Optimal Bank Liability Structure

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Abstract

We develop a model of capital and liability structure of banks that optimally respond to changes of regulatory environment. The model produces the following results.

In the absence of regulation, banks in our model take high leverage, both in the form of deposits and subordinated debt. We find that subordinated debt is important in banks’ liability structure—holding zero subordinated debt is never optimal for a bank. However, a bank optimizes its liability structure should not have too much subordinated debt; the optimal level is to set the endogenous default of debt to coincide with the point at which the depositors will choose to run. In this optimal choice of liability structure, the subordinated debt does not protect deposits from default.

The introduction of FDIC regulation and insurance raises the market value of banks, even when banks are charged for fair insurance premium. With deposit insurance, banks issue more deposits but reduce their subordinated debt to ensure that endogenous default still coincides with bank closure. In this optimal liability structure, subordinated debt does not protect FDIC from losses in covering deposits when the bank is closed. This particular response by banks dampens the reduction of expected bankruptcy loss potential that could be brought by the FDIC insurance program: if banks were not able to adjust their liability structure in response to the introduction of FDIC insurance, the reduction in expected losses would have been higher. Another important finding is that charging a fair insurance premium takes away the incentive of risk shifting.

The optimal response of banks to equity requirement are more complicated to analyze. Obviously, there should be no response if a bank’s optimal liability structure automatically satisfies the equity requirement imposed by the regulators. For a set of reasonable parameters for typical banks, we find Basel’s 7-percent equity requirement is mostly binding. Subject to binding capital requirement, banks need to trim both deposits and subordinated debt, as expected. However, in some cases it is optimal for banks to trim more subordinated debt than deposits so that the endogenous default is shielded by bank run/closure. In other cases, it is optimal for banks to set endogenous default exactly at the point of bank run/closure. Again, banks with optimal liability structure issue some subordinated debt, but not so much that it protects deposits.

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# Contents

1 Introduction .................................................. 3

2 Liability Structure and Regulation ............................. 5
   2.1 Bank Assets and Liabilities .................................. 6
   2.2 Bank Run, FDIC, and Equity Requirement ...................... 8

3 Valuation and Optimization .................................... 11
   3.1 Bank Valuation and Endogenous Insurance Premium .......... 11
   3.2 Optimal Liability Structure ................................... 14

4 Optimal Response of Banks to Regulation Changes .............. 19

5 Conclusion ..................................................... 20

A Appendix ......................................................... 23
   A.1 Proof of Theorem 1 ............................................. 27
   A.2 Proof of Theorem 2 ............................................. 28
   A.3 Proof of Lemma 1 .............................................. 29
   A.4 Proof of Lemma 2 .............................................. 30
   A.5 Proof of Lemma 3 .............................................. 32
1 Introduction

The banking industry has experienced several major crises in the past, and many of these crises have focused regulators’ attention on the inadequacy of equity capital held by the banks, which might have exacerbated the risk of bank runs or bank failure. Consequently, regulations were often introduced and revised after the banking crises. After the frequent bank runs experienced during the Great Depression, the Banking Act of 1933 was signed into law to create the Federal Deposit Insurance Corporation (FDIC). After the world-wide financial crisis and Great Recession of 2007–2009, the banking industry faced a sweeping change in its regulatory environment. In the U.S., the Dodd-Frank Wall Street Reform and Consumer Protection Act (often referred to as the Dodd-Frank Act) was signed into law in 2010 to bring regulatory reforms in a wide range of areas from FDIC deposit insurance to stress tests of banks’ capital adequacy. Internationally, regulators agreed in 2011 on more stringent capital adequacy standards for banks, which are collectively named as Basel III. It lifts the tier-1 equity capital requirement for all banks from 4 percent in Basel II to 7 percent and to even higher levels for banks that are designated as systemically important.

Since the introduction of Dodd-Frank Act and Basel III, regulators around the world have gradually rolled out changes to the regulation of banks’ capital structure, and the shape of bank regulation is still evolving. The Swiss National Bank has raised the capital requirement to 19 percent for its two largest banks. The European Union has set up its additional capital requirement in accordance with Basel III. The Federal Reserve, the FDIC, and the Office of Comptroller of the Currency (OCC) have issued the interim rule on raising capital requirement for the U.S. banks and foreign banks operating in the U.S. The Volcker Rule drafted by the U.S. regulators aims to control the risk of bank assets more effectively. Proposals for further regulatory changes are abundant in the academic literature. Some have argued for raising equity requirement to a level as high as 20 percent.\(^1\) There are also antithetical views on whether banks should hold subordinated debt.\(^2\) A drastic proposition is to reduce corporate tax for banks.\(^3\)

The recent changes of regulation and the proposals for additional changes call for careful analyses of the effects of regulatory changes. Each regulatory change tends to fix a particular observed broken factor in the bank’s assets or liability. For example, the deposit insurance intends to address bank runs caused by the fear of losing deposits, which are the major source of financing for banks. The equity requirement intends to address the high leverage

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\(^1\)See the book by Admati and Hellwig (2013) and a non-technical article by Admati, DeMarzo, Hellwig, and Pfleiderer (2011).

\(^2\)For example, Bulow and Klemperer (2013) suggest that banks should hold no subordinated debt. In their view, the only securities issued by banks should be equity or securities convertible to equity. In contrast, the Fed governor Daniel Tarullo (2013) argued that a requirement of holding long-term subordinated debt would improve capital structure and resolution of banks.

\(^3\)Citing academic studies of corporate tax as incentives of taking debt, Fleischer (2013) suggests that reducing corporate tax for banks will make banks safer.
of banks. The Volcker Rule addresses the risks of the assets held by the banks and intends
to cut down the risk of large losses in assets that bank equity must absorb. However, these
intentions implicitly assume that banks will keep other things the same, ignoring the overall
optimal response of banks that can change its assets and liabilities. With deposit insurance,
it can be optimal for a bank to finance assets with more deposit, exposing the bank to a
higher risk of being closed if it incur large losses. With higher equity requirement, a bank
may find it more optimal to reduce banking services. With lower asset risk, banks may be
better off by increasing leverage. More broadly, it is unclear whether the optimal response of
the entire bank capital structure will undo or significantly diminish the intended effects of a
regulatory change. It is even possible that the regulatory changes may result in unintended
consequences.

In order to understand banks’ optimal responses to regulatory changes, we develop a
parsimonious model of optimal capital/liability structure of banks. Following the traditional
literature in corporate finance, we assume a bank maximizes its total market value by
financing its cash-flow generating assets through issuing debt and equity. Our assumption
is consistent with the principle that the bank management should act in the interest of its
claim-holders. We do not assume that banks choose their liability structure to maximize
a measure of social welfare such as reducing systemic risk and increasing banking services.
Analysis of social welfare associated with alternative bank regulations is unquestionably
important, but understanding the optimal response of banks to regulatory changes is a
necessary first step for proper social welfare analysis of bank regulation. This is the primary
goal of our paper.

In our model, banks take deposits, issue subordinated debt, and are owned by equity
holders. Banks’ assets are risky and they generate cash flows. A distinctive characteristic
of banks is that banks take deposits and provide account services to their depositors. Bank
deposit is different from other forms of debt partly because banks earn income from the
provision of account services to depositors. More striking differences between deposit and
other debt are that depositors can run, deposit can be FDIC insured, and deposit-taking
banks are subject to special regulations. Our model incorporates these features of banks
explicitly. Most importantly, bank run by rational depositors and bank closure by rule-
following regulators are fully reflected in equity holders’ endogenous choice of defaulting on
debt in order to maximizing bank’s market value. Deposit insurance premiums are endoge-
nously determined by FDIC based on the risk in the bank’s asset and liability structure.
Extending the framework pioneered by Merton (1974, 1977) and Leland (1994), we analyt-
cally solve for the optimal bank liability structure under various regulatory environments,
which allows us examine the optimal responses of banks to the changes in regulation.

In the absence of regulation, banks in our model take high leverage, both in the form
of deposits and subordinated debt. We find that subordinated debt is important in banks’
liability structure—holding zero subordinated debt is never optimal for a bank. However,
a bank optimizes its liability structure should not have too much subordinated debt; the
optimal level is to set the endogenous default of debt to coincide with the point at which the depositors will choose to run. In this optimal choice of liability structure, the subordinated debt does not protect deposits from default.

The introduction of FDIC regulation and insurance raises the market value of banks, even when banks are charged for fair insurance premium. With deposit insurance, banks issue more deposits but reduce their subordinated debt to ensure that endogenous default still coincides with bank closure. In this optimal liability structure, subordinated debt does not protect FDIC from losses in covering deposits when the bank is closed. This particular response by banks dampens the reduction of expected bankruptcy loss potential that could be brought by the FDIC insurance program: if banks were not able to adjust their liability structure in response to the introduction of FDIC insurance, the reduction in expected losses would have been higher. Another important finding is that charging a fair insurance premium takes away the incentive of risk shifting.

The optimal response of banks to equity requirement are more complicated to analyze. Obviously, there should be no response if a bank’s optimal liability structure automatically satisfies the equity requirement imposed by the regulators. For a set of reasonable parameters for typical banks, we find Basel’s 7-percent equity requirement is mostly binding. Subject to binding capital requirement, banks need to trim both deposits and subordinated debt, as expected. However, in some cases it is optimal for banks to trim more subordinated debt than deposits so that the endogenous default is shielded by bank run/closure. In other cases, it is optimal for banks to set endogenous default exactly at the point of bank run/closure. Again, banks with optimal liability structure issue some subordinated debt, but not so much that it protects deposits.

The road-map for this paper is as follows. In section 2, we develop the model of bank liability structure in the context of several regulatory environments. In section 3, we characterize bank optimal liability structures in those regulatory environments. In section 4, we examine the effects of regulatory changes when banks are assumed to adjust liability structure optimally. Section 5 concludes the paper by placing our work in the context of the literature on bank capital.

