the steady state level. The reason is that, in the steady state, there is complete separation between the risk-sharing problem between the large and small shareholders that determines the stock price, and the incentive problem between the large shareholder and the manager. Consequently, the steady state ownership level of the large shareholder is entirely determined by risk-sharing between the large and small shareholders so that it coincides with the competitive equilibrium level. The stock price in the steady state continues to vary stochastically because of the need to provide incentives to the manager.

We exploit our analytical characterization of the equilibrium to derive testable implications for the relations among the large shareholder’s ownership stake, stock returns and managerial incentives. The sensitivity of the manager’s pay to the stock return is inversely related to the expected excess return, volatility and Sharpe ratio of the stock. The effects of the large investor’s ownership on stock returns and managerial incentives, as well as the impact of firm-specific parameters—the mean and volatility of earnings—on the equilibrium are difficult to pin down analytically for general parameter values. We, therefore, investigate these effects numerically by calibrating the model to match key moments.

Our analysis of the calibrated model shows that block ownership has a negative effect on the expected excess return, volatility and Sharpe ratio of the stock. The large shareholder’s ownership has direct and indirect effects on the stock return. The direct effect stems from the fact that her ownership stake reduces the stock’s liquidity available to small shareholders, thereby increasing the
current stock price and, thus, lowering the expected stock return. The indirect effect arises from the effects of the large shareholder’s ownership stake on the manager’s incentive compensation. The direct effect outweighs the indirect effects in the baseline calibrated model. Because the manager’s pay-performance sensitivity is positively related to the expected stock return and volatility, the large shareholder’s ownership stake has a positive effect on the manager’s pay-performance sensitivity. This finding, which suggests that block ownership and explicit contracts complement each other in corporate governance, is consistent with the evidence in Almazan, Hartzell and Starks (2005) and Kim (2010). We also obtain the intuitive results that the productivity of the manager’s effort has a positive effect on incentives, and a negative effect on the excess stock return and volatility. The cash flow volatility has a negative effect on incentives and a positive effect on the excess stock return and volatility.

Interestingly, the dynamics of the large investor’s ownership stake are insensitive to firm-specific parameters—the mean and volatility of earnings—and are primarily determined by the risk aversions of the large and small shareholders as well as the manager. In contrast, the large shareholder’s ownership dynamics are significantly influenced by firm-specific parameters in the benchmark owner-manager model. Indeed, the separation of ownership and control, and the facilitation of risk-sharing through the manager’s incentive contract effective insulates the large investor from variations in firm-specific parameters.

As mentioned earlier, we contribute to the burgeoning literature that studies the interactions between agency conflicts and asset prices. Gorton et al. (2014) abstract away from non-marketed managerial incentive contracts and assume that agents do not have access to a savings technology. Ou-Yang (2005) considers a contracting framework with lump-sum compensation at the terminal date as in Holmstrom and Milgrom (1987). We consider a more general contracting problem in which agents can consume inter-temporally and have access to a savings technology. As in studies in the dynamic contracting literature (see the survey in Bolton and Dewatripont (2005)), a recursive characterization of the contracting problem is facilitated by including the manager’s promised payoff as an additional state variable.

Among studies in the vast literature on corporate governance, our study is also related to the ongoing literature initiated by Gompers, Ishii and Metrick (2003) that explores the effects of corporate governance mechanisms on equity prices. Cremers and Nair (2005) show complementary
interactions between internal (active shareholders) and external (market for corporate control) governance mechanisms in generating long-term abnormal returns. Parigi, Pelizzon, and von Thadden (2013) theoretically and empirically show that the quality of corporate governance, which is endogenously chosen, correlates positively with CAPM beta and idiosyncratic volatility and negatively with returns on assets. Lastly, in the asset pricing literature, there are several studies that examine the effects of large shareholders (Cvitanic (1997), El Karoui, Peng and Quenez (1997), Cuoco and Cvitanic (1998), Subramanian and Jarrow (2001)). In this literature, however, the effects of large investor trades on asset prices are exogenous.

2 The Model

We consider an economy with a representative all-equity firm. The firm’s shareholders are comprised of two groups: “large” shareholders or blockholders who hold a block of shares in the firm and a continuum of dispersed small shareholders who collectively hold the remaining equity stake. Because our focus is on asset pricing implications, we assume that both groups of shareholders are risk averse. Based on prior research on firm ownership, we consider different holding periods for the two groups of investors. As in DeMarzo and Urošević [DU] (2006), small shareholders trade shares of the firm continuously, while the large shareholder trades the firm’s shares at a discrete set of dates. The time interval between the successive trading dates could be arbitrary. In addition to the firm’s shares, both types of investors can trade a risk-free bond (savings account) continuously.

In reality, major strategic corporate decisions require the approval of corporate boards that are significantly influenced by large shareholders (e.g., see Shleifer and Vishny (1997), Holderness (2003), Tirole (2006)). Cronqvist and Fahlenbrach (2009) document evidence for significant blockholder effects on a number of important corporate decisions such as investment, financial and executive compensation policies. For simplicity, we ignore strategic behavior among different blockholders, that is, they behave as a monolithic unit in their collective interest, so that we refer to them as a single representative blockholder. The large shareholder hires a risk-averse manager to operate the firm who affects the firm’s output through his costly, unobservable effort. The large shareholder influences the manager’s effort through a long-term incentive contract that is contingent on the firm’s observable output process. The two parties can renegotiate their existing contract after
trades by the large shareholder, but commit to the contract until the next trading date. Renegotiations must be weakly Pareto improving to be accepted. We now describe the various elements of the model in detail.

2.1 Firm’s Output Process

We consider a continuous-time model in which the uncertainty is generated by a standard Brownian motion $Z_t$ on a probability space $(\Omega, \mathcal{F}, P)$. The firm’s cumulative output $X_t$ evolves as follows:

$$dX_t = \mu(a_t, t)dt + \sigma dZ_t,$$

where $a_t \in [0, A]$ with $0 < A < \infty$ is the manager’s effort at date $t$ and the constant $\sigma > 0$ is the output volatility. The output process is publicly observable. We denote the (complete and augmented) information filtration generated by the cumulative output process, $X_t$, by $\{\mathcal{F}_t\}$. All instantaneous cash flows net of the manager’s compensation are paid out to the firm’s shareholders.

2.2 Large and Small Shareholders

The firm has a representative large shareholder $L$ and a continuum of small, dispersed shareholders $S$ (uniformly indexed over the unit interval, that is, $S \in [0, 1]$). We normalize the total number of the firm’s shares outstanding to one. As in DU, small shareholders trade continuously, while the large shareholder trades on the discrete set of dates $\{t_i; i = 1, 2, \ldots, N\}$ with $t_1 = 0$ and $t_{N+1} = \infty^1$ that are not necessarily equally spaced. A small shareholder $S$’s shareholding process is an $\{\mathcal{F}_t\}$-adapted process $\theta^S_t$, where $\theta^S_t$ is the number of shares held by $S$ at date $t$. The large shareholder, $L$’s ownership process is an $\{\mathcal{F}_t\}$-adapted process $\Theta^L_t$, where $\Theta^L_t$ is $L$’s holding at date $t$. $L$ and $S$’s ownership levels are publicly observable. $L$’s shareholding process is piecewise constant (it is constant between successive trading dates), and is right continuous with left limits. If $L$’s shareholdings just prior to her trade at date $t_i$ are $\Theta^L_{t_i-}$, the difference $\Theta^L_{t_i} - \Theta^L_{t_i-} > 0$ ($< 0$) represents the number of shares that she purchases (sells) at date $t_i$. As in DU, $L$ cannot commit to

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1 Using finite horizon does not qualitatively change our results. In the appendix we present the solution for our benchmark model under the finite horizon. The only difference from this generalization is that most coefficients become time-varying but deterministic instead of constants. Interesting results under the finite horizon can be obtained from different terminal and boundary conditions, as being shown in Sannikov (2008). Further extension along this direction may result in possible fruitful future research.
her trading policy. Due to the market clearing condition for the firm’s shares, the total number of shares collectively held by small shareholders at any date $t \in [t_i, t_{i+1})$ is $\int_0^1 \theta_t^S dS = 1 - \Theta_t^L = 1 - \Theta_{t_i}^L$.

Investors also have access to a risk-free bond that is in perfectly elastic supply so that it pays a continuously compounded constant return $r > 0$. The investors are initially endowed with wealth $Y^j(0) \geq 0$, for $j \in \{L, S\}$. Both groups of investors are risk averse and their preferences are described by the utility function $u^j(c_t^j)$ for $j \in \{L, S\}$ that is twice continuously differentiable, strictly increasing and strictly concave in the instantaneous consumption rate, $c_t^j$, at date $t$.

### 2.3 Preferences and Contracting

The large shareholder hires a manager, $M$, to operate the firm and offers him a long-term contract. The contract can be renegotiated by both parties at any trading date of $L \{t_i; i = 1, 2, \ldots, N\}$ provided it is in the interests of both parties to do so with $t_1 = 0$. Both parties, however, commit to the terms of the contract over a trading period, $[t_i, t_{i+1}); i = 1, \ldots, N$. The time line of events in any trading period, $[t_i, t_{i+1})$ is as follows. At trading date $t_i$, given her prior shareholdings $\Theta_{t_i-1}$, $L$ makes a trading decision $\Theta_{t_i}$. $L$ and $M$ then renegotiate the terms of their existing contract. In making her trade, $L$ rationally anticipates its effects on the terms of the renegotiated contract, the manager’s effort, the firm’s output and stock price. However, $L$ cannot commit to the terms of the renegotiated contract before she makes her trade. Small shareholders $S$ competitively trade shares of the company at each date — that is, they take the stock price as given — rationally anticipating $L$’s trading and contracting decisions. The stock price, $P_t$, at any date $t$ is determined by market clearing for the firm’s shares.

Because any contractual terms that are renegotiated in the future can be rationally incorporated in the original contract, we can, without loss of generality, restrict consideration to long-term contracts that are renegotiation-proof (RP) at each date $\{t_i; i = 1, 2, 3, \ldots, N\}$ (see also Laffont and Martimort (2002)). A long-term contract is RP at each date $t_i$ if it is weakly Pareto optimal among the set of continuation contracts that are themselves RP at future trading dates $\{t_{i+1}, t_{i+2}, \ldots\}$ (see, for example, Wang (2000)).

A long-term contract $\Pi$ can be formally expressed as $\Pi \equiv \{c^M, a^M\}$, where we augment the

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\(^2\)We can alternatively assume that the risk-free bond is in zero net supply, which would endogenously determine the risk-free rate over time without altering our main results.
definition of the manager’s contract to include the recommended effort process \( \{a^M_t\} \) and the manager’s compensation process, \( \{c^M_t\} \). The manager’s effort and compensation processes are \( \mathcal{F}_t \)-adapted stochastic processes. The manager continuously chooses an unobservable effort level \( a^M_t \in [0, A] \) given his contractual compensation that affects the firm’s output \( X_t \) as shown in (1). He incurs the instantaneous effort cost, \( \Psi(a_t, t) \), that is twice continuously differentiable, increasing and convex in the effort level, \( a^M_t \). Throughout, we assume that the upper bound, \( A \), on the manager’s effort is large enough that the manager’s optimal effort choices take interior values. 

\( M \) can also invest in the risk-free bond and the stock, but his savings are observable. Without loss of generality, therefore, we can restrict consideration to “savings proof” contracts in which \( M \)’s contract directly specifies his consumption at each date. Therefore, the process \( c^M_t \) specified by the contract is the manager’s consumption process.

The manager’s total utility function, \( u^M(M_t, a^M_t) \), satisfies \( u^M_c \equiv \partial u^M / \partial c > 0 \), \( u^M_{cc} \equiv \partial^2 u^M / \partial c^2 < 0 \), \( u^M_a \equiv \partial u^M / \partial a < 0 \) and \( u^M_{aa} \equiv \partial^2 u^M / \partial a^2 < 0 \). The cost of effort is a monetary cost at the same unit of the consumption. The manager’s continuation value or promised payoff at any date \( t \) — that is, his expected utility from his future consumption and effort under the contract \( \Pi \) — is given by

\[
W^M_t(\Pi) \equiv E^M_t \left[ \int_t^\infty e^{-\delta^M(t-\tau)} u^M(c^M_\tau, a^M_\tau) d\tau \right],
\]

where \( E^M_t[.] \) denotes the conditional expectation at date \( t \) with respect to the probability distribution induced by \( M \)’s effort choices \( a^M, \delta^M \) is \( M \)’s time discount rate.

The contract must be incentive compatible for the manager at each date \( t \). That is, given the stream of future compensation, it is optimal for the manager to choose the effort levels specified by the contract. By (2),

\[
a^M = \arg \max_{\Pi' \in \{c^M, a^M'\}} W^M_t(\Pi')
\]

\[
= \arg \max_{a^M'} E^{a^M'}_t \left[ \int_t^\infty e^{-\delta^M(t-\tau)} u^M(c^M_\tau, a^M_\tau') d\tau \right].
\]

Next, the contract must satisfy the manager’s participation constraint at date zero, that is,

\[
W^M_0(\Pi) \geq W_0,
\]
where \( W_0 \) is \( M \)'s reservation utility at date zero. A contract is *incentive feasible* if it is incentive compatible and satisfies the manager’s participation constraint at date zero.

Finally, we characterize the renegotiation-proofness (RP) constraints. Let \( \Pi_{[t, \infty)} \) be the restriction of the contract, \( \Pi \), to the period after date \( t \), where \( t \) is a trading date for \( L \). The RP constraints can be recursively characterized as follows (e.g., see Wang (2000), Giat and Subramanian (2013)).

**Renegotiation Proofness:**

1. The contract, \( \Pi_{[t_N, \infty)} \), over the last trading period, \([t_N, \infty]\), is weakly Pareto optimal among all incentive compatible, continuation contracts, \( \Pi'_{[t_N, \infty)} \).