2 Liability Structure and Regulation

Banks share some common characteristics with non-financial firms: both have access to cash flows generated by their assets and both finance their assets by issuing debt and equity. Banks, however, differ from non-financial firms in that they take deposits and provide liquidity services to their depositors through check writing, ATMs, and other transaction services such as wire transfers, bill payments, etc. The banking business of taking deposit and serving accounts is heavily regulated in most countries. In the U.S., a large part of deposit accounts is insured by the Federal Deposit Insurance Corporation (FDIC), which
imposes additional regulations on banks. Deposits and the associated account services, FDIC insurance, and bank regulation distinguish banking business from other non-financial corporate business and set the bank capital decision apart from that of other firms.

A typical firm operates in a market with two important frictions: corporate taxes and costs associated with default/bankruptcy. These two frictions are the most crucial for a general firm in its choice of capital and liability structure, as recognized in the literature originating from Modigliani and Miller (1963) and Baxter (1967) and analyzed more recently in Leland (1994). While banks face these same frictions, they have to also simultaneously incorporate other considerations such as the potential of a run by depositors, FDIC deposit insurance premium, charter authority’s closure rules, and bank regulations on capital requirements in determining their optimal capital and liability structure. Figure 1 illustrates assets and liabilities of a typical bank. We will analyze each part of bank liability structure thoroughly after discussing our model of bank assets.

2.1 Bank Assets and Liabilities

In our model, the bank owns a portfolio of risky assets that generate cash flows. The portfolio of assets is valued at \( V \), which is the major part on the asset side of Figure 1. The asset is risky, and its value follows a stochastic process.\(^4\) The instantaneous cash flow of the asset is \( \delta V \), where \( \delta \) is referred to as the rate of the cash flow. The risk of the asset is represented by the volatility of the asset value and denoted by \( \sigma \). We assume that the portfolio of assets is given exogenously. Although this assumption rules out interesting issues of endogenous asset substitution,\(^5\) we will later examine the incentives for banks to alter asset riskiness or cash flow. Following Merton (1974) and Leland (1994), we assume that all investors have the full information about the asset portfolio value.\(^6\)

The most distinctive feature of the liability structure of a bank is its function to take deposits from households or businesses and pay interest on such deposits. Deposits, which are shown as the first part on the liability side in Figure 1, are often the single most important source of funds to finance the bank’s assets. We denote by \( D \) the total amount of deposits that the bank takes. There are two ways to make deposit a safe debt: (1) depositors withdraw their money early enough to ensure that the bank has enough assets to redeem depositors, or (2) the bank purchase insurance that guarantees the depositors in

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\(^4\)Following Merton (1974) and Leland (1994), we assume the stochastic process is a geometric Brownian motion, which is described by equation (19) in Appendix.

\(^5\)The literature has pointed out that debt may create incentives to substitute assets with higher risk (e.g., Green, 1986, and Harris and Raviv, 1991) and FDIC insurance may also make for such incentive (e.g., Pennacchi, 2006, and Schneidar and Tornell, 2004).

\(^6\)In reality only accounting values of assets are reported quarterly, and thus the assumption of full information sets aside the disparity between accounting value and intrinsic value. Under this assumption, we may interpret \( V \) as the fair accounting value. Furthermore, if the assets have relatively homogenous risk, we may also interpret \( V \) as the value of risk-weighted assets.
full. The first way induces bankruptcy cost due to a bank run, and the second way requires that the bank pay insurance premium, denoted by $I$. We will later discuss bank run and deposit insurance in more detail.

If the deposit can be withdrawn at anytime without risk, the fair interest rate on deposit should be the risk-free rate. On deposit accounts, however, banks typically pay lower interest rate. Depositors accept lower interest rate because they receive the service of maintaining accounts and transacting payments, such as writing checks and withdrawing cash at ATM. Bank also charge fees for services such as money transfers, overdrafts, etc. Suppose the risk-free interest rate is $r$. If the net income (from the bank’s perspective) for serving deposit accounts is $\eta$ per dollar of deposit, the net interests on deposit is $C = (r - \eta)D$. The parameter $\eta$ plays a crucial role in our model. It represents a sacrifice to the required rate of return that the households are willing to accept. The sacrifice distinguishes deposit from other form of debt. If the deposit is risk-free because of deposit insurance or due to the ability of depositors to run before the bank fails, the bank’s total cash outflow on deposit is $I + C$.

An important form of debt, in addition to deposit, issued by banks is the subordinated debt. It is the second part of the liability side in Figure 1. Subordinated debt pays coupon until bankruptcy, at which it has a lower priority than deposits in its claims on the liquidation value of a bank. The lower priority potentially protects deposit at bankruptcy. For that reason, long-term subordinated debt is viewed as tier-2 capital of the bank by regulators. A perpetual subordinated debt like “noncumulative preferred equity” is labeled as “additional tier-1 capital” in Basel III. This, however, comes at a price: the long-term debt yields will capture a credit spread above the risk-free rate to compensate them for bearing the risk of bankruptcy. This credit spread arises endogenously in our model, depending on the asset risk and the (endogenous) leverage of the bank. Thus, a bank’s choice of liability structure affects the credit spread. Let $s$ be the credit spread, which we will solve in our model endogenously, and $D_1$ be the face value of subordinated debt. The coupon on subordinated debt is $C_1 = (r + s)D_1$, which lasts until bankruptcy. We will discuss bankruptcy later in more detail.

The bank is owned by its common equity holders, who garner all the residual value and earnings of the bank after paying the contractual obligations on deposits and subordinated debt. The first slice of value that equity owners lay claim to is the difference between assets and debts: $V - (D + D_1)$, which is also on the liability side in Figure 1. This is sometimes referred to as tangible equity or book-value of equity, which is the value the equity holders would receive if bank assets are liquidated at fair value and all debts are paid off at par. A larger book-value of equity means deposit and subordinated debt are less likely suffer a loss. Hence, regulators regard it as bank capital of the highest quality and refer it as core tier-1 capital.

Equity holders are also rewarded by all future earnings of the bank. The present value
of the future earnings is the bank’s charter value, which is the bottom part on asset side in Figure 1. Part of the earnings comes from the service income on deposit accounts: $\eta D$, but the earnings associated with deposit account should exclude insurance premium $I$ if deposits are insured. Another part of the earnings is the saving from corporate tax. Let $\tau$ be corporate tax rate. Since costs of debt financing are deductible from earnings for tax purposes, the total tax saving is $\tau(I + C + C_1)$. Therefore, the dividend paid to equity holders is the difference between asset cash flow and the net total outflow of cash associated with deposit and subordinated debt: $\delta V - (1 - \tau)(I + C + C_1)$. Since equity value depends on its dividend, it is affected by the liability structure. In banks with deposit insurance, the liability structure is characterized by the triplet $(I, C, C_1)$, whereas in banks without deposit insurance, the pair $(C, C_1)$ typifies the liability structure.

2.2 Bank Run, FDIC, and Equity Requirement

A natural consequence of borrowing using deposits is the risk that the depositors may run. This is an important way in which banks differ from most non-financial firms. As experienced in the crises of U.S. banking history and theorized by Diamond and Dybvig (1983), depositors may “run” if they believe that the banks may have difficulty in repaying their deposits in a prompt and timely fashion upon their demand. When depositors run, the bank will be closed and its assets have to be liquidated. Assume that the market liquidation costs through bankruptcy courts, which includes dead-weight losses associated with the liquidation process and legal expenses, is a fraction, $\alpha$, of the asset value $V_a$ at bankruptcy, leading to a liquidation value of $(1 - \alpha)V_a$. While deposit brings the benefit of account service income $\eta$, the cost associated with deposit is bankruptcy. With full information, it is rational for the depositors to run just before the bank decides to liquidate. The depositors will never wait if they believe that the value after liquidation is below the total deposit $D$. In addition, it may be reasonable to assume that the depositors, worrying about the delays that may ensue when the bank files for bankruptcy, might actually wish to run even earlier. We assume that depositors will run when asset value drops to $V_a = D/(1 - \alpha)$. Since a bank is closed when depositors run, by letting $\kappa = 1/(1 - \alpha)$ we can interpret $\kappa D$ as the threshold for bank to close due to bank run.

The establishment of the Federal Deposit Insurance Corporation (FDIC) is widely regarded as a deterrent to bank runs by insuring that the deposits (up to a limit) will be paid in full when a bank is closed by its charter authority. A charter authority is typically

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7 In September 2007, Norther Rock, a U.K. Bank, experienced a run on its deposits, and had to be nationalized in 2008. See Shin (2008) for a cogent analysis of Northern Rock run.

8 Our analysis actually works for the case of $V_a > D/(1 - \alpha)$. A structure where $V_a > D/(1 - \alpha)$ can be interpreted as a situation in which depositors are risk averse or demand cash immediately for their liquidity needs.

9 This belief was reflected during the credit crisis, when the FDIC deposit insurance limit was raised from $100000 to $250000 on October 3, 2008.
either the bank’s state banking commission or the Office of the Comptroller of the Currency (OCC). The charter authority will close a bank if the bank’s capital is deemed to be too low to be sustainable. The charter authority closes the bank when it believes that bank is insolvent. Suppose a bank is believed to be insolvent when the capital that protecting deposit drops to a threshold as a specified percent (say, 2%) of asset value. The total capital is the sum of tier-1 and tier-2 capital, which is the sum of tangible equity and subordinated debt. This amounts to \( V - (D + D_1) \) + \( D_1 = V - D \). Let \( V_a \) be asset value at the time when charter authority closes the bank. Then, \( V_a - D = 2\%V_a \), implies \( V_a = D/0.98 \). In general, charter authority closes a bank when its asset value reaches \( V_a = \kappa D \), where \( \kappa \) is a positive number.

The FDIC functions both as a receiver of the closed bank and an insurer of the deposits. As a receiver, the FDIC liquidates the assets of the closed bank in its best effort to pay back the bank’s creditors. Following the tradition in structural models, we assume that the insurance corporation’s liquidation cost, \( \beta V_a \), is proportional to the asset value \( V_a \) when the bank is closed. It is possible that the insurance corporation’s liquidation costs are different from the costs of liquidating the bank through the bankruptcy procedures, and thus we admit \( \beta \neq \alpha \). Without going through bankruptcy court, it is indeed likely that \( \beta < \alpha \).

As an insurer, the FDIC pays \( D \) to depositors. Consequently, the insurance corporation lose \( D - (1 - \beta)V_a \) if it is a positive number and 0 otherwise. This loss function can be written as \( [D - (1 - \beta)V_a]^+ \), where \([x]^+\) returns only the positive part of its argument.\(^{10}\) Since \( V_a = \kappa D \), the loss function is positive if \( \kappa < 1/(1 - \beta) \), in which the insurance corporation expects to suffer a loss after a bank closure.\(^{11}\) To cover the loss, the FDIC charges insurance premium on banks. In 2006, Congress passed reforms that permitted the FDIC to charge banks risk-based premium. It also allowed FDIC some authority to manage the Deposit Insurance Fund (DIF) into which the premiums are invested. At present, for deposit insurance assessment purposes, an insured depository institution is placed into one of four risk categories each quarter, determined primarily by the institution’s capital levels and supervisory evaluation. Hence a riskier bank pays higher insurance premium than a less risky bank. Recall that \( I \) denotes the total deposit insurance premium a bank pays.\(^{12}\)

The economic role of FDIC and charter authority in a full information model presented

\(^{10}\)That is, \([x]^+ = x \) if \( x > 0 \) and \([x]^+ = 0 \) if \( x \leq 0 \).

\(^{11}\)In practice, the FDIC expects a loss because liquidation cost is uncertain. To keep the analysis tractable, we assume a fixed \( \beta \) instead. Under a fixed cost \( \beta \), the FDIC expects a loss if and only if \((1 - \beta)V_a < D \). Then, in view of \( V_a = \kappa D \), the FDIC expects a loss if and only if \( \kappa < 1/(1 - \beta) \).

\(^{12}\)Until 2010, the FDIC assess the insurance premium based on total deposits. The assessment rate of the insurance is \( a \) such that \( I = aD \). However, there have long been concerns that banks shift deposits out of account temporally at quarter-ends to lower the assessment base. As required by the Dodd-Frank Act (Section 331), the FDIC have changed the assessment base to the difference between the risk-weighted assets and tangible equity since April 2011. If \( V \) equals the risk-weighted assets, the new assessment base equals \( D + D_1 \), which implies the new assessment rate is \( b \) such that \( I = b(D + D_1) \). The actual premium calculations may also depend on the credit ratings and the proportion of long-term debt to deposits. See Federal Register, Vol. 76, No. 38, Friday, February 25, 2011, Rules and Regulations, for more details.
in the paper wherein the depositors can run at the right time to make their deposits risk-free can be explained as follows: the charter authority will close the bank at a later point than the depositors choose to run were there no deposit insurance or regulation. The FDIC increases the expected life of the bank. In turn, the bank will pay the insurance premium to FDIC in “good states” when it is solvent. This transfer of payments across the states can improve the overall value of the bank by increasing the present value of the tax shields and reducing the expected costs of default. With taxes and costs associated with bankruptcy, our analysis will show that the combined actions of the charter authority and the FDIC coupled with these important market frictions creates additional value to the bank, and lowers the dead-weight losses associated with bankruptcy.

Equity holders can potentially choose to default debt, which consists of deposit and subordinated debt, before bank run or closure. Absent bank run and closure, there is an optimal point for equity holders to default. This default decision maximizes equity value, given a liability structure. The optimal default of debt was referred to as endogenous default and derived by Leland (1994) for firms with long-term debt but without deposit. In the presence of deposit, we can derive the point of endogenous default. Let $V_d$ be the critical point for endogenous default, i.e., equity holders choose to default if and only if asset value $V$ reaches to or below $V_d$, if there were no bank run or closure. Then bankruptcy happens if either equity holders endogenously choose to default debt or the bank is closed due to bank run or by charter authority. Let $V_b$ be the critical point for bankruptcy, then $V_b = \max\{V_d, V_a\}$.

In summary, there can be three types of bankruptcy. The first type is endogenous default chosen by equity holders before a bank is closed due to bank run or by charter authority. In this case, liquidation of assets have to go through bankruptcy court and bankruptcy cost is $\alpha V_d$. The second type is bank run, which happens before endogenous default. Bankruptcy cost in the case of bank run is $\alpha V_a$, as it goes through bankruptcy court. The last type is bank closure by charter authority under FDIC insurance and regulation. In this case, bankruptcy cost is $\beta V_a$. Concisely, we denote bankruptcy cost by $\phi V_b$, where $\phi$ equals $\alpha$ or $\beta$ depending on the type of bankruptcy. At bankruptcy when the bank’s assets are liquidated, depositors are paid first, and the subordinated debt holders are paid the next if there is value left over for them. Consequently, the payoff to debt holders is $[(1 - \phi) V_b - D]^+$. In order to control the likelihood of bankruptcy, banks around the world are subject to equity requirements which form the core part of the international accord, commonly referred to as the Basel III, agreed in 2011 by bank regulators. In Basel III, all banks are required to hold core common equity above 7 percent of the risk-weighted assets. This is a substantial increase from a 4-percent requirement in Basel II. For large banks that are identified by regulators as “systemically important,” the core common equity requirement can be 0~2.5 percent higher. In our model, core common equity is $V - (D + D_1)$ and the value of risk-weighted assets is $V$. If the equity requirement is $\zeta$, a bank must choose a capital structure that satisfies $[V - (D + D_1)]/V \geq \zeta$. This imposes a limit on leverage.
because it implies $D + D_1 \leq (1 - \zeta)V$. If a bank’s capital structure does not meet the equity requirement, its regulator instructs the bank to adjust its capital structure, but the regulator will not close the bank if the bank is not insolvent. Therefore, we model equity requirement as constraint on banks’ choice of liability structure.

3 Valuation and Optimization

Before proceeding with bank valuation, it is useful to summarize the exogenous parameters in our model and our assumptions on the parameters. The parameters and assumptions are listed in Table 1. The third column presents our assumption on the range of values for the parameters. Particularly, we assume account service income is positive but doesn’t cover the entire cost of taking deposit: $0 < \eta < r$. We also assume the existence of tax: $0 < \tau < 1$. The bankruptcy and FDIC liquidation are both costly in our model: $0 < \alpha < 1$ and $0 < \beta < 1$. We believe that these assumptions are realistic, and they are the requisite mathematical conditions to carry out valuation and optimization.

3.1 Bank Valuation and Endogenous Insurance Premium

Since deposit is either withdrawn before bank run or FDIC insured, it is a safe security. Consequently, the value of deposit is its par value $D$. Since the interests paid on deposit is $C = (r - \eta)D$, the value of deposit is related to deposit interests by $D = C/(r - \eta)$.

The value of subordinated debt and equity are affected by the risk of default, leading to bankruptcy. Consequently, the Arrow-Debreu price of bankruptcy plays a key role in bank valuation. Consider a security that pays $1 when default occurs, and 0 otherwise. The price of this security is the Arrow-Debreu price of bankruptcy, which is also the risk-neutral probability of bankruptcy. It is well known that the state price $P_b = [V_b/V]^\lambda$, where $\lambda$ is an increasing function of $r$ and decreasing function of $\delta$ and $\sigma$. If cash flow of the assets is zero, $\delta = 0$, we have $\lambda = 2r/\sigma^2$, which is proportional to $r$ and inversely proportional to $\sigma^2$. The exact function of $\lambda$ is given in equation (21) in Appendix. The state price $P_b$ can be derived as solution to Merton’s (1974) no-arbitrage pricing equation (20) and $\lambda$ is the positive root of a quadratic equation (22). The details of the derivation can be found in Appendix.

Bank value depends on its liability structure $(I, C, C_1)$ because the liability structure affects bankruptcy boundary and its state price. The following theorem summarizes the precise relation between bank value and liability structure.

**Theorem 1** Given liability structure $(I, C, C_1)$, bank run/closure boundary is $V_a = \kappa C/(r - \eta)$, endogenous default boundary is

$$V_d = (1 - \tau)[\lambda/(1 + \lambda)](I + C + C_1)/r,$$

(1)
and bankruptcy boundary is $V_b = \max\{V_a, V_d\}$. The equity, subordinated debt, and bank values are

\[
E = V - (1 - P_b)(1 - \tau)(I + C + C_1)/r - P_bV_b \tag{2}
\]

\[
D_1 = (1 - P_b)C_1/r + P_b[(1 - \phi)V_b - D]^+ \tag{3}
\]

\[
F = V + (1 - P_b)\left\{\left[\tau + \eta/(r - \eta)\right]C + \tau C_1\right\}/r
- (1 - P_b)(1 - \tau)I/r - P_b\min\{\phi V_b, V_b - D\}. \tag{4}
\]

The liability structure in the theorem includes insurance premium, but if we set $I = 0$, these formulas in the theorem also apply to banks without deposit insurance.

The endogenous credit spread of subordinated debt, which depends on the liability structure, can be obtained from Theorem 1. The endogenous credit spread is $s = C_1/D_1 - r$, where $D_1$ is a function of $C_1$ as given in equation (3). This credit spread takes the probability of bankruptcy into account through state price $P_b$, which is affected by the bank liability structure. It is worth pointing out that the insurance premium also affects the credit spread, even though $I$ does not appear in equation (3) explicitly. The insurance premium has an influence on endogenous default boundary in equation (1). Thus it affects bankruptcy boundary $V_b$ and the state price of bankruptcy, through which it influences the credit spread.

Theorem 1 shows the role of account service income and deposit insurance in bank valuation. Along with tax savings on debt, account service income (positive $\eta$) increases the bank value as shown by the second term on the right-hand side of equation (4). The ability of the bank to attract deposits at a rate lower than the risk-free rate comes at a price: the bank has to close and incur bankruptcy cost if depositors run or charter authority closes the bank. The last term of the equation reflects the value lost due to bankruptcy. For a bank with deposit insurance, insurance premium also reduces the bank value, which is evidenced by the third term of the equation. Notice that even though insurance premium does not appear in equation (3), it affects the subordinated debt value through its influence on default boundary $V_b$, on which debt value hinges.