2. At any trading date, \( t_i; i < N \), the contract, \( \Pi_{[t_i, \infty)} \), is weakly Pareto optimal among all incentive compatible, continuation contracts, \( \Pi'_{[t_i, \infty)} \), that are themselves RP at future trading dates, \( \{t_{i+1}, \ldots, t_N\} \).

The key consequence of renegotiation proofness for our analysis is *sequential optimality*, that is, the manager’s contract must be sequentially optimal at each trading date \( \{t_1, t_2, \ldots\} \).

To solve for the equilibrium fully, we assume the following for the rest of the paper. First, all the players have constant absolute risk aversion (CARA) preferences over their consumption and their risk aversion parameters are \( \gamma_L \), \( \gamma_M \) and \( \gamma_S \), respectively. More specifically, their utility functions are given by

\[
\begin{align*}
u_M(c^M_t, a^M_t) &= -\frac{1}{\gamma_M} e^{-\gamma_M(c^M_t - \Psi(a^M_t))}, \quad V_M(c^M_T) = -\frac{1}{\gamma_M} e^{-\gamma_M c^M_T} \\
u^j(c^j_t) &= -\frac{1}{\gamma^j} e^{-\gamma^j c^j_t}, \quad V^j(c^j_T) = -\frac{1}{\gamma^j} e^{-\gamma^j c^j_T} \quad \text{for} \quad j = L, S.
\end{align*}
\] (5)

Second, the firm’s mean cash flow and the cost of exerting effort are given by

\[
\mu(a_t, t) = \mu(a_t) = \mu_0 + \mu_1 a_t; \quad \Psi(a_t, t) = \Psi(a_t) = \frac{1}{2} \psi a^2_t; \quad (6)
\]

where the constants \( m \) and \( \psi \) are positive.
2.4 The Objectives of Large and Small Shareholders

$L$ chooses its ownership policy $\Theta^L$, the manager’s incentive feasible RP contract, $\Pi$, and its consumption policy, $c^L$ to maximize its expected utility subject to its budget constraint, that is, $L$ solves

$$\max_{(\Theta^L,c^L,\Pi)} E^0_0 \left[ \int_0^{\infty} e^{-\delta^L \tau} u^L(c^L_\tau) d\tau \right], \quad (7)$$

where $\delta^L$ is $L$’s time discount rate, and $E^a_0$ denotes the expectation with respect to the probability measure induced by the effort process $a$ specified by the contract. The above optimization program is subject to $L$’s budget constraint.

To characterize $L$’s budget constraint, we need to specify $L$’s proceeds from trading at a trading date $t_i$ both on and off-equilibrium. Let $\Theta^L(t_i-)$ and $\Theta^L(t_i)$ be $L$’s share ownership before and after trading, where $\Theta^L(t_i-)$ and $\Theta^L(t_i)$ could be on or off the equilibrium path. We follow DeMarzo and Urosevic [DU] (2006) and Gorton et al. [GHH] (2014) by assuming that $L$’s proceeds from trading are given by

$$L’s \text{ trading proceeds} = [\Theta^L(t_i-) - \Theta^L(t_i)] P_{t_i}(\Theta^L(t_i)), \quad (8)$$

where $P_{t_i}(\Theta^L(t_i))$ is the stock price after trading. As we discuss in more detail in Section 6, however, the stock price, $P_{t_i}(\Theta^L(t_i))$, is the price at which the marginal trade occurs and is not necessarily the price at which supra- or infra-marginal trades occur. The specification (8) can be justified if we assume that $L$ changes its ownership position from $\Theta^L(t_i-)$ to $\Theta^L(t_i)$ by making a single block trade in firm stock. If $L$ can change her ownership position incrementally, however, the proceeds from trading differ from (8) that changes the results quite significantly.

We can characterize $L$’s budget constraint by the evolution of her money market account balance as follows.

$$dB^L_t = (rB^L_t - c^L_t) dt + \Theta^L (dX_t - c^M_t dt) - P_t(\Theta) d\Theta$$

$$= (rB^L_t - c^L_t + \Theta^L(\mu(a_t) - c^M_t)) dt + \Theta^L \sigma dZ_t - P_t(\Theta) d\Theta, \quad (9)$$

11
where the differential, $P_t(\Theta)\,d\Theta$, is nonzero only at $L$’s trading dates, and is given by (8). Between any two consecutive trading dates $t \in [t_i, t_{i+1})$ for $L$, during which $L$’s shareholdings remain the same, the budget constraint reflects a change in $L$’s risk-free money market account balance in a time interval $(t, t + dt)$ due to her instantaneous consumption and dividend payment (the change in the firm’s cumulative output net of $M$’s instantaneous compensation payment in (1)). When $L$ trades at each trading date $t_i$ (that is, when she changes her shareholdings from $\Theta_{t_{i-1}}^L$ to $\Theta_{t_i}^L$), her money market account balance also changes due to the proceeds from trading new shares. Note that the stock price is affected by $L$’s subsequent ownership and contracting decisions, and $L$ incorporates the effects of her trades on the stock price in making these decisions. We make the following assumption about the observability of $L$’s ownership, contracting and consumption decisions.

**Assumption 1** $L$’s ownership process, $\Theta^L$, money market account balance $B^L$, consumption process, $c^L$, and the manager’s contract, $\Pi$, are publicly observable.

As mentioned earlier, $L$ cannot commit to her ownership policy. Further, the manager’s contract must be sequentially optimal for $L$ at each trading date $t_i$. In addition, as in GHH, we restrict consideration to Strong Markov Perfect Public Equilibria by imposing the stronger restriction that $L$’s ownership, contracting and consumption decisions must be sequentially optimal at each trading date $t_i$ both on and off the equilibrium path. In particular, the stock price, $P_t(\Theta)$ in (9) is the stock price at the possibly off-equilibrium ownership choice, $\Theta$, incorporating the fact that $L$’s consumption policy and continuation of the manager’s contract $\Pi$ are sequentially optimal given the ownership level, $\Theta$. Consequently, $L$’s ownership, contracting and consumption decisions must solve

$$\max_{(\Theta^L_{[t_i, \infty)}, c^L_{[t_i, \infty)}, \Pi_{[t_i, \infty)})} E_{t_i}^a \left[ \int_{t_i}^{\infty} e^{-\delta L(\tau - t_i)} u^L(c^L_{\tau}) d\tau \right].$$

(10)

In the above, $(\Theta^L_{[t_i, \infty)}, c^L_{[t_i, \infty)}, \Pi_{[t_i, \infty)})$ denotes the restriction of the vector of processes, $(\Theta^L, c^L, \Pi)$, to the interval $[t_i, \infty)$ with the understanding that $L$’s decisions must be sequentially optimal on or off the equilibrium path, that is, for any past history.

Each small shareholder, $S$, chooses its consumption and ownership policies to maximize its expected utility taking the stock price process as given, and rationally anticipating $L$’s trading and
contract choices, that is, $S$ solves

$$
\max_{(\theta^S, c^S)} E^S_0 \left[ \int_0^\infty e^{-\delta^S \tau} u^S(c^S) d\tau \right],
$$

(11)

where $\delta^S$ is $S$'s time discount rate. $S$’s money market balance evolves according to

$$
dB^S_t = (rB^S_t - c^S_t) dt + \theta^S_t (dX_t - c^M_t dt) - P_t d\theta^S_t.
$$

(12)

As $S$ changes its shareholdings continuously, its total wealth process, $Y^S_t = B^S_t + \theta^S_t P_t$, evolves as

$$
dY^S_t = (rY^S_t - c^S_t) dt + \theta^S_t (dX_t - c^M_t dt) + dP_t - rP_t dt.
$$

(13)

In the above, we explicitly indicate the fact that $S$, unlike $L$, takes the stock price process $P$ as given in making its trading decisions.

2.5 Equilibrium Characterization

An equilibrium of the model is described by the vector of processes

$$
\{(\Theta^{L*}, B^{L*}, c^{L*}) ; (\theta^{S*}, Y^{S*}, c^{S*}) ; \Pi^* ; P^* \}
$$

(14)

where $(\Theta^{L*}, B^{L*}, c^{L*})$ is the vector of processes representing $L$’s share ownership, money market account balance and consumption, respectively; $(\theta^{S*}, Y^{S*}, c^{S*})$ is the vector of processes representing a small shareholder $S$’s share ownership, total wealth and consumption, $\Pi^*$ is the manager’s contract, and $P^*$ is the stock price process.

The processes must satisfy the following equilibrium conditions.

1. $L$’s share ownership, money market account balance and consumption $(\Theta^{L*}, B^{L*}, c^{L*})$ as well as the manager’s incentive feasible RP contract, $\Pi^*$, solve (10) subject to the budget constraint (9). In particular, $L$’s decisions are sequentially optimal at each trading date on or off the equilibrium path.

2. The small shareholder, $S$’s ownership, total wealth and consumption, $(\theta^{S*}, Y^{S*}, c^{S*})$, solve (11) subject to the budget constraint (13).
3. The stock price process $P^*$ clears the market at each date, that is, at each date $\int_0^1 \theta_t^* dS = 1 - \Theta_t^L$. In particular, the stock price process must clear the market on or off the equilibrium path.

3 A Benchmark Model: Owner-Manager

We first analyze a benchmark model in which $L$ directly runs the firm by exerting costly effort herself, that is, there is no agency problem between $L$ and $M$. We then turn to the main analysis of the full model with the agency problem.

As discussed in Section 2.3, $L$’s consumption and ownership decisions must be sequentially optimal given any past history. Suppose that $L$’s ownership policy in the firm is given by $\Theta = \{\Theta_t; i = 1, \ldots, N\}$. Adapting (10) to the case where $L$ also makes effort choices, her problem is to solve

$$
\max_{(\Theta_t^L, c_t^L, a_t^L)} \mathbb{E}_t^a \left[ \int_{t_i}^{\infty} e^{-\delta \tau} u^L(c_t^L, a_t^L) d\tau \right]
$$

subject to the budget constraint

$$
\begin{align*}
  dB_t^L &= (rB_t^L - c_t^L)dt + \Theta_t^L dX_t - \chi_{t=t_i} P_t(\Theta) d\Theta \\
  &= (rB_t^L - c_t^L + \Theta_t^L \mu(a_t)) dt + \Theta_t^L \sigma dZ_t - P_t(\Theta) d\Theta. 
\end{align*}
$$

In (15), $\mathbb{E}_t^a$ signifies the expectation with respect to the probability distribution induced by $L$’s effort choice process.

A small shareholder $S$ chooses its ownership and consumption policies rationally anticipating $L$’s ownership, effort and consumption decisions to solve

$$
\max_{\Theta_t^S, c_t^S} \mathbb{E}_t^a \left[ \int_{t_i}^{\infty} e^{-\delta \tau} u^S(c_t^S) d\tau \right]
$$

subject to the budget constraint

$$
\begin{align*}
  dB_t^S &= (rB_t^S - c_t^S)dt + \Theta_t^S dX_t - \chi_{t=t_i} P_t(\Theta) d\Theta \\
  &= (rB_t^S - c_t^S + \Theta_t^S \mu(a_t)) dt + \Theta_t^S \sigma dZ_t - P_t(\Theta) d\Theta. 
\end{align*}
$$
subject to the budget constraint

\[ dY_t^S = (rY_t^S - c_t^S)dt + \theta_t^S(dX_t + dP_t - rP_tdt) \]
\[ = (rY_t^S - c_t^S)dt + \theta_t^S(\mu(a_t)dt + dP_t - rP_tdt) + \theta_t^S \sigma dZ_t. \] (18)

As in Gorton, He and Huang (2014), we consider Strong Markov Public Perfect Equilibria (SMPPE) where the public state vector is \((t, B_t^L, X_t)\). The following proposition describes \(L\)’s optimal consumption and effort choices. We provide detailed proofs of all results in Appendix A.

**Proposition 1 (Large Shareholder’s Optimal Policies for Given Ownership Level)**

Let \(\Theta\) be any level (on- or off-equilibrium) of \(L\)’s ownership at time \(t \in [t_i, t_{i+1})\). \(L\)’s value function at time \(t\) has the form

\[ W_t^L = F(t, B_t^L) = -\frac{1}{\gamma^L} e^{-\gamma^L \left[ r(B_t^L + G(t, \Theta)) + \delta^L - r \right] \frac{\Theta}{\gamma^L}}. \] (19)

\(L\)’s optimal effort and consumption policies are

\[ a_t^L = \frac{\mu_1}{\psi} \Theta, \] (20)
\[ c_t^L = r(B_t^L + G(t, \Theta)) + \frac{\delta^L - r}{\gamma^L} + \Psi(a_t^L). \] (21)

In the above, the time-deterministic certainty-equivalent payoff, \(G(t, \Theta)\), satisfies the recursion

\[ G(t, \Theta) = \phi(t)V(\Theta) + (1 - r\phi(t))G(t_{i+1}, \Theta), \] (22)

where \(\phi(t) = \frac{1}{r} \left( 1 - e^{-r(t_{i+1}-t)} \right)\),

\[ \lim_{t \to \infty} G(t, \Theta) = 0. \] (24)

\(V(\Theta)\) is the net benefit flow to \(L\) from holding \(\Theta\) that is given by

\[ V(\Theta) = \Theta \mu(a_t^L) - \Psi(a_t^L) - \frac{1}{2} \gamma_L r \Theta^2 \sigma^2 = \mu_0 \Theta + \frac{1}{2} \Theta^2 \left[ \frac{\mu_1^2}{\psi} - \gamma_L r \sigma^2 \right]. \] (25)

By (20), we note that, not surprisingly, \(L\) exerts greater effort when she holds a larger block of shares in the firm, when the productivity of effort is higher or the unit cost of effort is lower. \(L\)’s
net benefit flow from holding $\Theta$ captures the trade-off between expected payoff, which is reduced by the effort cost solely borne by $L$ in the owner-manager case, and the cost of the risk she bears from her ownership stake in the firm. The function $G$, defined by (22) is $L$’s certainty equivalent payoff from holding the firm’s shares that is useful to obtain $L$’s optimal trading policy.

The following proposition describes the representative small shareholder, $S$’s, value function as well as optimal consumption and trading policies.