A comparison of our bank valuation with Leland’s (1994) firm valuation is revealing. If we set $C = 0$ and $I = 0$ but $C_1 > 0$, all formulas in Theorem 1 reduce to those derived for the capital structure of firms with only debt—the word “subordinated” drops because there is no deposit in this case, and hence there is no subordination. If we set $I = 0$, $\eta = 0$, and $C_1 = 0$ but $C > 0$, the valuation formulas in Theorem 1 coincides with Leland’s case for debt protected at level $\kappa D$. Leland’s seminal capital structure theory is for firms; the theory does not apply to banks that take deposit, earn account service income, pay deposit insurance premium, and subject to bank run or closure. Our theory extends Leland’s to banks and offers a consistent framework to understand the similarities and differences between banks and other firms.

Deposit insurance premium in Theorem 1 is exogenously given, but it should endogenously depend on the amount of deposit under insurance and the risk involved, among
other things. In principle, the deposit insurance corporation should charge each bank the fair insurance premium, which makes the insurance contract worth zero. The next Theorem characterizes the fair insurance premium.

**Theorem 2** Given deposit $D$, the fair insurance premium is

$$I^\circ = r[1 - (1 - \beta)\kappa]^+DP_a/(1 - P_a),$$

where $P_a = [\kappa D/V]^{\lambda}$ is the state price of bank closure.

An alternative way to write the insurance pricing equation is

$$(1 - P_a)(I^\circ/r) = P_a[1 - (1 - \beta)\kappa]^+D,$$

which says that the expected present value of the insurance premium paid to the insurance corporation equals the expected present value of the insurance obligations when the insured bank is closed by its charter authority.

The fair insurance premium, $I^\circ$, increases with $D$. Not only the insurance premium increases in deposit, the rate of insurance premium on each dollar deposit also increases in $D$. By Theorem 2, the rate of insurance premium is

$$h \equiv I^\circ/D = r[1 - (1 - \beta)\kappa]^+P_a/(1 - P_a).$$

It is increasing in $D$ because $P_a$ is increasing in $D$. This makes sense because expanding deposits increases the risk exposure of insurance corporation. If $\kappa < 1/(1 - \beta)$, the fair premium $I^\circ$ is positive. It converges to zero when $\kappa$ rises to $1/(1 - \beta)$. If $\kappa \geq 1/(1 - \beta)$, fair premium is zero because the bank will be closed with enough asset value to fully cover the losses of depositors.

Some academics have argued that FDIC has provided subsidized insurance to banks by charging a premium which is lower than its fair rate.\(^{13}\) To allow for potentially subsidized insurance premium, we assume that the subsidized FDIC insurance premium is $I = \omega I^\circ$, where $\omega = 1$ represents fair insurance premium and $\omega < 1$ represents the insurance subsidy. Relating to the net deposit interest by $D = C/(r - \eta)$, we have $I = iC$, where

$$i = \omega[1 - (1 - \beta)\kappa]^+[r/(r - \eta)]P_a/(1 - P_a).$$

If the insurance corporation gives an insurance subsidy, then such a subsidy increases the bank value because the bank pays lower insurance premium while receiving the risk-free value of deposits.

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\(^{13}\)See Duffie et al. (2003). On the other hand, it can be argued that the subsidy is necessary to compensate the insured banks for the costs of reporting requirements and tight regulation.
With endogenous insurance premium, liability structure is characterized by the pair 
\((C, C_1)\) because \(C\) determines \(I\). Imposing endogenous insurance premium \(I = iC\) in bank 
value formula (4), it is straightforward to obtain

\[
F = V + (1 - P_b)[\tau(1 + i) + \eta/(r - \eta) - i]C/r \\
+ (1 - P_b)\tau C_1/r - P_b \min\{\phi V_b, V_b - D\}.
\] (9)

On the right-hand side of equation (9), the second term is the value of tax deduction 
and account service income, netted off against the insurance premium that the bank pays 
for its deposits. The third term is the value of tax benefits to the bank for its interest expense 
on subordinated debt. The last term is the loss of bank value due to bankruptcy, for which 
bankruptcy cost \(\phi\) takes the value of \(\alpha\) or \(\beta\), depending on the type of bankruptcy.

### 3.2 Optimal Liability Structure

Now we proceed to examine how a value-maximizing bank will choose its liability structure, 
but we first consider the case of a regulated bank without FDIC deposit insurance. In 
this case, the bank has to take into account the potential for the depositors to run at any 
time to keep their loans risk-free. This “unregulated market” benchmark serves as a useful 
counter-factual for evaluating the effects of adding regulatory mandates such as the FDIC 
insurance, charter authority to close troubled banks, and capital adequacy requirements. 
This allows us to examine how a bank might rearrange its liability structure when different 
regulatory tools are mandated by the regulators. Later, we are able to gain some insights 
into the “optimal response” of a bank to regulatory mandates. For example, we will be 
able to make both qualitative and quantitative statements about the economic implications 
of the actions taken by the charter authority and the FDIC on the optimal response of 
the bank is adjusting their leverage, liability structure, and default decisions, relative to 
the counter-factual in which the banks make value-maximizing choices, unfettered by any 
government interventions.

As pointed out earlier, liability structure of an unregulated bank is completely de- 
scribed by the pair \((C, C_1)\). An optimal liability structure is the deposit liability \(C^*\) and 
subordinated debt liability \(C_{1*}\) that maximizes bank value. The theorem below provides 
a characterization of the optimal liability structure for unregulated bank without deposit 
insurance.

**Theorem 3** The optimal liability structure of an unregulated bank is unique. In the optimal 
liability structure, \(V_{a*} = V_{d*}\), and the state price of bankruptcy \(P_b^*\) is given in equation (41). 
The optimal deposit and subordinated debt liabilities are

\[
C^* = (r - \eta)V P_b^{1/\lambda}(1 - \alpha)
\] (10)

\[
C_{1*} = r V P_b^{1/\lambda}\left[\frac{1 + \lambda}{(1 - \tau)\lambda} - \frac{r - \eta}{r/(1 - \alpha)}\right].
\] (11)
This theorem is a direct consequence of Lemma 1 in Appendix. Equation (41) in Lemma 1 gives the exact formula of $P^*_b$; it is an elementary algebraic function of the following exogenous parameters: $r$, $\sigma$, $\delta$, $\tau$, $\eta$, and $\alpha$.

The theorem characterizes the optimal liability structure of a bank that faces taxes, bankruptcy cost, and issues deposits at a rate lower than the risk-free rate. Combining Theorem 3 with Theorem 1, we can obtain analytical solutions for deposit $D^*$, subordinated debt $D^*_1$, equity $E^*$, bank value $F^*$, bankruptcy boundary $V^*_b$, and credit spread $s^*$ in the optimal capital structure of unregulated banks without FDIC deposit insurance. The optimal structure gives an optimal ratio $x^* = C^*_1/C^*$ of the subordinated debt liability to deposit liability. Equation (40) in Appendix presents the formula of the optimal liability ratio in terms of exogenous parameters. Whenever the liability ratio $x$ is given along with the deposit liability $C$, the subordinated debt liability can be calculated as $C_1 = xC$.

A key insight of the theorem is that it is optimal for banks to choose the precise amount of subordinated debt such that endogenous default boundary coincides with bank-run boundary. While this is proved mathematically in Appendix, we provide below the economic reasoning for this result, which is intuitive. For a bank, a special benefit of issuing deposit is the discounted interest rate that the bank gets for providing account services, in addition to tax savings, whereas the cost of issuing deposit is the loss associated with a bank-run. In contrast, subordinated debt brings tax savings but produces no account service income, while its cost is the loss associated with bankruptcy. Therefore, at the margin, the bank should use deposit, not debt, to balance the tax and liquidity benefits with the cost of bankruptcy. With this balance, the bank should take as much debt as possible for availing the tax benefits but should avoid the costs of bankruptcy resulting from endogenous default. To avoid the cost associated with endogenous default, the bank should avoid setting the endogenous default boundary above the bank-run boundary. As a result, the optimal subordinated debt should be set so that default occurs at exactly the same point as the bank-run.

The assumption rational bank run, $\kappa = 1/(1 - \alpha)$, is essential for Theorem 3 to hold. In fact, it can be proved that the unique optimal liability structure can lead to $V^*_d > V_a$ if $\kappa < 1/(1 - \alpha)$, which would imply that the depositors run from the bank at a point when the bank would not have enough assets to refund the deposit in full after bankruptcy proceedings. Such a late bank-run is clearly not rational if depositors know the bank asset value.

Account service associated with deposits is very important for Theorem 3. In fact, if all assumptions of the theorem hold except that $\eta = 0$, then it can be shown that for every liability ratio $x \geq x^*$ there exists an optimal structure, and setting $\eta = 0$ in the theorem’s formulas gives the optimal structure with the largest deposit. We can think of these optimal structures for a firm that faces tax but does not receive account service income.\(^{14}\) In this

\(^{14}\)Without account service income, deposit in our model bears resemblance to secured debt in Leland’s
case, the optimal liability structure is not unique. A lower liability ratio in the optimal structure corresponds to a larger deposit. The optimal capital structure is not unique because deposit and subordinated debt have the same tax benefits and bankruptcy cost in the absence of account service. This suggests that a structural model without considering bank account service is not appropriate for understanding bank decision about its optimal combination of deposits and subordinated debt.

Taxes play an important role in Theorem 3. If all assumptions of the theorem hold except $\tau = 0$, it can be shown that for every liability ratio $x \leq x^*$ there exists an optimal structure. Setting $\tau = 0$ in the theorem gives the optimal structure with the smallest deposit. This special case corresponds to a bank which account service but receives no tax benefit. In fact, this is the typical setup of banks in the banking literature that focus on the role of bank account services. Without tax benefit, banks have no incentive to issue subordinated debt because it exposes banks to bankruptcy cost. In this situation, as long as debt level is low enough so that default never happens before bank run, default cost is irrelevant to bank valuation. Therefore, as long as the liability ratio sets default boundary below bank-run boundary, any liability structure with subordinated debt is optimal. This indeterminacy of subordinated debt in the absence of tax benefit suggests that a model of a bank without consideration of tax savings is not appropriate for understanding bank optimal liability structure.

Theorem 3 ignores deposit insurance and FDIC regulation of banks, but it will serve as a useful counterfactual case to examine the effects of deposit insurance and FDIC regulation. If we consider banks under FDIC, the optimal liability structure is a pair of $C^*$ and $C_1^*$ that maximizes the bank value in equation (9) subject to equation (8). The value-maximizing bank in our framework is fully aware that any decision pertaining to leverage and liability structure will have a consequence on the FDIC insurance premium. It will therefore be very mindful of this channel in its choice of leverage and liability structure. The endogenous determination of FDIC premium, leverage and the liability structure allows us to capture the feedback channel from FDIC to the banks and vice versa.

The next theorem, which is a direct consequence of Lemma 2, characterizes the conditions for a liability structure to be optimal. The theorem first presents a necessary condition. The condition is also sufficient if the closure rule is ironclad enough or the FDIC subsidy is generous enough to keep the insurance premium low relative to the account service income.

**Theorem 4** Consider a bank that has deposit insurance and regulated by FDIC and its charter authority. The liability structure $(C^*, C_1^*)$ of the bank is optimal only if endogenous default boundary is not below bank closure boundary, i.e., $V_d^* \geq V_a^*$. Furthermore,

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(1994) because deposit is protected by bank run. Leland (1994) considers the optimal capital structure of firms that take either secured or unsecured debt but not the optimal mix of the two. Leland analytically solves the optimal capital structure of firms that take unsecured debt, but for firms that takes secured debt, he solves their optimal structure numerically.
1. For any given \( \omega \), there exists \( \bar{\kappa} < 1/(1 - \beta) \) such that for each \( \kappa > \bar{\kappa} \), a liability structure \((C^*,C^*_1)\) is optimal if and only if \( V^*_a = V^*_d \);

2. Alternatively, for any given \( \kappa \), there exists \( \bar{\omega} \) such that for each \( \omega < \bar{\omega} \), a liability structure is optimal if and only if \( V^*_a = V^*_d \);

3. In an optimal structure with \( V^*_a = V^*_d \), the state price of bankruptcy \( P^*_b \) satisfies equation (42), and the optimal deposit and subordinated debt liabilities are

\[
C^* = (r - \eta)VP^*_b^{1/\lambda}/\kappa \tag{12}
\]
\[
C^*_1 = rVP^*_b^{1/\lambda} \left[ \frac{1 + \lambda}{(1 - \tau)\lambda} - \frac{r - \eta}{r\kappa} - \omega[1/\kappa - (1 - \beta)]^{+} \right. \frac{P^*_b}{1 - P^*_b} \right]. \tag{13}
\]

The first part of theorem gives a necessary conditions for the optimal structure, which imposes a lower bound on optimal debt liability relative to deposit liability. A level of debt liability below this bound cannot be optimal. In particular, the theorem shows that zero subordinated debt \( C_1 = 0 \) is not optimal for a bank as long as insurance premium is not too high. If \( C_1 = 0 \), since insurance premium is \( I = iC \), it follows from Theorem 1 that

\[
V_a = \kappa C/(r - \eta) \tag{14}
\]
\[
V_d = (1 - \tau)[\lambda/(1 + \lambda)](1 + i)C/r. \tag{15}
\]

Then, \( V_a > V_d \), meaning the structure is suboptimal, if \( i \) is very large.

The three items of Theorem 4 show that facing high critical point for bank closure or large insurance premium subsidy relative to account service income, it is optimal for banks to leverage so that endogenous default of debt will be as late as the closure of banks. If this is the best for banks, we can analytically derive the optimal liability structure. The exact formula of \( P^*_b \) is given by equation (42) of Lemma 2 in appendix. It is a function of the following exogenous parameters: \( r, \sigma, \delta, \tau, \eta, \beta, \kappa \), and \( \omega \). Combining this theorem with Theorem 1 and Theorem 2, we obtain analytical solutions for the deposit \( D^* \), subordinated debt \( D^*_1 \), equity \( E^* \), bank value \( F^* \), bankruptcy boundary \( V^*_b \), credit spread \( s^* \), and insurance premium \( I^* \) in the optimal capital structure of a bank under FDIC regulation.

The most distinctive feature of Theorem 4 is optimization with endogenous insurance premium. Besides considering the tradeoff among tax benefit, account service income, bank closure, and bankruptcy cost, banks in this theorem also consider insurance premium. If we set \( \omega = 0 \) and \( \kappa = 1/(1 - \alpha) \), the formulas in this theorem reduce to those in Theorem 3. With positive \( \omega \) and more general \( \kappa \) in this theorem, the insurance premium rate \( \omega h \) is an increasing function in \( D \), which causes \( i \) to be increasing in \( C \). Therefore, banks have to consider increase in insurance premium caused by \( D \) and by the associated increase in the rate. In later part of the paper, we will show the impact of endogenous insurance premium on banks’ optimal choice of capital structure.
Optimization in Theorem 4 ignores equity requirement. Subject to equity requirement, banks have to maximize bank value $F$ by choosing $(C, C_1)$ subject to constraint $D + D_1 \leq (1 - \zeta)V$, where $D = C/(r - \eta)$ and $D_1$ is a function of $C_1$ defined by equation (3). The sum of deposit and subordinated debt is the total debt of the bank, and its ratio to total assets measures the bank’s leverage, which is referred to as leverage ratio: $l = (D + D_1)/V$. Equity requirement imposes a limit on leverage ratio: $l \leq 1 - \zeta$.

If an optimal capital structure in Theorem 3 or 4 happen to meet equity requirement, i.e., $D^* + D_1^* \leq (1 - \zeta)V$, then, the optimal liability structures in those theorems are also the optimal structures for banks subject to equity requirement. In this case, the equity requirement is not binding. Therefore, all the features of the optimal structures in those theorems remain when equity requirement is not binding. However, we lost some features when the requirement is binding, as summarized in the next two theorems.

For a bank that is not insured but subject equity requirement, the optimal liability structure is characterized as follows.

**Theorem 5** Assume a bank is not insured or regulated by FDIC but subject to equity requirement. There is a unique optimal liability structure that maximizes the bank value subject to the constraint of equity requirement. If equity requirement is binding, the optimal liability structure $(C^\dagger, C_1^\dagger)$ of the bank sets $V^\dagger_a \geq V^\dagger_d$. In the optimal structure, the state price of bankruptcy $P_b^\dagger$ is given by equation (48) for the case of $V^\dagger_a > V^\dagger_d$ and by equation (49) for the case of $V^\dagger_a = V^\dagger_d$, and the optimal liabilities are

\[
C^\dagger = (r - \eta)VP_b^\dagger 1/\lambda(1 - \alpha) \quad (16)
\]

\[
C_1^\dagger = rV[1 - \zeta - (1 - \alpha)P_b^\dagger 1/\lambda]/(1 - P_b^\dagger) \quad \text{if} \quad V^\dagger_a > V^\dagger_d \quad (17)
\]

\[
C_1^\dagger = rVP_b^\dagger 1/\lambda r(1 + \lambda) \quad \text{if} \quad V^\dagger_a = V^\dagger_d \quad (18)
\]

This theorem is a direct consequence of Lemma 3 in Appendix. Equations (48) and (49) of the lemma give the exact formulas of the state price of bankruptcy. In the formulas, the state price in the optimal structure is a function of following exogenous parameters: $r$, $\sigma$, $\delta$, $\tau$, $\eta$, $\alpha$, and $\zeta$.

The most important effect of a binding equity requirement is to push endogenous default boundary $V^\dagger_d$ potentially below bank closure boundary $V^\dagger_a$. This means that an optimal response to a binding equity requirement is associated with a reduction in subordinated debt. When endogenous default boundary is below bank run boundary, subordinated debt is at a level so that tax benefit is not fully exploited by the bank. However, this helps in meeting the equity requirement and taking advantage of deposit account service income. It is still possible that the optimal structure sets $V^\dagger_d$ at the same level of $V^\dagger_a$, in which cases the bank reduce both deposit and subordinated debt proportionally. In either case,
one unvarnished truth in the optimal structure is that subordinated debt does not protect deposit.

4 Optimal Response of Banks to Regulation Changes

In Table 2 we compare the counterfactual referred to as “Free market” with two cases. We present the results for the FDIC insurance case with no equity requirements. Then we present the equity requirements case with no FDIC insurance. The final column is where we hope to present the FDIC insurance case with equity requirements. This part of our research is still work in progress.

Note that when FDIC insurance is imposed, the bank chooses to increase its deposit ratio, and decreases its level of subordinated debt. The equity capital ratio goes up. Perhaps the most important outcome is the fact that the bank’s default boundary goes down, and the expected losses associated with default goes down. This result comes about despite the fact that the bank pays the fair insurance premium. For the parameter values chosen, the FDIC premium was 9 basis points.

Table 3 explores the implications of increasing the bankruptcy costs from 25 percent to 35 percent in Panel A. For the counterfactual, the default boundary goes down, and the level of deposits go down pushing down the run boundary as well. The resulting lower credit spreads lead to a higher level of subordinated debt. With FDIC insurance in place, the bank reduces all its liabilities, and the equity value actually goes up. Panel B reports our findings with a lower corporate tax rate. Leverage goes down, but the equity value actually improves, even though the overall charter value goes down. When equity capital requirements are imposed, the reduction in taxes affects the subordinated debt much more adversely.

Panel C of Table 3 reports the consequences of delivering higher account services and charging a bigger fee. Somewhat surprisingly, most of the increase in bank’s charter value accrues to the subordinated debt investors, with or without FDIC insurance. The overall charter value goes up with increase in account services as one might expect. When equity capital requirements are imposed, we find that the deposits are affected much more significantly than subordinated debt. The increase in subordinated debt is exactly offset by the decrease in deposits.