**Proposition 2 (Small Shareholder’s Optimal Policies for Given Large Shareholder Ownership)**

**$S$’s value function at time $t$ is**

$$ W^S_t = Q(t, Y^S_t) = -\frac{1}{\gamma^S r} e^{-\gamma^S r [r(Y^S_t + J(t, \Theta)) + \frac{t^S}{\gamma^S r}]}.$$

(26)

**$S$’s optimal consumption and ownership policies are**

$$ c^S_t = r(Y^S_t + J(t, \Theta)) + \frac{\delta^S - r}{\gamma^S r} $$

(27)

$$ \theta^S_t = \frac{\mu_R(\Theta)}{\gamma^S r \sigma^2_R(\Theta)}.$$

(28)

Its certainty equivalent payoff satisfies the recursion

$$ J(t, \Theta) = \phi_i(t) \frac{1}{2} \frac{\mu^2_R(\Theta)}{\gamma^S r \sigma^2_R(\Theta)} + (1 - r \phi_i(t)) J(t_{i+1}, \Theta); $$

(29)

$$ \lim_{t \to \infty} J(t, \Theta) = 0, $$

(30)

where $\phi_i(t)$ is defined in (23).

Finally, the following proposition characterizes the equilibrium stock price.

**Proposition 3 (Stock Price for Given Ownership Level of Large Shareholder)**

The stock price given $L$’s ownership level, $\Theta$, is given by $P(t, \Theta)$ where

$$ P(t, \Theta) = \phi_i(t) k(\Theta) + (1 - r \phi_i(t)) P(t_{i+1}, \Theta); \lim_{t \to \infty} P(t, \Theta) = 0 $$

(31)

and

$$ k(\Theta) = \mu(a^L_t) - \gamma^S r (1 - \Theta) \sigma^2 = \mu_0 + \frac{\mu^2}{\psi} \Theta - \gamma^S r (1 - \Theta) \sigma^2. $$

(32)
The expectation and volatility of the excess dollar return of the stock are

\[ \mu_R(\Theta) = (1 - \Theta) \gamma_S r \sigma^2; \quad \sigma_R(\Theta) = \sigma. \]  \hfill (33)

The Sharpe ratio is

\[ \Sigma(\Theta) = \frac{\mu_R(\Theta)}{\sigma_R(\Theta)} = (1 - \Theta) \gamma_S r \sigma \]  \hfill (34)

The excess dollar return for holding a share of the firm’s stock within the time interval \((t, t + dt)\) is defined as

\[ dR_t = dX_t + dP_t - rP_t dt = (1 - \Theta) \gamma_S r \sigma^2 dt + \sigma dZ_t. \]  \hfill (35)

We, therefore, obtain the expected excess dollar return or risk premium of the stock, \(\mu_R\), and its volatility, \(\sigma_R\), as specified in (33). The following corollary summarizes how \(L\)’s ownership level is related to the stock return distribution.

**Corollary 1 (Large Shareholder Ownership and Stock Returns)**

The expected excess return of the stock and the Sharpe ratio decline with the large shareholder’s ownership.

\(L\)’s shareholdings affect the risk premium for the stock through its effect on the supply of shares. As \(L\) holds a larger block of the company’s stock, the stock’s liquidity available to small shareholders is lower, thereby increasing the current stock price and, thus, lowering the expected stock return as well as the Sharpe ratio.

We now discuss \(L\)’s optimal ownership choice at each trading date \(t_i\). Recall that \(L\)’s ownership process is piecewise constant, that is, her holdings in the firm are constant over a trading period, \([t_{i-1}, t_i)\). \(L\)’s ownership decision for the trading period, \([t_i, t_{i+1})\), is made at \(t_i^-\) when she holds \(\Theta_{t_{i-1}}^L\) shares in the firm and her money market balance is \(B_{t_i}^L\). By (19), \(L\)’s value function once she chooses the new equity stake \(\Theta\) is

\[ W_{t_i}^L = -\frac{1}{\gamma_{Lr}} e^{-\gamma_{Lr} \left[ r(B_{t_i}^L + G(t_i, \Theta)) + \frac{L_{t_i}}{\gamma_{Lr}} \right]}, \]  \hfill (36)

so that \(L\)’s optimal ownership choice maximizes \(B_{t_i}^L + G(t_i, \Theta)\). As noted earlier, her money market account balance changes from \(B_{t_i}^L\) to \(B_{t_i}^L\) by the proceeds from trading shares at \(t_i\).
as below:

\[ B_{t_i}^L = B_{t_i}^L - P(t, \Theta)(\Theta - \Theta_{t_{i-1}}). \]  

(37)

\L\text{’s optimal ownership choice for the period, } [t_i, t_{i+1}), \text{ thus solves}

\[ \Theta^L_{t_i} = \arg \max_{\Theta} B_{t_i}^L + G(t_i, \Theta) = B_{t_i}^L - P(t, \Theta)(\Theta - \Theta_{t_{i-1}}) + G(t_i, \Theta), \]

\[ = \arg \max_{\Theta} -P(t, \Theta)(\Theta - \Theta_{t_{i-1}}) + \phi_i(t_i)V(\Theta) + (1 - r\phi_i(t_i))G(t_{i+1}, \Theta), \]

\[ \Rightarrow \text{ FOC : } V'(\Theta) - k(\Theta) - (\Theta - \Theta_{t_{i-1}})k'(\Theta) = 0, \]

(38)

where the second line follows from (22), (31) and the continuity of \L\text{’s time-deterministic certainty}

\text{equivalent function } G \text{ at } t_{i+1}, \text{ and } \phi_i(t_i) \text{ is given by (23). The following proposition characterizes}

\L\text{’s optimal ownership path.}

**Proposition 4 (Large Shareholder’s Optimal Ownership Path)**

\L\text{’s optimal ownership stake in the firm evolves according to the following recursive form:}

\[ \Theta_{t_i}(\Theta_{t_{i-1}}) = \frac{\eta \Theta_{t_{i-1}} + \gamma^S r \sigma^2}{\eta + (\gamma^L + \gamma^S) r \sigma^2}, \]

(39)

where \( \eta \equiv \frac{\mu^2}{\sigma^2} + \gamma^S r \sigma^2 > 0. \)

By (39), we see that \L\text{’s ownership path is a deterministic function of time. It then follows from}

Proposition 3 that the equilibrium stock price is also a deterministic function of time. Because \L\text{’s ownership level and effort are constant between successive trading dates, the expected output and}

\S\text{’s cost of risk from her ownership stake are constant between successive trading dates. The stock}

price is determined by the net present value of the expected earnings net of the cost of risk and is, therefore, a deterministic function of time. As we show in the next section, however, the equilibrium stock price is stochastic when we introduce the agency conflict between \L \text{and } \M. \text{ As shown above, the equilibrium is similar to that obtained by DU, that is, the stock price and ownership path are deterministic functions of time.}

We now characterize the steady state equilibrium, which corresponds to the case where the number of trades by \L \text{goes to } \infty, \text{ that is, } N \to \infty \text{ and } t_N \to \infty. \text{ The following corollary describes}

the steady state equilibrium.
Corollary 2 (Steady State Equilibrium)

In the steady state, $L$’s ownership stake is

$$\tilde{\Theta} = \frac{\gamma_S}{\gamma_S + \gamma_L}.$$  \hspace{1cm} (40)

The steady state equilibrium stock price is

$$\tilde{P} = \frac{k(\tilde{\Theta})}{r}.$$  \hspace{1cm} (41)

The optimal policies of $L$ are given by Proposition 1 with $L$’s ownership level equal to $\tilde{\Theta}$.

In the steady state, $L$’s ownership level equals the competitive equilibrium level that depends only on the risk aversions of $L$ and $S$. By Propositions 4 and 3, $L$’s ownership level and the stock price converge deterministically to their competitive equilibrium values.

As we discuss in Section 6, if we adopt an alternate specification for $L$’s proceeds from trading—that is, we allow $L$ to trade incrementally—the resulting equilibrium is significantly different. In particular, we show that $L$’s ownership path is actually constant through time and equal her holdings under the competitive equilibrium. In other words, instead of gradually converging to the competitive equilibrium as in DU, $L$ instantly trades to the competitive equilibrium level.

4 Contracting

We now examine the main model that incorporates the agency conflict between $L$ and $M$. As in the owner-manager case, we consider SMPPE and derive the equilibrium using backward induction.

4.1 Optimal Contracting

As noted earlier, we consider renegotiation-proof contracting along the lines of Wang (2000) and Giat and Subramanian (2013). The incorporation of the possibility of renegotiation is consistent with the assumption that $L$ cannot commit to her future trading policy at the outset. The contract specifies the instantaneous compensation payoff and recommended effort level that maximize $L$’s expected utility (7) subject to the incentive compatibility and participation constraints for the manager as specified in (3) and (4).
Following the dynamic contracting literature (see the survey by Cvitanić and Zhang (2013)), the contracting problem can be characterized recursively if we take the manager’s continuation value or promised payoff, $W^M_t$, defined in (2) as an additional state variable whose value evolves according to the following stochastic process

$$dW^M_t = \left(\delta^M W^M_t - u^M(c^M_t, a^M_t)\right) dt - \chi^M_t \sigma dZ_t. \tag{42}$$

In the above, $\chi^M_t$ is a $\mathcal{F}_t$-adapted process that represents the sensitivity of the manager’s continuation value to the exogenous shock in the firm’s output and plays a key role in the provision of incentives.

As shown by previous studies such as Williams (2009), the incentive compatibility constraint for the manager can be replaced by the following local incentive compatibility constraint under technical conditions that are satisfied under CARA preferences for the principal and agent:

$$\chi^M_t = -\frac{u^M_t(c^M_t, a^M_t)}{\mu'(a^M_t)} = \frac{\Psi'(a^M_t)}{\mu'(a^M_t)} H(c^M_t, a^M_t) > 0, \tag{43}$$

where $H(c^M_t, a^M_t) \equiv e^{-\gamma M(c^M_t-\frac{1}{2}\psi(a^M_t)^2)}$. The manager’s contract is publicly observable so that we can assume (without loss of generality) that his promised payoff, $W^M$, is publicly observable.

**Assumption 2** $W^M$ is public observable.

As we now show, the Strong Markov Perfect Public Equilibrium (SMPPE) is completely characterized by the state vector, $(t, B^L_t, W^M_t, X_t)$.

**Proposition 5 (Large Shareholder’s Optimal Policies for Given Ownership Level)**

Let $\Theta$ be any level (on- or off-equilibrium) of $L$’s ownership at $t \in [t_i, t_{i+1})$. Suppose that $M$’s promised payoff at date $t$ is $W^M_t$. $L$’s value function at time $t$ has the form of

$$W^L_t = F(t, B^L_t, W^M_t) = -\frac{1}{\gamma^L r} e^{-\gamma L r} \left[ \left(r(B^L_t + G^*(t, \Theta)) + \frac{\Theta}{\gamma M} \ln(-W^M_t) + \frac{\delta^L - r}{\gamma L r} \right) \right]. \tag{44}$$

$L$’s optimal consumption, $M$’s optimal effort and compensation are given by

$$c^L_t = r(B^L_t + G^*(t, \Theta)) + \frac{\Theta}{\gamma M} \ln(-W^M_t) + \frac{\delta^L - r}{\gamma L r}. \tag{20}$$
\[ a_t^M = \alpha, \]
\[ c_t^M = -\frac{1}{\gamma^M} \ln(\beta) - \frac{1}{\gamma^M} \ln(-W_t^M) + \Psi(a_t^M), \] (45)

In the above \( \beta \) is given by
\[ \beta = \gamma^M r \left[ \gamma^M \frac{\psi \alpha - \mu_1}{\beta} + 1 \right], \]

and the manager’s optimal effort, \( \alpha \), satisfies the following equation:
\[ \nu_6(\Theta) \alpha^6 + \nu_5(\Theta) \alpha^5 + \nu_4(\Theta) \alpha^4 + \nu_3(\Theta) \alpha^3 + \nu_2(\Theta) \alpha^2 + \nu_1(\Theta) \alpha + \nu_0(\Theta) = 0, \] (46)

where the coefficients \( \nu_i, i = 0, \ldots, 6 \) are defined in the proof of the proposition in Appendix A and are functions of \( \Theta \). In (44), the time-deterministic certainty-equivalent payoff \( G^* \) satisfies the recursion,
\[ G^*(t, \Theta) = \frac{1}{r} \left( 1 - e^{-r(t_{i+1}-t)} \right) V^*(\Theta) + e^{-r(t_{i+1}-t)} G^*(t_{i+1}, \Theta); \lim_{t \to \infty} G^*(t, \Theta) = 0, \] (47)

where the net benefit flow to \( L \) from holding \( \Theta \) is
\[ V^*(\Theta) = \Theta \left( \mu(\alpha) - \left( -\frac{1}{\gamma^M} \ln(\beta) - \frac{1}{\gamma^M r} \left( \mu_W - \frac{1}{2} \sigma_W^2 r = \frac{1}{2} \gamma^M \Theta^2 \left( \sigma - \frac{\sigma_W}{\gamma^M r} \right)^2, \right) \right) \right) \] (48)

and
\[ \mu_W = \delta^M - \frac{\beta}{\gamma^M}; \quad \sigma_W = \frac{\psi}{\mu_1} \alpha^2 \] (49)

In the above, both \( L \)'s and \( M \)'s consumption processes are linear in the log of \( M \)'s promised utility process \( W_t^M \). We note that \( M \)'s effort level \( \alpha \) and the fixed component \( \beta \) of in \( M \)'s compensation are constant within the time period \([t_i, t_{i+1})\) and depend nonlinearly on \( L \)'s ownership level, \( \Theta \), during the period. The evolution of the manager’s promised payoff process, (42), is described by the following corollary.
Corollary 3 (Manager’s Promised Payoff Process)

$M$’s promised utility process $W_t^M$ follows a geometric Brownian motion,

$$d\ln(-W_t^M) = \left(\mu_W - \frac{1}{2}\sigma_W^2\right) dt - \sigma_W dZ_t,$$

(50)

where $\mu_W, \sigma_W$ are defined in (49).

The manager’s dollar pay-performance sensitivity (PPS), $PPS_X$, is the dollar change in the manager’s pay for a dollar change in the firm’s cumulative output $X_t$. From (45) and (50), we have

$$dc_{M,t} = -\gamma_M d\ln(-W_t^M) = -\frac{1}{\gamma_M} \left(\mu_W - \frac{1}{2}(\sigma_W)^2\right) dt + \frac{\sigma_W}{\gamma_M} dZ_t.$$  

(51)

We then obtain

$$PPS_X = \frac{dc_t}{dX_t} = \frac{\sigma_W}{\gamma_M \sigma} = \frac{1}{\gamma_M} \left(\frac{\psi}{\mu_l} \alpha \beta\right),$$

(52)

where the last equality follows from the definition of $\sigma_W$ in (49). The PPS is positively associated with the volatility term $\sigma_W$ of the manager’s promised payoff process. As $\alpha$ and $\beta$ depend on $L$’s ownership level, $\Theta$, the manager’s incentives with respect to the firm’s cash flows depend on $L$’s ownership level. The relation between the manager’s incentives and $L$’s ownership level is difficult to pin down analytically because the manager’s optimal effort, $\alpha$, satisfies a sixth degree polynomial equation (see (46)) whose solutions cannot be characterized analytically. We calibrate the model in the next section and numerically analyze the relation between $L$’s ownership level and $M$’s incentives.

4.2 Large Shareholder Ownership, Stock Prices and Returns

We now derive the stock price for a given ownership level of $L$ by analyzing the representative small shareholder, $S$’s portfolio choice problem. $S$ makes his portfolio choice taking the stock price as given, but rationally anticipating $L$’s ownership stake and her decision on the manager’s optimal contract. The following proposition characterizes the stock market equilibrium derived from the optimal portfolio and consumption problem of small shareholders and the stock market clearing condition.
Proposition 6 (Stock Price for Given Large Shareholder Ownership)

Given any level (on- or off-equilibrium) of $L$’s ownership, $\Theta$, and $M$’s promised utility, $W_t^M$, at time $t \in [t_i, t_{i+1})$, the stock price is given by

$$P^*(t, \Theta, W_t^M) = \Lambda^*(t, \Theta) + \frac{1}{\gamma^M} \ln(-W_t^M), \quad (53)$$

where

$$\Lambda^*(t, \Theta) = \phi_i(t)k^*(\Theta) + (1 - r\phi_i(t))\Lambda^*(t_{i+1}, \Theta) + \lim_{t \to \infty} \Lambda^*(t, \Theta) = 0, \quad (54)$$

$$k^*(\Theta) = \mu(\alpha) - \left(-\frac{1}{\gamma^M} \ln \beta - \frac{1}{\gamma^M} \left(\mu_W - \frac{1}{2}\sigma_W^2 + \Psi(\alpha)\right) - \gamma^S r(1 - \Theta) \left(\sigma - \frac{\sigma_W}{\gamma^M} \right)^2 \right), \quad (55)$$

The excess dollar return for holding a share of the firm’s stock within the time interval $(t, t + dt)$ is

$$dR_t = dP^*_t + c^M_t dt - rP^*_t dt = \mu_R^*(\Theta) dt + \sigma_R^*(\Theta) dZ_t, \quad (56)$$

where

$$\mu_R^*(\Theta) = (1 - \Theta) \gamma^S r \left(\sigma - \frac{\sigma_W}{\gamma^M} \right)^2; \quad \sigma_R^*(\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma^M} \right). \quad (57)$$

The Sharpe ratio is

$$\Sigma^*(\Theta) = \frac{\mu_R^*(\Theta)}{\sigma_R^*(\Theta)} = (1 - \Theta) \gamma^S r \left(\sigma - \frac{\sigma_W}{\gamma^M} \right), \quad (58)$$

we see that the stock price is stochastic as it depends on the manager’s promised payoff that evolves stochastically. Consequently, in sharp contrast with the benchmark owner-manager model (and with DU), the presence of the agency conflict between $L$ and $M$ makes the stock price evolve stochastically (see Proposition 3)). As one can easily see from (57), there is a positive relation between the stock market variables,

$$\mu_R^* = (1 - \Theta) \gamma^S r \sigma_R^*^2, \quad (59)$$

which arises because the collective demand for the stock by small shareholders with CARA preferences is competitively determined by the expected stock returns adjusted for the investor’s risk premium, $\mu_R^*/(\gamma^S r \sigma_R^*^2)$, which must equal the supply of the shares, $1 - \Theta$, to ensure market
clearing. The following corollary compares the stock return characteristics with the benchmark owner-manager scenario.

**Corollary 4 (Stock Returns in Agency and Benchmark Models)**

*For a given ownership level, \( \Theta \), the expected excess return, volatility and Sharpe ratio are lower in the agency model than in the benchmark owner-manager model.*

The possibility of risk-sharing between the large shareholder and the manager through the manager’s incentive contract makes the manager’s compensation payments volatile. As shareholders receive the dividend streams net of the manager’s compensation payments, the sharing of risk with the manager makes the residual cash flows less volatile and, thereby, lowers the stock return volatility. The expected excess return of the stock and the Sharpe ratio are also lower than in the benchmark scenario due to the agency costs of risk-sharing between \( L \) and \( M \). The observation that the owner-manager case involves a more volatile stock return and a higher risk premium partially explains the higher equity risk premium in emerging markets, in which owner-manager firms, that is, family-controlled public firms are more prevalent, than in developed markets.

Note that, in contrast to the owner-manager case, \( L \)’s ownership level, \( \Theta \), has both direct and indirect influence on the expected stock return \( \mu^*_R \) and the volatility, \( \sigma^*_R \). Her ownership stake reduces the stock’s liquidity available to small shareholders, thereby increasing the current stock price and, thus, lowering the expected stock return, as shown by the term \( (1 - \Theta) \). By Proposition 5 and Corollary 3, however, her ownership also affects the mean and volatility of the manager’s promised payoff process and compensation payments that, in turn, influence the stock returns. The net effects are difficult to pin down analytically for general parameter values so we explore the relations by numerically analyzing the calibrated model in the next section.

As shown by Propositions 5 and 6, the manager’s optimal compensation payment \( c^M_t \) and the equilibrium stock price \( P^*_t \) are negatively and positively related to the log of the state variable, \( \ln(-W^M_t) \), respectively, which suggests a negative relation between CEO pay and the contemporaneous stock price. This negative relation appears because the current stock price reflects the costs of compensating the manager that are shared by all shareholders.
Combining (57) with (51), we obtain the sensitivity of managerial pay to the stock return:

\[ PPS_R = \frac{dc^M}{dR_t^*} = \frac{r\sigma_W}{\gamma^M r\sigma - \sigma_W}. \]  

(60)

By (57), (58) and (60), the expected excess return, volatility and Sharpe ratio of the stock decrease with the quantity, \( \sigma_W \), while the managerial pay-stock return sensitivity increases. We immediately obtain the following corollary.

**Corollary 5 (Pay-Performance Sensitivity and Stock Return)**

The sensitivity of managerial pay to the stock return is negatively correlated with the expected excess return, volatility and Sharpe ratio of the stock.

### 4.3 Large Shareholder’s Optimal Ownership Policy

\( L \)'s optimal ownership choice at each trading date \( t_i \) maximizes \( L \)'s value function (44). As in the owner-manager case, \( L \)'s optimal trading policy at \( t_i \), given \( M \)'s promised utility \( W^M_{t_i} \), solves

\[ \Theta_{t_i} = \arg \max_{\Theta} \left[ B^L_{t_i} - P(t_i, \Theta, W^M_{t_i})(\Theta - \Theta_{t_{i-1}}) + G^*(t_i, \Theta) + \frac{\Theta}{\gamma^M} \ln(-W^M_{t_i}) \right]. \] 

(61)

The following proposition describes \( L \)'s optimal ownership path.

**Proposition 7 (Large Shareholder’s Ownership Path)**

\( L \)'s optimal ownership level, \( \Theta^*_t \), in period \( [t_i, t_{i+1}) \) satisfies the following equation:

\[ \left[ V^{\star'}(\Theta^*_{t_i}) - k^*(\Theta^*_{t_i}) - (\Theta^*_{t_i} - \Theta^*_{t_{i-1}})k^*(\Theta^*_{t_i}) \right] \]

\[ = r\sigma_R^2(\Theta^*_{t_i}) \left[ (\gamma^S(1 + \Theta^*_{t_{i-1}}) - (\gamma^L + 2\gamma^S)\Theta^*_{t_{i-1}}) - (\Theta^*_{t_i} - \Theta^*_{t_{i-1}})(\gamma^L + 2\gamma^S)\Theta^*_{t_i} - \gamma^S) \frac{\sigma^*_R(\Theta^*_{t_i})}{\sigma_R(\Theta^*_{t_i})} \right] = 0, \] 

(62)

In the above, we see two forces that determine \( L \)'s optimal ownership choices. The first force, which also exists in the owner-manager case, is \( L \)'s risk-sharing with \( S \) as their risk aversions directly affect \( L \)'s ownership choice. The other is \( L \)'s costly risk-sharing with \( M \) through his optimal contract, which is captured by the last term, the rate of increase in the firm’s stock return volatility.
with respect to $L$’s equity stake. In the steady state, the latter force disappears as described in the following corollary.

**Corollary 6 (Steady State Equilibrium)**

In the steady state ($t \geq T$), $L$’s ownership stake is

$$
\tilde{\Theta} = \frac{\gamma S}{\gamma S + \gamma L}.
$$

(63)

$L$’s optimal consumption and contract choices are given by Proposition 5 with $L$’s ownership level equal to $\tilde{\Theta}$. The stock price is given by

$$
P^*(t, W^M_t) = \frac{k^*(\tilde{\Theta})}{\gamma M^r} + \frac{1}{\gamma M^r} \ln(-W^M_t).
$$

(64)

Comparing (63) with (40), we see that, in the steady state, $L$’s ownership level is the same as in the benchmark case with no agency conflicts, that is, it is the ownership level in the competitive equilibrium where $L$ takes the stock price as given in making her ownership choice. Agency conflicts, therefore, affect the convergence of $L$’s ownership levels to the competitive equilibrium level. However, as is apparent from (64), the stock price evolves stochastically in the steady state equilibrium because of the stochastic evolution of the manager’s promised payoff process. As $L$’s steady-state ownership level is the same as in the benchmark owner-manager scenario, we can compare other equilibrium variables in the steady state.

**Corollary 7 (Comparison of Steady State Equilibria)**

In the steady state, the manager’s effort, the expected excess stock return and the volatility are lower than in the benchmark owner-manager scenario.

As emphasized above, the optimal incentive provision to the manager plays a significant role of risk-sharing, which effectively lowers the firm’s stock return volatility.

## 5 Numerical Analysis

We calibrate the model and numerically analyze it to obtain further implications that are difficult to derive analytically mainly because the manager’s optimal effort is the solution of a polynomial equation (see (46)) whose solution cannot be analytically characterized.
5.1 Baseline Parametrization

We determine the baseline parameters for our analysis by setting some of the parameters directly using guidance from previous studies and calibrating the remaining parameters to match salient output variables in the steady state equilibrium described in the previous section. Table 1 reports the baseline parameters of the model. We set the annual risk-free rate \( r \) to 2% and each economic agent’s time discount rate \( \delta_i \) to 0.0404 so that the annual discount factor \( e^{-\delta_i} = 0.96 \) for \( i = L, M, S \) (Guvenen, 2009)). The time interval between the large shareholder’s successive trading dates corresponds to 1 year. We set the unit cost \( \psi \) of effort to 1 as this parameter appears together with the productivity of effort, \( \mu_1 \).

We calibrate the remaining parameters to match selected variables in the steady state equilibrium of the model. To approximate the steady state, the parameters that we calibrate are the coefficients of absolute risk aversion for the manager and for the (large and small) shareholders; the parameters, \( \mu_0, \mu_1 \), that determine the impact of the manager’s effort on the mean output; the output volatility, \( \sigma \), and the manager’s mean promised payoff in the steady state \( \ln(-\tilde{W}^M) \).

We match the following empirical moments reported in Table 2: (i) the median block ownership in firms that have been public for at least five years as reported in Foley and Greenwood (2009); (ii) the median volatility of stock returns; (iii) the median Sharpe ratio; (iv) the median market to book ratio; (v) the median dividend-price ratio; (vi) the median sensitivity of CEO pay to shareholder value; and (vii) the median ratio of CEO pay to market value.

The model counterpart of \( L \)'s ownership is given by (63). We define the model-predicted ratio \( (\tilde{MB}^*) \) in the steady-state using the fact that the stock price of a firm represents its total market value. The market value of equity \( (\tilde{MV}^*) \) is thus the steady-state equilibrium stock price, which is given by

\[
\tilde{MV}^* = \frac{1}{r} k_S^*(\tilde{\Theta}) + \frac{1}{\gamma MW} \ln(-\tilde{W}^M). \tag{65}
\]

Our proxy for the book value of equity \( (\tilde{BV}^*) \) is the stock price of the hypothetical firm in the absence of the manager’s human capital inputs (e.g., Ou-Yang (2005), Jung and Subramanian (2014) ).

\[
\tilde{BV}^* = \frac{1}{r} \hat{k}_S^*(\tilde{\Theta}) \text{ where } \hat{k}_S^*(\tilde{\Theta}) = \mu_0 - (1 - \tilde{\Theta}) \gamma S r \sigma^2. \tag{66}
\]
By matching their ratios to the median book-to-market ration from data, we can pin down the parameters, $\mu_0$ and $\mu_1$, that determine the mean output. To match the observed stock return volatility (the standard deviation of the dollar return on the stock normalized by the stock price), we divide the dollar return volatility $\sigma^*_R$ in (57) by the steady-state equilibrium stock price ($\tilde{MV}^*$).