Table 4 reports the changes associated with an increase in asset volatility (from 10 percent to 15 percent in panel A) and an increase in total cash payout ratio (from 2 percent to 3 percent, in Panel B). A key result in panel A is the effect of increased volatility on the liability structure and leverage: both the deposit and subordinated debt levels go down relative to the base case. Equity value goes up much more under the FDIC insurance case. The FDIC premium has gone up by 4 basis points to reflect the increased risk of assets.

Increased cash payouts (panel B) factor equity holders at the expense of creditors: the
residual dividends are much higher, and the payouts leave the creditors with lower level of assets when the bank defaults: this is reflected by the loss in value. The FDIC premium increases by 3 basis points.

In Table 5 we characterize the optimal response of the banks with respect to FDIC liquidation costs (panel A), insurance subsidies (panel B) and the charter authority’s closure rules (panel C). Lower costs of liquidation leads to higher recovery for creditors leading to a lower FDIC insurance premium. The gains accrue to creditors at the expense of equity holders, relative to the base case. Subsidy in insurance premium leads to greater use of both deposits and subordinated debt, and higher expected losses relative to the base case. Note that the FDIC premium is higher: this reflects the higher level of deposits.

Panel C of Table 5 examines the implications of a tougher closure rule on the optimal response by the bank. FDIC premium goes down as the liquidation value is higher. The bank’s deposits fall but the subordinated debt levels go up. Note that there is only a marginal change in the optimal default boundary.

Figure 2 explores the relationship between bankruptcy costs and the optimal default boundary in the absence of any regulation. The key observation here is that when the bank optimally responds, the default and run boundaries are the same.

TO BE COMPLETED

5 Conclusion

We conclude by briefly summarizing the issues that we have addresses in our paper, and by placing our work in the perspective of previous literature that is pertinent to our work.

Our paper provides a setting in which the following questions can be addressed. How much equity capital should a bank hold? What should be the ideal composition of insured deposits and long-term unsecured debt in the capital structure of banks? What should be the optimal response of the bank in choosing its default policy on its long-term debt when it is subjected to the potential of a run by depositors and closure by the charter authority? Why should regulators impose bank equity capital standards, and what should be the optimal response of the banks to such standards? We have provided a framework to study these questions in this paper. We believe that these questions are of first order importance to banks, regulators, FDIC, and investors. After the credit crisis of 2008 banks were significantly recapitalized and the regulators have since been grappling with the question of the
level and composition of capital that banks should hold.\textsuperscript{15} Basel III has recently announced more stringent capital adequacy standards for banks, and the Swiss National Bank has enforced capital standards that are tougher than Basel III. Admati, et.al (2013) have argued that banks should hold significantly higher equity capital than the levels proposed by Basel III.\textsuperscript{16}

By the nature of the questions that we seek to answer, the literature pertinent to our paper falls under three categories: bank run, optimal capital structure, and bank capital regulation. The bank run literature was pioneered by Diamond and Dybvig (1984) who constructed a formal model in which bank run emerges as one equilibrium, and the FDIC insurance of bank deposits enables the economy to avoid this equilibrium. This literature has subsequently been extended in significant ways by Allen and Gale (1998) and others. In our paper depositors can run in order to make their claims risk-free. There is no panic risk in our full-information model as depositors observe exactly the value of the assets of the bank, and can implement their optimal run (withdrawal) strategy without any risk. In this sense we do not have a situation, in which a fraction of the depositors who arrive too late after the onset of a crisis, are left with losses. The challenge of a run in our model, is that the bank must decide how much deposits to hold and how to optimally respond to the ability of the depositors to withdraw any time to make their loans risk-free, in choosing their leverage, debt composition, and optimal default strategy with respect to their long-term creditors. This is similar to the run problem considered by Auh and Sundaresan (2013) in the context of repo financing, but they do not model the FDIC insurance, the charter authority’s closure rules, and capital adequacy standards, which are non-trivial institutional features of any banking environment.

A number of papers have recently contributed to the theory of banks’ optimal capital structure, including the ones by Harding, Liang, and Ross (2009), DeAngelo and Stulz (2013), Allen and Carletti (2013), and Gornall and Strebulaev (2013). Harding, Liang and Ross (2009) consider only deposits, but do not model the issuance of long-term debt.\textsuperscript{17} They also assume that the bank does not pay any insurance premium on its deposits. Garnall and Strebulaev (2013) posit that the leverage decisions are jointly made by the banks and its borrowers, and argue that high leverage of banks arises because of low volatility of banks’ assets due to diversification.\textsuperscript{18} DeAngelo and Stulz (2013) provide a rationale as to why high leverage may be essential for banks by positing that banks provide liquidity services to financial constrained households and firms that value insurance against liquidity shocks

\textsuperscript{15}Regulators in Europe, for example, are arguing for mandatory bail-in debt. See Coeure (2013), for example.
\textsuperscript{16}In a forthcoming paper, we explore the role of contingent capital in the context of the framework developed in this paper.
\textsuperscript{17}Aydıjiev, Ketaševa, and Bogdanova (2013) report that during the 2009-2013 period alone, banks have issued $4.1 trillion of unsecured long-term debt. This has always been an important source of funding for banks.
\textsuperscript{18}They consider a mix of deposits and long-term debt, but assume that this mix is exogenous.
and are prepared to pay a liquidity premium to the banks. We also explore the link between the liquidity services provided by a bank and its value-maximizing endogenous capital and liability structure decisions. Chen, Glasserman and Nouri (2012) focus on contingent capital and bail-in issues in the context of a model of endogenous default where the cash flow process is driven by a mixed diffusion and jump dynamics.

Our paper differs from these contributions in the following ways: first, we allow the equity holders of the bank to arrive at value-maximizing optimal capital and liability structure, which includes both deposits and non-convertible long-term debt: in this sense the composition of deposits and long-term debt as well as the optimal equity capital is endogenously determined. Second, the bank optimally chooses its default strategy on its long-term debt in response to the run risk presented by the presence of depositors. Finally, we determine the optimal equity to asset ratios that banks will choose to hold when it acts to maximize its value, and link this to FDIC’s insurance premium (fair or subsidized) and the charter authorities closure policies. To our knowledge, our paper is the first to examine the optimal response of the bank (in terms of default boundary, equity capital and the composition of liabilities) to the potential for a run by depositors and closure by the charter authority in the presence of FDIC insurance. The FDIC insurance premium depends on the leverage and liability structure, and the banks’ decisions on leverage and liability structure depend on FDIC premium. We explicitly model this feedback channel. We believe that this is of first order importance in assessing any regulatory policies pertaining to equity capital ratios that banks should be subjected to.

Finally, our paper contributes to the theory of bank regulation. A recent survey of this literature by Santos (2000) motivates regulation as a policy arising out of market failure, in which regulation plays a key role in minimizing run risk, and mitigating banks from taking excessive leverage. We motivate regulation as follows: in an unregulated markets, with full information, value-maximizing banks are shown to take excessive leverage to increase their market values. This leads to a high expected costs of defaults which are dead-weight losses to the society. Provision of FDIC insurance, and closure by charter authority, can serve to lower leverage and thus reduce the expected costs of bank failures. But to bring down further the expected costs of defaults and runs, we argue that bank capital requirements may be necessary.

Our paper is also closely related to the structural framework of Merton (1974, 1977) and Leland (1994). The latter considers unprotected long-term debt and secured debt, which Leland suggests may be interpreted as short-term debt. Leland considers each class of debt separately, and we model the endogenous choice of deposits and long-term debt simultaneously. In addition, we model the deposit insurance provided by the FDIC explicitly with the closure strategy of the charter authority. This allows us to derive the endogenous FDIC insurance premium, taking into account the optimal closure policy and the default boundary. A number of papers, notably Merton (1977), Ronn and Verma (1986) have derived the risk-adjusted FDIC insurance policies, and our paper extends their insights.
when default and closure policies are endogenous, and the bank optimally responds to FDIC insurance by choosing its liability structure and leverage to maximize its value.

We have not addressed the issue of bank dynamically changing the liability structure as well as its composition of assets. This question is much more ambitious and one that warrants further research.

A Appendix

Our model of bank liability structure builds on the framework pioneered by Merton (1974) and extended by Leland (1994) for firms, which issue perpetual debt but do not take deposit. We extend their framework to the capital structure of banks, which takes deposit, offer account service, and subject to deposit insurance and regulations.

Following their framework, the value of assets portfolio follows a stochastic process: a geometric Brownian motion in the risk-neutral probability measure:

\[
    dV = (r - \delta)V dt + \sigma V dW.
\]

where \( r \) is the risk-free interest rate, \( \delta \) is the rate of cash flow, \( \sigma \) is the volatility of asset value, and \( W \) is a Wiener process.

For given \( V_b \), consider a security that pays one dollar if and only if \( V \) hits \( V_b \) for the first time. According to Merton (1974), the price of this security, \( P_b \), satisfies the following differential equation

\[
    \frac{1}{2} \sigma^2 V^2 P_b'' + (r - \delta)V P_b' - r P_b = 0
\]

where \( P_b' \) and \( P_b'' \) are the first and second partial derivatives of \( P_b \) with respect to asset value \( V \), and boundary conditions \( P_b(V_b) = 1 \) and \( \lim_{V \to \infty} P_b(V) = 0 \). It is well known that the solution of this equation is \( P_b = [V_b/V]^\lambda \), where

\[
    \lambda = \sqrt{\left[ \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right]^2 + \frac{2r}{\sigma^2} + \left[ \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right]}.
\]

The solution can be verified directly, observing that \( \lambda \) is the positive root of the following quadratic equation,

\[
    \frac{1}{2} \sigma^2 \lambda(1 + \lambda) - (r - \delta)\lambda - r = 0.
\]

The quadratic equation has two roots in general—one positive and the other negative, but only the positive root is relevant to valuation.