Table 2 reports the model-predicted and actual values of the moments. We see that the baseline model matches the moments reasonably well. In the baseline model, the manager is more risk-averse than the large shareholder who is, in turn, more risk-averse than the representative small shareholder, which conforms to the intuition that outside shareholders are well-diversified relative to blockholders who are, in turn, better diversified than managers who have significant human capital invested in their firms. We should note, however, that the calibration exercise is only intended to provide a reasonable set of baseline parameter values for our analysis.

Table 3 shows key output variables in the steady state in the contracting and owner-manager cases. As discussed in Section 4, we notice that the steady state ownership stakes of the large shareholder are identical in the two scenarios. The effort, expected dollar return, volatility, and Sharpe ratio of the stock are all lower in the contracting case. Although earnings and dividends are lower in the contracting case due to agency conflicts, the stock price is also lower than in the benchmark owner-manager case. The decline in the stock price relative to the owner-manager case outweighs the decline in dividends so that the dividend-price ratio is higher in the contracting case.

5.2 Large Shareholder Ownership, Stock Returns and Incentives

Figure 1 shows the effect of varying the large shareholder’s ownership level about its baseline value on the expected excess return, volatility and Sharpe ratio of the stock as well as the two PPS measures; the sensitivities of managerial pay to earnings ($PPS_X$) and stock returns ($PPS_R$), respectively. The expected excess return, volatility and Sharpe ratio of the stock all decline with the large shareholder’s ownership level. As discussed in the previous section, the large shareholder’s ownership has direct and indirect effects on the stock returns. The direct effect stems from the fact that her ownership stake reduces the stock’s liquidity available to small shareholders, thereby increasing the current stock price and, thus, lowering the expected stock return. The indirect effect stems from the effects of the large shareholder’s ownership stake on the manager’s incentive compensation. Figure 1 shows that the direct effects outweigh the indirect effects in the baseline
The large shareholder’s ownership stake has a positive impact on the two PPS measures suggesting that block ownership and incentive contracting are complementary mechanisms in effecting corporate governance. This finding is consistent with the evidence in Almazan, Hartzell and Starks (2005) and Kim (2010) that the sensitivity of CEO compensation to firm performance is positively associated with the level of outside block ownership.

5.3 Productivity, Risk, Stock Returns and Incentives

Figure 2 shows the effects of the productivity of effort, $\mu_1$ on the expected stock return, volatility, Sharpe Ratio and the manager’s pay-performance sensitivity measures. The expected excess return, volatility and Sharpe ratio of the stock decline with the productivity of effort, while the PPS measures increase. An increase in the effort productivity increases the power of incentives that can be provided to the manager and, therefore, the manager’s effort. Consequently, the stock price increases, thereby lowering the expected return. The return volatility declines because of the increase in the power of incentives to the manager so that the volatility of the residual cash flows to shareholders declines. The Sharpe ratio also declines because the decrease in the expected return outweighs the decrease in the return volatility.

Figure 3 shows the effects of the output volatility, $\sigma$. The expected excess return, volatility and Sharpe ratio of the stock all increase sharply, while the PPS measures decline. An increase in the output volatility increases costs of risk-sharing, thereby lowering the power of incentives and lowering the manager’s effort. Consequently, the stock price decreases so that the expected return of the stock increases. The volatility of the stock also increases because of the decline in the incentive power of the manager’s contract. The Sharpe ratio increases because, as with the effect of $\mu_1$, the increase in the expected return outweighs the increase in the volatility.

5.4 Productivity, Risk and Large Shareholder Ownership

Figures 4 and 5 show the effects of productivity and risk on the dynamics of the large shareholder’s ownership stake in the contracting case and the benchmark owner-manager case. We set $L$’s initial ownership stake to the median ownership of blockholders for IPO firms, which is 60%. Both parameters have a significant impact on the large shareholder’s optimal ownership choices.
in the benchmark scenario, but the ownership choices are largely insensitive to the parameters in the contracting scenario. Indeed, the separation of ownership and control, and the facilitation of risk-sharing through contracting with the manager, effectively insulates the large shareholder from fluctuations in firm-specific parameters. Consequently, the large shareholder’s ownership choices are largely determined by the risk aversions of the large and small shareholders as well as the manager. In other words, in the contracting case, risk-sharing between the large and small shareholders that determines the stock price is effectively separated from the contracting problem that determines the manager’s effort and the firm’s earnings. In sharp contrast with the benchmark owner-manager scenario, the large shareholder’s ownership stake plays less of an incentive role so that it is largely determined by risk-sharing with small shareholders.

6 Alternate Trading Protocol for the Large Shareholder

In our analysis so far, we have followed DU and GHH by specifying $L$’s proceeds from trades by (8). However, the stock price, $P_{t_i}(\Theta^L(t_i))$, is the price at which the marginal trade occurs and is not necessarily the price at which supra- or infra-marginal trades occur. The specification (8) can be justified if $L$ changes its ownership position from $\Theta^L(t_{i-})$ to $\Theta^L(t_{i})$ by making a single block trade in firm stock. However, it can be easily shown that it is optimal for $L$ to alter its ownership incrementally rather than by making a single block trade. If $L$ can trade incrementally, then her proceeds from trading are given by the area under the inverse demand curve, that is,

$$L’s \text{ trading proceeds } = \int_{\Theta^L(t_{i-})}^{\Theta^L(t_{i})} P_{t_i}(\Theta) d\Theta,$$

The following proposition describes the equilibrium in the benchmark owner-manager case with the above specification for $L$’s trading proceeds.

Proposition 8

[Equilibrium in Owner-Manager Case] If $L$’s trading proceeds are given by (67), $L$’s equilibrium ownership stake in the firm is constant over time and given by

$$\Theta^L = \frac{\gamma^S}{\gamma^L + \gamma^S}.$$

30
The value functions of $L$ and $S$, their optimal policies, and the equilibrium stock price are given by Propositions 1, 2 and 3 with $L$’s ownership level given by (68).

In sharp contrast with the case in which $L$’s trading proceeds are given by (8), $L$’s equilibrium ownership level is the price-taking (or competitive) equilibrium that is determined when $L$ does not take into account the impact of her trading policy on the stock price when choosing an optimal equity stake. In other words, the gradual divestiture or accumulation of shares toward the price-taking outcome in DeMarzo and Urošević [DU] (2006) is critically dependent on $L$’s proceeds from trading that, as we discussed earlier, hinges on the assumption that $L$ can only make block trades. The intuition is that, if $L$ can trade incrementally in infinitesimal amounts, she can negate the adverse price impact associated with block trade and, thereby, mimic the competitive trading outcome.

The next proposition shows that the equilibrium in the presence of the agency conflict between $L$ and $M$ also features a constant ownership level for $L$ over time.

**Proposition 9**

[Equilibrium in Agency Case] In the presence of an agency conflict between $L$ and $M$, $L$’s equilibrium ownership level is constant through time and solves the following equation:

\[
[k^*(\Theta^*) - V^*(\Theta^*)] = r \left[ (\gamma_L + \gamma_S)\Theta^* - \gamma_S \right] \sigma_R^2(\Theta^*) - \Theta^* \frac{\partial [\mu(\alpha) - \nu(\Theta^*)]}{\partial \Theta} + \gamma_L r \Theta^* \sigma_R(\Theta^*) \sigma_R'(\Theta^*) = 0,
\]

where $\nu(\Theta^*) \equiv - \left( -\frac{1}{\gamma_M} \ln \beta - \frac{1}{\gamma_M r} \left( \mu_W - \frac{1}{2} \sigma_W^2 \right) + \Psi(\alpha) \right)$ represents the expected cost of instantaneous compensation payment to $M$ over the interval $(t, t + dt)$. In the above, the functions $k^*(\cdot), V^*(\cdot)$ and $\sigma_R(\cdot)$ are defined in the statements of Propositions 5 and 6. The value functions of $L$ and $S$, their optimal policies, $M$’s optimal contract and the equilibrium stock price are given by Propositions 5 and 6 with $L$’s ownership level set to $\Theta^*$.

As in the owner-manager case, $L$’s ownership level is constant through time, but differs from the ownership level in the owner-manager case because of the agency conflict between $L$ and $M$. Intuitively, the equilibrium ownership level also reflects the costs of risk-sharing between $L$ and $M$. Under certain conditions, we can pin down the relationship between the equilibrium in the owner-manager and contracting cases.
Proposition 10

[Comparison with the Benchmark Owner-Manager Case] (i) Provided that $L$’s ownership increases the expected dividend flow \(\frac{\partial [\mu(\alpha) - \nu(\Theta)]}{\partial \Theta} > 0\) and increases the volatility of $M$’s promised utility process \(\sigma'_W > 0\), $L$’s ownership is larger and the expected excess dollar return of the stock is lower in the agency case than in the owner-manager case. (ii) The volatility of the stock return is lower in the agency contracting case than in the owner-manager case.

7 Conclusion

We present a simple tractable dynamic framework that embeds a standard managerial moral hazard problem into an asset pricing model with a large shareholder. Using a simple CARA-normal continuous-time framework, we fully characterize the endogenous determination of a representative firm’s ownership structure, managerial compensation and stock return characteristics and examine their equilibrium interactions. We compare the equilibrium solutions in the owner-manager and agency contracting cases and then discuss the impact of optimal contracting on the large shareholder’s ownership dynamics and the firm’s equity prices.

We show that, in contrast to the benchmark owner-manager case, incorporating the optimal contracting problem results in a stochastically evolving stock price. Agency conflicts between large shareholders and managers lead to less volatile stock returns, lower expected stock returns and lower Sharpe ratios, ceteris paribus. Our results could partly explain the higher equity risk premium in emerging markets in which owner-managed (or family controlled) firms are more prevalent than in developed markets. The separation between ownership and control, and the facilitation of risk-sharing through optimal contracts, effectively insulate the ownership stakes of large shareholders from firm-specific parameters such as the mean and volatility of earnings, and largely determined by the risk aversions of large and small shareholders. We also obtain testable implications for the equilibrium relations among block ownership stakes, expected excess returns and volatilities of stock prices, and managerial incentives. In particular, we highlight the importance of examining the relations among these endogenous variables in an equilibrium framework.

There are several directions in which the relatively simple structure in this paper can be extended. First, we consider a representative firm in the current paper so that there are only two
assets available to investors — the risk-free bond and the firm’s stock. We intend to extend this
simple model to a general framework of multiple firms in a subsequent paper that would allow
us to present a CAPM with optimal managerial contracting and and cross-sectional implications
for large shareholder ownership, managerial incentives, and stock return characteristics (expected
returns, volatilities, and stock betas). More importantly, these theoretical predictions should be
supported by empirical asset pricing tests.

Second, we adopt a simple CARA-normal setting in this paper. This enables us to obtain
a closed-form solution for the optimal contract, equilibrium asset price and ownership structure.
There are two theoretical limitations to the generalization of the framework. As Sannikov (2008)
presents numerical solutions to the contracting problems with different non-CARA preferences,
it would be difficult to find a closed-form solution even to the optimal contracting problem. In
addition, under different preferences and cumulative payoff structures, it would not be plausible to
guess and verify the specific form of the equilibrium price. More work would be necessary to obtain
the general form of the price process in such a generalization.

Lastly, we do not consider an endogenous distinction between the large shareholder and small
shareholders as in Admati et. al (1994) and DeMarzo and Urošević (2006). It would be interesting
to further explore an ex-ante identical investor’s choice to be a large shareholder in a particular firm
in equilibrium, possibly by taking into account a fee structure that the large shareholder charges
small shareholders for the extra risk she bears.

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**Appendix: Proofs**

There are three cases that we have studied in the main text, owner-manager case, contracting case, and contracting case with alternative trading mechanism for $L$. We prove the results in each case recursively, that is, we show that the results hold for the last period and then for earlier periods in general.
Proofs of Propositions 1-4 and Corollary 1

Owner-Manager Case

Consider the last period \( t \in [t_i, t_{i+1}) \) with \( i = N \) during which \( L \)'s shareholdings are held constant: \( \Theta_t = \Theta \). We first determine \( L \)'s optimal effort and consumption choices given her ownership stake \( \Theta \) that maximize the expected utility

\[
W_t^L = \max_{c_t^L, a_t^L} E_t^{c_t^L, a_t^L} \left[ \int_t^T e^{-\delta (\tau - t)} u^L(c_{\tau}, a_{\tau}) d\tau \right] = E_t^{a_t^L} \left[ -\frac{1}{\gamma L} \int_t^T e^{-\delta (\tau - t) - \gamma L (c_{\tau} - \Psi(a_{\tau}^L))} d\tau \right]
\]  

subject to the budget constraint

\[
dB_t^L = (rB_t^L - c_t^L) dt + \Theta dX_t = \left[ rB_t^L - c_t^L + \Theta \mu(a_t^L) \right] dt + \Theta \sigma dZ_t.
\]  

Let \( F(t, B_t^L) \) be the format of \( L \)'s value function \( (W_t^L) \) representing the expected utility from her optimal policy, given the evolution of her risk-free account \( B_t^L \). With CARA preferences, we conjecture that

\[
W_t^L = F(t, B_t^L) = -\frac{1}{\gamma r} e^{-\gamma L} \left[ r(B_t^L + G(t, \Theta)) + \frac{\delta L - r}{\gamma L} \right].
\]  

The Hamilton-Jacobi-Bellman (HJB) equation is then given by

\[
F_t + \max_{c_t^L, a_t^L} \left[ F_B(rB_t^L - c_t^L + \Theta \mu(a_t^L)) + \frac{1}{2} F_{BB} \Theta^2 \sigma^2 + u^L(c_t^L, a_t^L) - \delta L F \right] = 0.
\]  

The first order conditions (FOCs) for \((c_t^L, a_t^L)\) are then

\[
-F_B + u_c^L(c_t^L, a_t^L) = 0,
\]

\[
F_B \Theta \mu'(a_t^L) + u_a^L(c_t^L, a_t^L) = 0.
\]  

By the equations above and using the guessed format of \( F \), in particular, \( F_B = -\gamma L r F \), we obtain the following optimal policies

\[
c_t^L = r(B_t^L + G(t, \Theta)) + \frac{\delta L - r}{\gamma L} + \Psi(a_t^L),
\]

\[
a_t^L = \frac{\mu_1}{\psi} \Theta,
\]  

where the second equation follows from the definition of time-invariant mean and volatility terms in the firm's cash flow process: \( \mu(a) = \mu_0 + \mu_1 a \) and \( \Psi(a) = \frac{1}{2} \psi a^2 \). Substituting the optimal policies and the derivatives of \( F \) into the HJB equation (102) yields

\[
G_t(t, \Theta) = rG(t, \Theta) - V(\Theta)
\]  

where \( V(\Theta) \) is the net benefit flow to \( L \) from holding \( \Theta \) that is given by

\[
V(\Theta) = \Theta \mu(a_t^L) - \Psi(a_t^L) - \frac{1}{2} \gamma L r \Theta^2 \sigma^2 = \Theta \mu_0 + \frac{1}{2} \Theta^2 \left[ \frac{\mu_0^2}{\psi} - \gamma L r \sigma^2 \right].
\]
By integrating (106) over the period, we obtain

$$G(t, \Theta) = \frac{1}{r} V(\Theta),$$

(79)

where we use the fact that the terminal payoff to \( L \), \( \lim_{t \rightarrow \infty} G(t, \Theta) = 0 \).

We now turn to each small shareholder \( S \)'s portfolio problem and determine the equilibrium stock price from the market clearing condition. Let \( \hat{\Theta} \) and \( \hat{a}_t^L \) denote \( L \)'s trading policy and effort choice anticipated by \( S \). \( S \) maximizes his expected utility by solving the optimal consumption and portfolio problem, given the stock price process and anticipated \( L \)'s policies. More specifically, \( S \)'s problem is specified by

$$W_t^S = \max_{c_t^S, \theta_t^S} E^S_t \left[ \int_t^T e^{-\delta^S(r-t)} u^S(c_t^L)dt \right] = E^S_t \left[ -\frac{1}{\gamma^S} \int_t^T e^{-\delta^S(r-t)-\gamma^S c_t^S} dt \right],$$

(80)

subject to the budget constraint that captures the evolution of \( S \)'s stock and risk-free bond balances. Specifically, \( S \)'s bond balance evolves according to

$$dB_t^S = (rB_t^S - c_t^S)dt + \theta_t^S dX_t - P_t d\Theta_t^S,$$

(81)

\( S \)'s total wealth \( Y_{S,t} \) from his portfolio holdings is \( Y_t^S = B_t^S + \theta_t^S P_t \) and evolves according to

$$dY_t^S = (rY_t^S - c_t^S)dt + \theta_t^S(dX_t + dP_t - rP_t dt).$$

(82)

As \( dR_t \equiv dX_t + dP_t - rP_t dt \) in (111) represents the excess dollar return from holding a share of the firm’s stock within the time interval \( (t, t + dt) \). As will be shown below, this process is expressed as the linear in the firm’s cash flow stochastic process: \( \mu_R(t, \hat{\Theta})dt + \sigma_R(t, \hat{\Theta})dZ_t, \)

Let \( Q(t, Y_t^S) \) be the format of \( S \)’s value function \( (W_t^S) \) that depends on the evolution of his total wealth \( Y_t^S \). With CARA preferences, we conjecture and verify the following exponential form as in \( L \)'s problem above,

$$W_t^S = Q(t, Y_t^S) = -\frac{1}{\gamma^S r} e^{-\gamma^S \left[ r(\hat{Y}_t^S + J(t, \hat{\Theta})) + \frac{\delta^S - \gamma^S r}{\gamma^S r} \right]},$$

(83)

The HJB equation is then

$$Q_t + \max_{c_t^S, \theta_t^S} \left[ Q_Y \left( rY_t^S - c_t^S + \theta_t^S \mu_R(t, \hat{\Theta}) \right) + \frac{1}{2} Q_{YY} \left( \theta_t^S \sigma_R(t, \hat{\Theta}) \right)^2 + u^S(c_t^S) - \delta^S Q \right] = 0,$$

(84)

whose first order conditions (FOCs) with respect to \( c_t^S \) and \( \theta_t^S \) are

$$Q_Y = e^{-\gamma^S c_t^S}; \quad \theta_t^S = -\frac{Q_Y \mu_R(t, \hat{\Theta})}{Q_{YY} \sigma_R^2(t, \hat{\Theta})},$$

(85)

Using the guessed format of \( Q \) (that is, \( Q_Y = -\gamma^S r Q \) and \( Q_{YY} = (\gamma^S r)^2 Q \)), the FOCs above imply the following optimal choices of \( S \):

$$c_t^S = r(Y_{t}^S + J(t, \hat{\Theta})) + \frac{\delta^S - r}{\gamma^S r};$$

(86)
\[
\theta_t^S = \frac{\mu_R(\hat{\Theta})}{\gamma^S r \sigma^2_R(\hat{\Theta})}.
\]  

(87)

In equilibrium, \(S\) rationally anticipate \(L\)’s optimal policies (\(\hat{\Theta} = \Theta\) and \(\hat{a}_L^t = a_L^t\)) and the stock market clears \(\int_S \theta_t^S dS = 1 - \Theta\), so that

\[
\mu_R(t, \Theta) = \gamma^S r (1 - \Theta) \sigma^2_R(t, \Theta).
\]

(88)

By plugging the optimal policies and the derivatives of \(Q\) into the HJB equation (113),

\[
J_t(t, \Theta) = rJ_t(t, \Theta) - \frac{1}{2} \frac{\mu^2_R(t, \Theta)}{\sigma^2_R(t, \Theta)} - \frac{1}{2} \frac{\sigma^2_R(t, \Theta)}{\sigma^2_R(t, \Theta)}.
\]

(89)

Then one can easily show that the equilibrium stock price is simply a time-deterministic function that depends on \(L\)’s ownership as below:

\[
P(t, \Theta) = \Lambda(t, \Theta); \quad d\Lambda_t - r\Lambda_t dt = -k(\Theta) dt
\]

(90)

where

\[
k(\Theta) = \mu(a_L^t) - \gamma^S r (1 - \Theta) \sigma^2 = \mu_0 + \frac{\mu^2_R}{\psi} \Theta - \gamma^S r (1 - \Theta) \sigma^2,
\]

(91)

The time deterministic function \(\Lambda(t, \Theta)\) and, therefore, the stock price can be shown to follow the recursive format of

\[
\Lambda(t, \Theta) = \frac{1}{r} k(\Theta),
\]

(92)

where the terminal price \(\lim_{t \to \infty} \Lambda(t, \Theta) = 0\). Also, the expected excess dollar return of the stock and stock return volatility are

\[
\mu_R(t, \Theta) = \gamma^S r (1 - \Theta) \sigma; \quad \sigma_R(t, \Theta) = \sigma.
\]

(93)

We now consider \(L\)’s optimal ownership choice at date \(t_N\), more exactly, \(t_{N-1}\). By (101), \(L\)’s value function at \(t_N\) right after she chooses the ownership level of \(\Theta\) is

\[
W_{t_N}^L = -\frac{1}{\gamma^L} e^{-\gamma L \left[ r(B_{t_N}^L + G(t_N, \Theta)) + \frac{\mu L - r}{\gamma^L} \right]}.
\]

(94)

We denote \(L\)’s risk-free account balance and prior shareholdings at \(t_{N-1}\) by \(B_{t_{N-1}}^L\) and \(\Theta_{t_{N-1}} = \Theta_{t_{N-1}}\), respectively. The stock price right before and after \(L\)’s trade of shares are denoted by \(P(t_N, \Theta)\) and \(P(t_N, \Theta)\), respectively. \(L\)’s trading decision is made in a way that maximizes \(B_{t_N}^L + G(t_N, \Theta)\) in the value function above. The proceeds from trading shares at \(t_N\) is given by \((\Theta_{t_{N-1}} - \Theta) P(t_N, \Theta)\). Then the optimal choice of \(\Theta_{t_N}\) is:

\[
\Theta_{t_N} = \arg \max_\Theta \quad B_{t_N}^L + G(t_N, \Theta) = B_{t_N}^L + (\Theta_{t_{N-1}} - \Theta) P(t_N, \Theta) + G(t_N, \Theta),
\]

FOC: 

\[
- P(t_N, \Theta) + (\Theta_{t_{N-1}} - \Theta) \frac{\partial P(t_N, \Theta)}{\partial \Theta} + \frac{\partial G(t_N, \Theta)}{\partial \Theta} = 0,
\]

\[
\Rightarrow \Theta_{t_N} = \frac{\mu_{t_{N-1}} + \gamma^S r \sigma^2}{\nu + (\gamma^L + \gamma^S) r \sigma^2},
\]

(95)
where \( \nu = \nu_0^2 + \gamma S^r \sigma^2 > 0 \).

Once \( L \)'s optimal trading policy is determined, the stock price on the equilibrium path must be continuous before and after the trading date \( t_i \), at \( P(t_i; \Theta_{t_i}) = L(t_i; \Theta_{t_i}) \). By plugging the optimal ownership choice into (108), the equilibrium time-deterministic function \( G \) at time \( t_i = t_{N-1} \) is given by

\[
G(t_{N-1}, \Theta_{t_{N-1}}) = \phi_{N-1} V(\Theta_{t_{N-1}}),
\]

where \( V(\cdot) \) is defined by (107). By (140), the equilibrium stock price is given by

\[
P(t_{N-1}, \Theta_{t_{N-1}}) = \Lambda(t_{N-1}, \Theta_{t_{N-1}}) = \phi_{N-1} k(\Theta_{t_{N-1}}),
\]

where \( k(\cdot) \) is defined by (120). And \( L \)'s value function at \( t_i = t_{N-1} \) is then

\[
W^*_t \mid t_{N-1} = -\frac{1}{\gamma T} e^{-\gamma L} \left[ r(B^i_{t_{N-1}} + G(t_{N-1}, \Theta_{t_{N-1}})) + \frac{dL_r}{\gamma T} \right]
\]

By looking at the earlier years in general, we now provide the proofs of Propositions 1 to 4, which corresponds to the owner-manager case.

Proof of Proposition 1: Consider any period \( t \in [t_i, t_{i+1}) \) during which \( L \)'s shareholdings are held constant: \( \Theta_{r_t} = \Theta \). We first determine \( L \)'s optimal effort and consumption choices given her ownership stake \( \Theta \) that maximize the expected utility

\[
W^*_t = \max_{c^L_t, a^L_t} E^{L_t} \left[ \int_t^T e^{-\delta L} u^L(c^L_t, a^L_t) d\tau \right] = E^{L_t} \left[ \frac{1}{\gamma L} \int_t^T e^{-\delta L} u^L(c^L_t, a^L_t) d\tau \right]
\]

subject to the budget constraint

\[
dL_t = (rB^L_t - c^L_t) dt + \Theta dX_t = \left[ rB^L_t - c^L_t + \Theta \mu(a^L_t) \right] dt + \Theta \sigma dZ_t.
\]

Let \( F(t, B^L_t) \) be the format of \( L \)'s value function \( (W^*_t) \) representing the expected utility from her optimal policy, given the evolution of her risk-free account \( B^L_t \). With CARA preferences, we conjecture that

\[
W^*_t = F(t, B^L_t) = -\frac{1}{\gamma L} e^{-\gamma L} \left[ rB^L_t + G(t, \Theta) + \frac{dL_r}{\gamma T} \right].
\]

The Hamilton-Jacobi-Bellman (HJB) equation is then given by

\[
F_t + \max_{c^L_t, a^L_t} \left[ F_B(rB^L_t - c^L + \Theta \mu(a^L_t)) + \frac{1}{2} F_{BB} \Theta^2 \sigma^2 + u^L(c^L_t, a^L_t) - \delta L F \right] = 0.
\]

The first order conditions (FOCs) for \((c^L_t, a^L_t)\) are then

\[
-F_B + u^L_t(c^L_t, a^L_t) = 0,
F_B \Theta \mu'(a^L_t) + u^L_a(c^L_t, a^L_t) = 0.
\]
By the equations above and using the guessed format of $F$, in particular, $F_B = -\gamma L \cdot r F$, we obtain the following optimal policies

$$
c_t^L = r (B_t^L + G(t, \Theta)) + \frac{\delta L - r}{\gamma L r} + \Psi(a_t^L), \quad (104)
$$

$$
a_t^L = \frac{\mu}{\psi} \Theta, \quad (105)
$$

where the second equation follows from the definition of time-invariant mean and volatility terms in the firm’s cash flow process: $\mu(a) = \mu_0 + \mu_1 a$ and $\Psi(a) = \frac{1}{2} \psi a^2$. Substituting the optimal policies and the derivatives of $F$ into the HJB equation (102) yields

$$
G_t(t, \Theta) = r G(t, \Theta) - V(\Theta)
$$

where $V(\Theta)$ is the net benefit flow to $L$ from holding $\Theta$ that is given by

$$
V(\Theta) = \Theta \mu(a_t^L) - \Psi(a_t^L) - \frac{1}{2} \gamma L r \Theta^2 \sigma^2 = \Theta \mu_0 + \frac{1}{2} \Theta^2 \left[ \frac{\mu^2}{\psi} - \gamma L r \sigma^2 \right]. \quad (107)
$$

By integrating (106) over the period, we obtain

$$
G(t, \Theta) = \frac{1}{r} \left( 1 - e^{-r(t_{i+1} - t)} \right) V(\Theta) + e^{-r(t_{i+1} - t)} G(t_{i+1}, \Theta).
$$

**Proof of Proposition 2 and 3:** We now turn to each small shareholder $S$’s portfolio problem and determine the equilibrium stock price from the market clearing condition. Let $\tilde{\Theta}$ and $\tilde{a}_t^L$ denote $L$’s trading policy and effort choice anticipated by $S$. $S$ maximizes his expected utility by solving the optimal consumption and portfolio problem, given the stock price process and anticipated $L$’s policies. More specifically, $S$’s problem is specified by

$$
W_t^S = \max_{c_t^L, \Theta_t^S} E_t^{a_t^L} \left[ \int_t^T e^{-\delta^S (\tau - t)} u^S(c_{\tau}^L) d\tau \right] = E_t^{a_t^L} \left[ -\frac{1}{\gamma^S} \int_t^T e^{-\delta^S (\tau - t) - \gamma^S c_{\tau}^S} d\tau \right].
$$

subject to the budget constraint that captures the evolution of $S$’s stock and risk-free bond balances. Specifically, $S$’s bond balance evolves according to

$$
dB_t^S = (r B_t^S - c_t^S) dt + \theta_t^S dX_t - P_t d\Theta_t^S,
$$

$S$’s total wealth $Y_{S,t}$ from his portfolio holdings is $Y_t^S = B_t^S + \theta_t^S P_t$ and evolves according to

$$
dY_t^S = (r Y_t^S - c_t^S) dt + \theta_t^S (dX_t + dP_t - r P_t dt).
$$

As $dR_t \equiv dX_t + dP_t - r P_t dt$ in (111) represents the excess dollar return from holding a share of the firm’s stock within the time interval $(t, t + dt)$. As will be shown below, this process is expressed as the linear in the firm’s cash flow stochastic process: $\mu_R(t, \tilde{\Theta}) dt + \sigma_R(t, \tilde{\Theta}) dZ_t$.