Equity holders of the bank earn the after-tax cash flow, \( \delta V - (1 - \tau)(I + C + C_1) \), until bankruptcy. For given insurance premium \( I \), interests on deposit \( C \), and coupon
of subordinated debt \((C_1)\), equity value is a function of asset value, denoted by \(E(V)\). The no-arbitrage pricing equation for equity value before bankruptcy is

\[
\frac{1}{2} \sigma^2 V^2 E'' + (r - \delta) V E' - r E + \delta V - (1 - \tau)(I + C + C_1) = 0, \tag{23}
\]

where \(E'\) and \(E''\) are the first and second partial derivatives of \(E\) with respect to asset value \(V\). In this equation, we assume \(I \geq 0\) in general, but if setting \(I = 0\) gives the pricing equation of equity value in the case of unregulated banks without FDIC insurance.

If asset value is infinitely large, the cash flows are risk-free and thus equity value should become approximately the difference between asset value and after-tax value of cash outflows: \(V - (1 - \tau)(I + C + C_1)/r\). If asset value drops to endogenous default boundary \(V_d\), equity value satisfies the following two conditions: \(E(V_d) = 0\) and \(E'(V_d) = 0\), according to LeLand (1994). Recall that bankruptcy boundary is \(V_b = \max\{V_d, V_a\}\), where \(V_a = \kappa D\) is the boundary of bank run or bank closure. When bank is closed, equity is wiped out and thus \(E(V_a) = 0\). Therefore, the boundary conditions of equity value are

\[
\lim_{V \to \infty} E(V) - [V - (1 - \tau)(I + C + C_1)/r] = 0 \tag{24}
\]

\[
E(V_b) = 0 \tag{25}
\]

\[
E'(V_b) = 0 \text{ if } V_d > V_a. \tag{26}
\]

The value of subordinated debt \(D_1\) is also a function of asset value \(V\). Let \(D_1'\) and \(D_1''\) be the first and second partial derivatives of \(D_1\) with respect to \(V\). The no-arbitrage pricing restriction to the value of the debt \(D_1\) is

\[
\frac{1}{2} \sigma^2 V^2 D_1'' + (r - \delta) V D_1' - r D_1 + C_1 = 0. \tag{27}
\]

If asset value is infinitely large, the subordinated debt becomes risk-free, and thus the debt value approaches \(C_1/r\). Recall that at bankruptcy, payoff to debt holders is \([(1 - \alpha)V_b - D]^+\). Therefore, the boundary conditions of subordinated debt value are

\[
\lim_{V \to \infty} D_1(V) = C_1/r \tag{28}
\]

\[
D_1(V_b) = [(1 - \alpha)V_b - D]^+. \tag{29}
\]

The solutions to above equations and boundary conditions are presented in Theorem 1. Derivation of the solutions is similar to those in Leland (1994) as detailed in Section A.1. To simplify the derivation of optimal liability structure, we introduce the following notations:

\[
x = C_1/C, \quad c = C/(rV), \tag{30}
\]

\[
v_a = rV_a/C, \quad v_d = rV_d/C, \quad v_b = rV_b/C, \tag{31}
\]

\[
\iota = \eta/(r - \eta), \quad \theta = (1 - \tau)\lambda/(1 + \lambda). \tag{32}
\]
We refer to \( x \) as the liability ratio of a given liability structure and \( c \) as the deposit liability scaled by asset. With these notations, the state price of bankruptcy is simply \( P_b = (v_b c)^\lambda \).

Then, by Theorem 1, the rescaled boundaries are
\[
\begin{align*}
v_a &= \kappa (1 + \iota) \\
v_d &= \theta (1 + i + x) \\
v_b &= \max \{ \kappa (1 + \iota), \theta (1 + i + x) \}
\end{align*}
\]
(33-35)

Furthermore, equation (8) can be written as a function of \( c \)
\[
i = \omega [1 - (1 - \beta) \kappa]^+ (1 + \iota) (v_a c)^\lambda /[1 - (v_a c)^\lambda].
\]
(36)

We can also express deposit, subordinated debt, and bank values in Theorem 1 as ratios to asset value:
\[
\begin{align*}
D/V &= (1 + \iota) c \quad (37) \\
D_1/V &= \{ x[1 - (v_b c)^\lambda] + [(1 - \phi) v_b - (1 + \iota)]^+(v_b c)^\lambda \} c \quad (38) \\
F/V &= 1 + [1 - (v_b c)^\lambda] [(\iota + \tau (1 + x) - (1 - \tau) \iota) c - (v_b c)^\lambda \min \{ \phi v_b, v_b - (1 + \iota) \} c \\ &\equiv f(x, c). \quad (39)
\end{align*}
\]

The last equation is the bank value scaled by asset and we denote it by \( f(x, c) \). Choosing \((C, C_1)\) to maximize \( F \) is equivalent to choosing the duplet \((x, c)\) to maximize \( f \). Once we obtain the optimal \((x^*, c^*)\), the optimal \((C^*, C_1^*)\) can be obtained easily from \( C^* = c^* r V \) and \( C_1^* = x^* C^* \). Also notice that \( V_a < V_d \) if and only if \( v_a < v_d \).

For the case of unregulated bank without deposit insurance, we should setting \( \iota = 0 \) and \( \phi = \alpha \) in equations (33)–(39). Rational bank run implies we also have \( \kappa = 1/(1 - \alpha) \).

**Lemma 1** Assume \( \iota = 0 \), \( \phi = \alpha \), and \( \kappa = 1/(1 - \alpha) \). The scaled bank value \( f(x, c) \) has a unique maximum \((x^*, c^*)\). At the maximum, the liability ratio is
\[
x^* = r (1 + \lambda)/[\lambda (r - \eta)(1 - \alpha)(1 - \tau)] - 1,
\]
(40)
which implies \( v_a^* = v_d^* \), the state price is
\[
\pi = \frac{1}{1 + \lambda} \cdot \frac{\eta \theta (1 - \alpha) + r \tau}{\eta \theta (1 - \alpha) + r (\tau + \alpha \theta)},
\]
(41)
and the scaled deposit liability is \( c^* = \pi^{1/\lambda} (1 - \alpha)(r - \eta)/r \).

The proof of the lemma is presented in Appendix A.3. From this lemma, the optimal capital structure in an unregulated bank without deposit insurance can be solved analytically. Theorem 3 presents the economic characterization of the optimal liability structure.

Now, consider banks with deposit insurance and under FDIC regulation. We maximize the scaled bank value \( f \) in equation (39) by choosing \((x, c)\). In this case, \( i \) endogenously depends on \( c \) through equation (36).
Lemma 2 Assume $i$ is a function of $c$ as defined in equation (36). The liability structure $(x^*, c^*)$ maximizes $f(x, c)$ only if $v_a^* \geq v_d^*$. Furthermore,

1. for any given $\omega$, there exists $\bar{\kappa} < 1/(1-\beta)$ such that for each $\kappa > \bar{\kappa}$, $(x^*, c^*)$ is optimal if and only if $v_a^* = v_d^*$;
2. alternatively, for any given $\kappa$, there exists $\bar{\omega}$ such that for each $\omega < \bar{\omega}$, $(x^*, c^*)$ is optimal if and only if $v_a^* = v_d^*$;
3. if $v_a^* = v_d^*$ for the optimal $(x^*, c^*)$, the state price of bankruptcy $\pi$ satisfies the following quartic equation
   \[ \varphi_0 + \varphi_1\pi + \varphi_2\pi^2 + \varphi_3\pi^3 + \varphi_4\pi^4 = 0 \]  
   (42)

with each $\varphi_j$ being a simple algebraic function of the following exogenous parameters: $r$, $\sigma$, $\delta$, $\tau$, $\eta$, $\beta$, $\kappa$, and $\omega$, and optimal liability ratio and scaled deposit liability are

\[ x^* = \frac{r}{r-\eta} \left\{ \frac{\kappa(1+\lambda)}{(1-\tau)\lambda} - \omega[1-(1-\beta)\kappa] + \frac{\pi}{1-\pi} \right\} - 1 \]  
   (43)
\[ c^* = \frac{\pi^{1/\lambda}(r-\eta)/(r\kappa)}{} \]  
   (44)

Appendix A.4 presents the mathematical proof. This lemma prepares the mathematical facts for Theorem 4, whereas the theorem focuses on characterizing the economic features of the optimal liability structure for banks paying endogenous deposit insurance premium and being regulated by FDIC and its charter authority.

Equity requirement imposes a cap on leverage ratio. The leverage ratio is $l = (D + D_1)/V$. It follows from equations (37) and (38) that, given other parameters, the leverage ratio $l$ is a function of $(x, c)$:

\[ l(x, c) = c\{1 + \iota + x[1 - (v_b c)\lambda] + [(1 - \phi)v_b - (1 + \iota)]^+(v_b c)\lambda\}. \]  
   (45)

Then, the equity requirement is $l(x, c) \leq 1 - \zeta$. The optimal liability structure subject to equity requirement is

\[ (x^\dagger, c^\dagger) = \text{argmax}\{f(x, c) : l(x, c) \leq 1 - \zeta\}. \]  
   (46)

If the optimal structure $(x^\dagger, c^\dagger)$ subject to equity requirement is not the same as the unrestricted optimal structure $(x^*, c^*)$, then the constraint of equity requirement must be binding. In this case,

\[ (x^\dagger, c^\dagger) = \text{argmax}\{f(x, c) : l(x, c) = 1 - \zeta\}. \]  
   (47)
Lemma 3 Assume \( i = 0, \phi = \alpha, \) and \( \kappa = 1/(1 - \alpha) \). The scaled bank value \( f(x, c) \) has a unique maximum \((x^\dagger, c^\dagger)\) subject to equity requirement. If the equity requirement is binding at the maximum, then \( v_d^\dagger \leq v_a^\dagger \). In the case of \( v_d^\dagger < v_a^\dagger \), the state price of bankruptcy is

\[
\pi = \frac{1}{1 + \lambda} \cdot \frac{\eta(1 - \tau)(1 - \alpha)}{r + \tau(1 - \alpha)(r - \eta)},
\]

but in the case of \( v_d^\dagger = v_a^\dagger \), the state price of bankruptcy is \( \pi = z^\lambda \), where \( z \) solves the algebraic equation

\[
\left[1 - \frac{r(1 + \lambda)}{(r - \eta)(1 - \alpha)(1 - \tau)\lambda}\right]z^{1 + \lambda} + \left[\lambda + \frac{r(1 + \lambda)}{(r - \eta)(1 - \alpha)(1 - \tau)\lambda}\right]z - \frac{(1 - \zeta)r}{(r - \eta)(1 - \alpha)} = 0. \tag{49}
\]

The optimal liability structure is

\[
x^\dagger = \frac{r}{(r - \eta)(1 - \pi)} \left[\frac{1 - \zeta}{(1 - \alpha)\pi^{1/\lambda}} - 1\right] \quad \text{if} \quad v_d^\dagger < v_a^\dagger \tag{50}
\]

\[
x^\dagger = \frac{r(1 + \lambda)}{(r - \eta)(1 - \alpha)(1 - \tau)\lambda} \quad \text{if} \quad v_d^\dagger = v_a^\dagger \tag{51}
\]

\[
c^\dagger = \pi^{1/\lambda}(1 - \alpha)(r - \eta)/r. \tag{52}
\]

Appendix A.5 contains the proof of this lemma.