Let $Q(t, Y_t^S)$ be the format of $S$’s value function ($W_t^S$) that depends on the evolution of his total wealth $Y_t^S$. With CARA preferences, we conjecture and verify the following exponential form
as in $L$’s problem above,

$$W_t^S = Q(t, Y_t^S) = -\frac{1}{\gamma^S r} e^{-\gamma^S [r (Y_t^S + J(t, \hat{\Theta})) + \delta^S - r]}.$$  \hfill (112)

The HJB equation is then

$$Q_t + \max_{c^S, \theta^S} \left[ Q_Y \left( r Y_t^S - c^S + \theta^S \mu_R(t, \hat{\Theta}) \right) + \frac{1}{2} Q_{YY} \left( \theta^S \sigma_R(t, \hat{\Theta}) \right)^2 + u^S(c^S) - \delta^S Q \right] = 0,$$  \hfill (113)

whose first order conditions (FOCs) with respect to $c^S$ and $\theta^S$ are

$$Q_Y = e^{-\gamma^S c^S}; \quad \theta^S = -\frac{Q_Y \mu_R(t, \hat{\Theta})}{Q_{YY} \sigma_R^2(t, \hat{\Theta})}.$$  \hfill (114)

Using the guessed format of $Q$ (that is, $Q_Y = -\gamma^S r Q$ and $Q_{YY} = (\gamma^S r)^2 Q$), the FOCs above imply the following optimal choices of $S$:

$$c_t^S = r (Y_t^S + J(t, \hat{\Theta})) + \frac{\delta^S - r}{\gamma^S r};$$  \hfill (115)

$$\theta_t^S = \frac{\mu_R(\hat{\Theta})}{\gamma^S r \sigma_R^2(\hat{\Theta})}.$$  \hfill (116)

In equilibrium, $S$ rationally anticipate $L$’s optimal policies ($\hat{\Theta} = \Theta$ and $\hat{a}_t^L = a_t^L$) and the stock market clears ($\int_S \theta_t^S dS = 1 - \Theta$), so that

$$\mu_R(t, \Theta) = \gamma^S r (1 - \Theta) \sigma_R^2(t, \Theta).$$  \hfill (117)

By plugging the optimal policies and the derivatives of $Q$ into the HJB equation (113),

$$J_t(t, \Theta) = r J(t, \Theta) - \frac{1}{2} \frac{\mu_R^2(t, \Theta)}{\gamma^S r \sigma_R^2(t, \Theta)}.$$  \hfill (118)

Then one can easily show that the equilibrium stock price is simply a time-deterministic function that depends on $L$’s ownership as below:

$$P(t, \Theta) = \Lambda(t, \Theta); \quad d\Lambda_t - r \Lambda dt = -k(\Theta) dt$$  \hfill (119)

where

$$k(\Theta) = \mu(a_t^L) - \gamma^S r (1 - \Theta) \sigma^2 = \mu_0 + \frac{\mu_1^2}{\psi} \Theta - \gamma^S r (1 - \Theta) \sigma^2,$$  \hfill (120)

The time deterministic function $\Lambda(t, \Theta)$ and, therefore, the stock price can be shown to follow the recursive format of

$$\Lambda(t, \Theta) = \frac{1}{r} \left( 1 - e^{-r(t_{i+1} - t)} \right) k(\Theta) + e^{-r(t_{i+1} - t)} \Lambda(t_{i+1}, \Theta),$$  \hfill (121)

where we, for simplicity, assume that $P(t_{i+1}, \Theta) = 0$ for the case $i = N - 1$. Also, the expected excess dollar return of the stock and stock return volatility are

$$\mu_R(t, \Theta) = \gamma^S r (1 - \Theta) \sigma^2; \quad \sigma_R(t, \Theta) = \sigma.$$  \hfill (122)
Proof of Proposition 4: We now consider $L$’s optimal ownership choice at date $t_i$, more exactly, $t_i^-$. By (101), $L$‘s value function at $t_i$ right after she chooses the ownership level of $\Theta$ is

$$W_i^L = -\frac{1}{\gamma i}e^{-\gamma i \left[ r(B_{i-1}^L + G(t_i, \Theta)) + \frac{L_i - L_i^-}{\gamma i} \right]}.$$  \hspace{1cm} (123)

We denote $L$’s risk-free account balance and prior shareholdings at $t_i^-$ by $B_{t_i^-}$ and $\Theta_{t_i^-} = \Theta_{t_i-1}$, respectively. $L$ solves the following problem:

$$\Theta_{t_i} = \arg \max_{\Theta} B_{t_i}^L + G(t_i, \Theta) = B_{t_i}^L + (\Theta_{t_i-1} - \Theta)P(t_i, \Theta) + G(t_i, \Theta),$$

FOC: 

$$-P(t_i, \Theta) + (\Theta_{t_i-1} - \Theta) \frac{\partial P(t_i, \Theta)}{\partial \Theta} + \frac{\partial G(t_i, \Theta)}{\partial \Theta} = \phi_i \left[ V'(\Theta) - k(\Theta) - (\Theta - \Theta_{t_i-1})k(\Theta) \right] = 0,$$

$$\Rightarrow \Theta_{t_i} = \frac{\nu \Theta_{t_i-1} + \gamma^S r \sigma^2}{\nu + (\gamma^L + \gamma^S)r \sigma^2},$$ \hspace{1cm} (124)

where $\phi_i = \frac{1}{\gamma} (1 - e^{-r(t_{i+1}-t_i)})$ and $\nu = \frac{\mu^2}{\psi} + \gamma^S r \sigma^2 > 0$. \hspace{1cm} Q.E.D.

Proof of Corollary 1: It is clear from the above problem and the definitions of $V(\Theta)$ and $k(\Theta)$ that, in the steady-state equilibrium, $L$’s optimal ownership level $\Theta$ solves $V'(\Theta) - k(\Theta) = 0 \Rightarrow (\gamma^S - (\gamma^L + \gamma^S)\Theta)r \sigma^2 = 0$, which results in $\Theta = \gamma^S/\gamma^L + \gamma^S$. \hspace{1cm} Q.E.D.

Proofs of Propositions 5-9 and Corollaries 2-3

Contracting Case

The steps of proving the contracting case is similar as before. We start by solving the problem during the last trading period of $L$. Then we solve the problem recursively for the earlier periods. For expositional convenience, we only provide the proofs of the general results for the earlier periods.

Proof of Proposition 5 and Corollary 2: Suppose that $L$ holds an equity stake of $\Theta$ in the firm in the period $t \in [t_i, t_{i+1}]$. $L$’s optimal consumption and managerial contracting problem is now subject to $M$’s IC constraint:

$$\text{(IC)}: dW_t^M = (\delta^M W_t^M - u^M(e_t^M, a_t^M)) dt - \frac{u_a^M(e_t^M, a_t^M)}{\mu^M(a_t^M)} \sigma dZ_t$$

$$= \left( \delta^M W_t^M + \frac{1}{\gamma^M} H(e_t^M, a_t^M) \right) dt + \frac{\psi}{\mu^M} a_t^M H(e_t^M, a_t^M) \sigma dZ_t,$$ \hspace{1cm} (125)

where $H(e_t^M, a_t^M) = e^{-\gamma^M(e_t^M - \frac{1}{2}\psi(a_t^M)^2)}$. Given $M$’s incentive contract, $L$’s risk-free bond account now evolves according to

$$dB_t^L = (r B_t^L - c_t^L) dt + \Theta (dX_t - c_t^L dt) = (r B_t^L - c_t^L + \Theta (\mu(a_t^M) - c_t^M)) dt + \Theta \sigma dZ_t.$$ \hspace{1cm} (126)

We conjecture and verify that $L$’s value function has the following exponential format,

$$W_t^L = F(t, B_t^L, W_t^M) = -\frac{1}{\gamma^L}e^{-\gamma^L \left[ r(B_{t-1}^L + G(t, \Theta)) + \frac{L_t - L_t^-}{\gamma^L} \right]}.$$ \hspace{1cm} (127)
The Hamilton-Jacobi-Bellman (HJB) equation is then given by

\[
F_t + \max_{c^L, c^M, a^M} \left[ F_B(rB^L - c^L + \Theta(\mu(a^M) - c^M)) + F_W \left( \delta^M W^M + \frac{1}{\gamma^M} H(c^M, a^M) \right) + \frac{1}{2} F_{BB} \Theta^2 \sigma^2 + F_{BW} \Theta \sigma \left( \frac{\psi}{\mu_1} a^M H(c^M, a^M) \right) + \frac{1}{2} F_{WW} \left( \frac{\psi}{\mu_1} a^M H(c^M, a^M) \right)^2 + u^L(c^L) - \delta^L F \right] = 0.
\] (128)

The first order conditions (FOCs) for \((c^L, c^M, a^M)\) are then

\[
c^L : -F_B + u^L(c^L) = 0,
\]

\[
c^M : F_B \Theta + F_W H + F_{BW} \Theta \gamma^M \left( \frac{\psi}{\mu_1} a^M H \right) \sigma^2 + F_{WW} \gamma^M \left( \frac{\psi}{\mu_1} a^M H \right)^2 \sigma^2 = 0,
\]

\[
a^M : F_B \Theta \mu_1 + F_W(\psi a^M H) + F_{BW} \Theta \left( 1 + \gamma^M \psi(a^M)^2 \right) \left( \frac{\psi}{\mu_1} H \right) \sigma^2 + F_{WW} \left( 1 + \gamma^M \psi(a^M)^2 \right) \left( \frac{\psi}{\mu_1} H \right)^2 a^M \sigma^2 = 0.
\] (129)

Using the FOCs and the derivatives of \(F\), we obtain the following optimal policies

\[
c^L_t = r(B^L_t + G(t, \Theta)) + \frac{\Theta}{\gamma^M} \ln(-W^M_t) + \frac{\delta^L - r}{\gamma^L},
\]

\[
a^M_t = \alpha,
\]

\[
c^L_t = -\frac{1}{\gamma^M} \ln(\beta) - \frac{1}{\gamma^M} \ln(-W^M_t) + \psi(a^M_t),
\] (130)

where \(\alpha\) and \(\beta\) are determined by

\[
\beta = \gamma^M \left[ \gamma^M \alpha(\psi - \mu_1) + 1 \right],
\]

\[
\left( 1 + \frac{\gamma^L}{\gamma^M} \Theta \right) \sigma_W^2 - (\gamma^L r \Theta \sigma) \sigma_W - (r - \frac{\beta}{\gamma^M}) = 0.
\] (131)

where \(\sigma_W\) is defined below. Note that the constants \((\alpha\) and \(\beta\)) depend upon \(L\)’s ownership level \(\Theta\). By (125) and optimal policies, we show that \(M\)’s promised payoff process follows

\[
d\ln(-W^M_t) = \left( \mu_W - \frac{1}{2} \sigma_W^2 \right) dt - \sigma_W dZ_t,
\] (132)

where

\[
\mu_W = \delta^M - \frac{\beta}{\gamma^M}; \sigma_W = \frac{\psi}{\mu_1} \alpha \beta \sigma.
\] (133)

Substituting the optimal policies and the derivatives of \(F\) into the HJB equation (128) yields

\[
G_t(t, \Theta) = rG(t, \Theta) - V(\Theta),
\] (134)
where $L$’s net benefit flow from holding $\Theta$ is given

$$V(\Theta) = \Theta \left( \mu(\alpha) - \left( -\frac{1}{\gamma M} \ln(\beta) - \frac{1}{\gamma M r} \left( \mu W - \frac{1}{2} \sigma_W^2 \right) + \Psi(\alpha) \right) - \frac{1}{2} \gamma^L r \Theta^2 \left( \sigma - \frac{\sigma_W}{\gamma M r} \right)^2 \right).$$

By integrating (134) over the period, we obtain

$$G(t, \Theta) = \frac{1}{r} \left(1 - e^{-r(t+i+1-t)} \right) V(\Theta) + e^{-r(t+i+1-t)} G(t_{i+1}, \Theta).$$

For the last trading period $i = N$, we take the limit $t_{N+1} \to \infty$ and $\lim_{t \to \infty} G(t, \Theta) = 0$.

Proof for Proposition 6: As $L$’s optimal policies are linear in $\ln(-W_i^M)$, we conjecture and verify that the equilibrium stock price in the agency contracting case has the following form,

$$P(t, \Theta, W_i^M) = \Lambda(t, \Theta) + \frac{1}{\gamma M r} \ln(-W_i^M).$$

We follow a similar approach to solve $S$’s optimal portfolio problem except that $S$’s budget constraint now subtracts $M$’s compensation payment in the firm’s incremental dividend, and we need to additionally consider the stochastic component of the stock price process. In equilibrium, $S$ rationally anticipates $L$’s optimal policies and the stock market clears, which derives the time deterministic component of the stock price, $\Lambda(t, \Theta)$ :

$$d \Lambda_t - r \Lambda_t = -k(\Theta) dt,$$

where

$$k(\Theta) = \mu(\alpha) - \left( -\frac{1}{\gamma M} \ln(\beta) - \frac{1}{\gamma M r} \left( \mu W - \frac{1}{2} \sigma_W^2 \right) + \Psi(\alpha) \right) - \gamma S r (1 - \Theta) \left( \sigma - \frac{\sigma_W}{\gamma M r} \right)^2.$$

Note that the deterministic function $\Lambda(t, \Theta)$ can be expressed as the following recursive format:

$$\Lambda(t, \Theta) = \frac{1}{r} \left(1 - e^{-r(t+i+1-t)} \right) k(\Theta) + e^{-r(t+i+1-t)} \Lambda(t_{i+1}, \Theta).$$

For the last trading period, $\lim_{t \to \infty} \Lambda(t, \Theta) = 0$. Q.E.D.

Proof for Proposition 7: As shown in the prior section, $L$’s value function at $t_i$ right after she chooses the ownership level of $\Theta$ is then given by

$$W_{t_i}^L = -\frac{1}{\gamma L r} e^{-\gamma L \left[ r(B_{t_i}^L + G(t_i, \Theta)) + \frac{e^\frac{\alpha}{\gamma W} \ln(-W_{t_i}^M) + \frac{\gamma L}{\gamma M r} \left( \gamma M r W_{t_i}^M \right) L}{\gamma M r} \right]},$$

where $M$’s promised payoff from the incentive contract offered given $L$’s ownership choice $\Theta$ is $W_{t_i}^M$. We denote $L$’s risk-free account balance and prior shareholdings at $t_i$ by $B_{t_i}$ and $\Theta_{t_i} = \Theta_{t_{i-1}}$, respectively. The stock price right before and after $L$’s trade of shares are denoted by $P(t_i, \Theta_{t_i})$ and $P(t_i, \Theta)$ respectively. By (137),

$$P(t_i, \Theta, W_{t_i}^M) = \Lambda(t_i, \Theta) + \frac{1}{\gamma M r} \ln(-W_{t_i}^M).$$

$L$’s trading decision is made in a way that maximizes her certainty-equivalent continuation payoff.
after her trading of new shares in (141). Given the L’s trading proceeds \((\Theta_{t_i} - \Theta)P(t_i, \Theta)\), the optimal choice is given by:

\[
\Theta_{t_i} = \arg\max_{\Theta} r [B_{t_i}^L + G(t_i, \Theta)] + \frac{\Theta}{\gamma^M} \ln(-W_{t_i}^M),
\]

\[
\Theta_{t_i} = \arg\max_{\Theta} B_{t_i}^L + (\Theta_{t_i} - \Theta)P(t_i, \Theta, W_{t_i}^M) + G(t_i, \Theta) + \frac{\Theta}{\gamma^M} \ln(-W_{t_i}^M),
\]

**FOC:** \(-P(t_i, \Theta, W_{t_i}^M) + (\Theta_{t_i} - \Theta)\frac{\partial P(t_i, \Theta, W_{t_i}^M)}{\partial \Theta} + \frac{\partial G(t_i, \Theta)}{\partial \Theta} + \frac{1}{\gamma^M} \ln(-W_{t_i}^M) = 0\)

\[
\Rightarrow -\left(\Lambda(t_i, \Theta) + \frac{1}{\gamma^M} \ln(-W_{t_i}^M)\right) + (\Theta_{t_i} - \Theta)\phi \cdot k(\Theta) + \phi \cdot \gamma' + \frac{1}{\gamma^M} \ln(-W_{t_i}^M) = 0
\]

**\Rightarrow \phi_i [V'(\Theta) - k(\Theta) - (\Theta - \Theta_{t_i})k'(\Theta)] = 0.\]** (143)

By the definitions of \(V^*(\Theta)\) and \(k^*(\Theta)\) in (135) and (139),

\[
V^*(\Theta) = \Theta \Omega_1(\Theta) + \frac{1}{\gamma^M} \Omega_2(\Theta) - \frac{1}{2} \gamma^L r \Theta^2 \Omega_3(\Theta)
\]

\[
k^*(\Theta) = \Omega_1(\Theta) + \frac{1}{\gamma^M} \Omega_2(\Theta) - \gamma^S r (1 - \Theta) \Omega_3(\Theta),
\]

where \(\Omega_1(\Theta) \equiv (\mu(\alpha) + \frac{1}{\gamma^T} \ln(\beta) - \Psi(\alpha)), \Omega_2(\Theta) \equiv (\mu_W - \frac{1}{2} \sigma^2_W),\) and \(\Omega_3(\Theta) \equiv \left(\sigma - \frac{\sigma_W}{\gamma^M}\right)^2.\)

Note that the manager’s contractual variables, \(\alpha\) and \(\beta\), are also functions of \(\Theta\) and that \(\beta\) is directly obtained by its relation with \(\beta = \beta(\alpha) = \gamma^M \left[\gamma^M (\psi(\alpha - \mu_1) + 1)\right].\) Also note that the variables \(\mu_W\) and \(\sigma_W\) are functions of \(\alpha\). Then the derivative of \(V^*(\Theta)\) with respect to \(\Theta\) is

\[
V^*(\Theta) = \Omega_1(\Theta) + \Theta \frac{\partial \Omega_1(\Theta)}{\partial \Theta} + \frac{1}{\gamma^M} \left(\Omega_2(\Theta) + \Theta \frac{\partial \Omega_2(\Theta)}{\partial \Theta}\right) - \gamma^L r \Theta \Omega_3(\Theta) - \frac{1}{2} \gamma^L r \Theta^2 \frac{\partial \Omega_3(\Theta)}{\partial \Theta}\]

(144)

Consider the three derivative terms above.

\[
\Theta \frac{\partial \Omega_1(\Theta)}{\partial \Theta} + \frac{1}{\gamma^M} \Theta \frac{\partial \Omega_2(\Theta)}{\partial \Theta} - \frac{1}{2} \gamma^L r \Theta^2 \frac{\partial \Omega_3(\Theta)}{\partial \Theta}
\]

\[
= \Theta \left(\mu_1 \alpha' - \frac{1}{\gamma^M} \frac{\beta'(\alpha)}{\beta(\alpha)} - \psi(\Theta)\alpha'(\Theta)\right)
\]

\[
+ \frac{1}{\gamma^M} \Theta \left(\mu_W(\alpha) \alpha'(\Theta) - \sigma_W(\alpha) \sigma_W(\alpha) \alpha'(\Theta)\right) - \gamma^L r \Theta^2 \left(\sigma - \frac{\sigma_W(\alpha)}{\gamma^M}\right) \left(\frac{1}{\gamma^M} \frac{\partial \sigma_W(\alpha)}{\partial \alpha} \alpha'(\Theta)\right) = 0.
\]

(145)

By the envelope theorem, the above three terms in (145) that arise from the dependence of \(\Theta\) through \(\alpha\) should be zero. To see this more clearly, recall that we have chosen the manager’s optimal effort \(\alpha\) for a given \(\Theta\) to maximize \(L\)’s value function in (136), that is, \(V(\Theta)\). We thus have

\[
\frac{\partial V^*(\alpha)}{\partial \alpha} = \Theta \left(\mu_1 + \frac{1}{\gamma^M} \frac{\beta'(\alpha)}{\beta(\alpha)} - \psi(\Theta)\right)
\]

\[
+ \frac{1}{\gamma^M} \Theta \left(\mu_W(\alpha) - \sigma_W(\alpha) \sigma_W(\alpha)\right) - \gamma^L r \Theta^2 \left(\sigma - \frac{\sigma_W(\alpha)}{\gamma^M}\right) \left(\frac{1}{\gamma^M} \frac{\partial \sigma_W(\alpha)}{\partial \alpha}\right) = 0,
\]

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which results in

\[ V^{*'}(\Theta) = \Omega_1(\Theta) + \frac{1}{\gamma L \tau} \Omega_2(\Theta) - \gamma L \tau \Omega_3(\Theta). \]  

(146)

Similarly, we take the derivative of \( k^*(\Theta) \).

\[ k^{*'}(\Theta) = \frac{\partial \Omega_1(\Theta)}{\partial \Theta} + \frac{1}{\gamma M \tau} \frac{\partial \Omega_2(\Theta)}{\partial \Theta} + \gamma S r \Omega_3(\Theta) - \gamma S r (1 - \Theta) \frac{\partial \Omega_3(\Theta)}{\partial \Theta}. \]  

(147)

\[ = \frac{1}{2} \gamma L r \Theta \frac{\partial \Omega_3(\Theta)}{\partial \Theta} + \gamma S r \Omega_3(\Theta) - \gamma S r (1 - \Theta) \frac{\partial \Omega_3(\Theta)}{\partial \Theta}, \]  

(148)

where the second equality follows from (145).

As a result, we obtain the FOC by

\[ FOC = \left[ V^{*'}(\Theta) - k^*(\Theta) - (\Theta - \Theta_{t-1}) k^{*'}(\Theta) \right]. \]

Proof of Proposition 9: The proceeds from \( L \)'s trading becomes \(-\int_{\Theta_{t-1}}^{\Theta} P(t_i, \Theta) d\Theta\). The optimal choice is then given by:

\[ \Theta_{t_i} = \arg \max_{\Theta} \left[ B_{t_i}^L + G(t_i, \Theta) = B_{t_i}^L - \int_{\Theta_{t-1}}^{\Theta} P(t_i, \Theta) d\Theta + G(t_i, \Theta), \right. \]

\[ \text{FOC : } -P(t_i, \Theta) + \frac{\partial G(t_i, \Theta)}{\partial \Theta} = -\phi_i \left[ k(\Theta) - V'(\Theta) \right] = 0, \]

\[ \Rightarrow \Theta_{t_i} = \frac{\gamma S}{\gamma L + \gamma S}. \]  

(150)

Q.E.D.

Proof of Proposition 8: The only difference from the proof of Proposition 4 is that the proceeds from \( L \)'s trading is given by: Exact proceeds under the inverse demand curve: \(-\int_{\Theta_{t-1}}^{\Theta} P(t_i, \Theta, W_{t_i}^M) d\Theta\). Then the optimal choice follows:

\[ \Theta_{t_i} = \arg \max_{\Theta} \left[ B_{t_i}^L - \int_{\Theta_{t-1}}^{\Theta} P(t_i, \Theta) d\Theta + G(t_i, \Theta) + \frac{\Theta}{\gamma M} \ln(-W_{t_i}^M), \right. \]

\[ \text{FOC : } -P(t_i, \Theta, W_{t_i}^M) + \frac{\partial G(t_i, \Theta)}{\partial \Theta} + \frac{\Theta}{\gamma M} \ln(-W_{t_i}^M) = 0, \]  

Q.E.D.
\[ \Rightarrow - \left( \Lambda(t_i, \Theta) + \frac{1}{M_r} \ln(-W_{ti}^M) \right) + \phi_i V'(\Theta) + \frac{1}{M_r} \ln(-W_{ti}^M) = 0, \]
\[ \Rightarrow -\phi_i [k(\Theta) - V'(\Theta)] = 0. \quad (151) \]

Q.E.D.
Table 1: Baseline Parameters

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Baseline Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate ((r))</td>
<td>0.02</td>
</tr>
<tr>
<td>Subjective discount rate ((\delta_L = \delta_M = \delta_S))</td>
<td>0.0404</td>
</tr>
<tr>
<td>Unit cost of effort ((\psi))</td>
<td>1</td>
</tr>
<tr>
<td>Constant term ((\mu_0)) in the firm’s mean cash flow (\mu(a_t))</td>
<td>0.17</td>
</tr>
<tr>
<td>Productivity of effort ((\mu_1)) in the firm’s mean cash flow (\mu(a_t))</td>
<td>0.03</td>
</tr>
<tr>
<td>Firm output (cash flow) volatility (\sigma)</td>
<td>0.35</td>
</tr>
<tr>
<td>L’s absolute risk aversion ((\gamma_L))</td>
<td>224.25</td>
</tr>
<tr>
<td>M’s risk aversion ((\gamma_M))</td>
<td>297.79</td>
</tr>
<tr>
<td>S’s absolute risk aversion ((\gamma_S))</td>
<td>74.75</td>
</tr>
<tr>
<td>M’s mean promised payoff in steady state (\ln(-\tilde{W}^M))</td>
<td>-9.26</td>
</tr>
</tbody>
</table>

Table 2: Actual and Model-Predicted (Steady-State) Moments.

<table>
<thead>
<tr>
<th></th>
<th>L’s ownership</th>
<th>Stock return volatility</th>
<th>Sharpe ratio</th>
<th>Market-to-book ratio</th>
<th>Dividend-price ratio</th>
<th>PPS_R</th>
<th>CEO pay-to-market value ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>0.250</td>
<td>0.194</td>
<td>0.320</td>
<td>1.236</td>
<td>0.045</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>Predicted</td>
<td>0.250</td>
<td>0.126</td>
<td>0.320</td>
<td>1.235</td>
<td>0.062</td>
<td>0.004</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 3: Model-Predicted (Steady-state) Equilibrium in the Agency Contracting and Owner-Manager Cases.

<table>
<thead>
<tr>
<th></th>
<th>L’s ownership</th>
<th>Effort</th>
<th>Expected dollar return</th>
<th>Dollar return volatility</th>
<th>Sharpe ratio</th>
<th>Market-to-book ratio</th>
<th>Dividend-price ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agency Contracting Case</td>
<td>0.250</td>
<td>0.005</td>
<td>0.094</td>
<td>0.294</td>
<td>0.320</td>
<td>1.235</td>
<td>0.062</td>
</tr>
<tr>
<td>Owner-Manager Case</td>
<td>0.250</td>
<td>0.008</td>
<td>0.142</td>
<td>0.350</td>
<td>0.407</td>
<td>2.550</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Figure 1: Effects of Large Shareholder Ownership
Figure 5: Cash Flow Volatility and Large Shareholder Ownership

\[
\begin{align*}
\text{OM case} & : \sigma = 0.01 \quad \sigma = 0.17 \quad \sigma = 0.34 \\
\text{AC case} & : 
\end{align*}
\]