A.1 Proof of Theorem 1

The general form of solution to pricing equation (23) is

\[
E = \delta V - (1 - \tau)(I + C + C_1)/r + a_1 V + a_2 V^{-\lambda} \tag{53}
\]

where \( \lambda \) is a solution to equation (22), and \( a_1 \) and \( a_2 \) can be any two constants. The two constants and the default boundary together are determined by the three boundary conditions for equity value. Boundary condition (24) immediately implies \( \lambda > 0 \) and \( a_1 = 1 - \delta \). It then follows from boundary condition (26) that \( 1 - \lambda a_2 V_b^{-(\lambda + 1)} = 0 \), which gives \( a_2 = V_b^{1 + \lambda}/\lambda \). Further more, the boundary condition (25) implies \( V_b - (1 - \tau)(C + C_1)/r + a_2 V_b^{-\lambda} = 0 \). Substituting out \( a_2 \) in the preceding equation and solving for \( V_b \), we obtain equation (1). Substituting \( a_1 = 1 - \delta \) and \( a_2 = V_b^{1 + \lambda}/\lambda \) into equation (53), we arrive at equation (2).

The general form of solution to pricing equation (27) is

\[
D = C_1/r + b_1 V + b_2 V^{-\lambda}, \tag{54}
\]
where $\lambda$ is a solution to equation (22), and $b_1$ and $b_2$ can be any two constants. Given bankruptcy boundary $V_b$, the constants, $b_1$ and $b_2$, are determined by boundary conditions of subordinated debt. Boundary condition (28) implies $\lambda > 0$ and $b_1 = 0$. Boundary condition (29) then implies $C_1/r + b_2V_b^{-\lambda} = [(1 - \alpha)V_b - D]^+$, which gives

$$b_2 = \{(1 - \alpha)V_b - D]^+ - C_1/r\}V_b^\lambda. \tag{55}$$

Substituting equation (55) and $b_1 = 0$ into equation (54), we obtain equation (3).

Bank value is $F = D + D_1 + E$. Substituting equations (3) and (2), we obtain

$$F = V + (1 - P_b)[\tau C_1 - (1 - \tau)(I + C)]/r + D + P_b[(1 - \phi)V_b - D]^+ - P_bV_b. \tag{56}$$

Using $D = C/(r - \eta)$, we can express deposit as $D = (1 - P_b)[r/(r - \eta)]C/r + P_bD$, which is used to rewrite equation (56) into equation (4).

### A.2 Proof of Theorem 2

Let $Q$ be the value of the insurance product to banks. The pricing equation of the insurance product is

$$\frac{1}{2}\sigma^2 V^2 Q'' + (r - \delta) V Q' - r Q - I = 0, \tag{57}$$

where $Q'$ and $Q''$ represents the first and second partial derivatives of $Q$ with respect to $V$. The general solution to the differential equation is $Q(V) = -I/r + a_1 V + a_2 V^{-\lambda}$, where $a_1$ and $a_2$ can be any constants.

The boundary conditions of the value of the insurance product are $\lim_{V \to \infty} Q = -I/r$ and $Q(V_a) = [D - (1 - \beta)V_a]^+$, where $V_a = \kappa D$. The upper boundary condition restricts $a_1$ to zero, and the lower boundary condition implies $-I/r + a_2 V_a^{-\lambda} = [D - (1 - \beta)V_a]^+$. Solving $a_2$ from the lower boundary condition and substituting it and $a_1 = 0$ back into the formula of $Q(V)$, we obtain

$$Q(V) = -(1 - P_a)I/r + [D - (1 - \beta)V_a]^+ P_a, \tag{58}$$

where $P_a = [V_a/V]^\lambda$.

Since fair insurance premium should make the insurance product worth zero, we should have $Q(V) = 0$. It follows that the fair insurance premium $I^\circ$ must satisfy

$$(1 - P_a)I^\circ = r[D - (1 - \beta)V_a]^+ P_a. \tag{59}$$

Finally, we obtain equation (5) by substituting $V_a = \kappa D$ and then factoring $D$ out from the truncation function.
A.3 Proof of Lemma 1

It follows from $i = 0$, $\phi = \alpha$, $\kappa = 1/(1 - \alpha)$, and equation (35) that $v_b = \max\{(1 + \iota)/(1 - \alpha), \theta(1 + x)\}$. This further implies $\phi v_b \leq v_b - (1 + \iota)$, which simplifies equation (39) to

$$f = 1 + \left\{\iota + \tau(1 + x) - (v_b c)^\lambda [\iota + \tau(1 + x) + \alpha v_b]\right\} c$$

It follows that the first-order condition for $c$ to be optimal is

$$\iota + \tau(1 + x) - (1 + \lambda) [\iota + \tau(1 + x) + \alpha v_b] (v_b c)^\lambda = 0.$$  \hspace{1cm} (60)

Notice that $v_b = \theta(1 + x)$ if and only if $x \geq x^*$, where $x^* = (1 + \iota)/[\theta(1 - \alpha)] - 1$. Otherwise, $v_b = (1 + \iota)/(1 - \alpha)$. If $x < x^*$, then the partial derivative of $f$ with respect to $x$ is

$$f'_x = \tau[1 - (b_b c)^\lambda] c > 0$$

which implies that $f$ is an increasing function in $x$ for $x < x^*$. If $x > x^*$, then the partial derivative of $f$ with respect to $x$ is

$$f'_x = \left\{\tau - \left[\lambda \iota/(1 + x) + (1 + \lambda)(\tau + \alpha \theta)\right] (v_b c)^\lambda\right\} c.$$  \hspace{1cm} (62)

In the case of $x > x^*$, condition (61) becomes

$$\iota + \tau(1 + x) - (1 + \lambda) [\iota + (\tau + \alpha \theta)(1 + x)] (v_b c)^\lambda = 0.$$  \hspace{1cm} (64)

Imposing this condition equation (63) gives

$$\left[f'_x\right]_{f'_x=0} = -\frac{\iota}{1 + x} [1 - (v_b c)^\lambda] c < 0.$$  \hspace{1cm} (65)

Thus, $f$ is a decreasing function in $x$ for $x > x^*$ if we always keep $c$ optimal relative to $x$.

Therefore, $x^*$ is the optimal point for $x$. At this point, $v^*_a = v^*_d = v^*_b = (1 + \iota)/(1 - \alpha)$. It follows that $P^*_a = P^*_d = P^*_b$. We can solve $P^*_b$ from equation (61) as

$$P^*_b = \frac{1}{1 + \lambda} \cdot \frac{\iota + \tau(1 + x^*)}{\iota + \tau(1 + x^*) + \alpha v^*_b}.$$  \hspace{1cm} (66)

After substituting out $x^*$ and $v^*_b$, the above formula gives

$$P^*_b = \frac{1}{1 + \lambda} \cdot \frac{\iota \theta(1 - \alpha) + \tau(1 + \iota)}{\iota \theta(1 - \alpha) + (\tau + \alpha \theta)(1 + \iota)}.$$  \hspace{1cm} (67)

Thus, we define the right-hand size of the above equation as $\pi$. From $(v^*_b c^*)^\lambda = \pi$, we obtain $c^* = \pi^{1/\lambda}(1 - \alpha)/(1 + \iota)$. After substituting out $\iota$ and $\theta$ using equation (32), we arrive at the formulas of $x^*$ and $c^*$ in the lemma, which completes the proof.
A.4 Proof of Lemma 2

We first show that \( v_a > v_d \) cannot be optimal. Observe that \( v_a > v_d \) if and only if \( \theta(1+i+x) < (1+i)\kappa \). We then have \( v_b = v_a = (1+i)\kappa \) in this case. Since \( v_b \) is independent of \( x \) in this case, the partial derivative of \( f \) with respect to \( x \) is

\[
f'_x = \tau[1 - (v_b c)^\lambda]c.
\]

(68)

Since the state price \((v_b c)^\lambda\) is smaller than 1 when the bank is not closed, we have \( f'_x > 0 \). This implies that bank can add value by raising \( x \), and thus such \( x \) cannot be optimal.

It follows from equation (36) that the derivative of \( i \) with respect to \( c \) is \( i'_c = \lambda i c^{-1}/[1 - (v_a c)^\lambda] \). If \( \kappa < 1/(1-\beta) \), both \( i \) and \( i'_c \) are positive. Moreover, both \( i \) and \( i'_c \) converge to zero uniformly for all \( c \) when \( \kappa \) rises to \( 1/(1-\beta) \) while other parameters are fixed. If \( \kappa \geq 1/(1 - \beta) \), both \( i \) and \( i'_c \) are zero. Also notice that \( i \) and \( i'_c \) converge to zero uniformly for all \( c \) when \( \omega \) goes down to zero while \( \kappa \) is fixed.

Since an optimal liability structure must satisfy \( v_d \geq v_a \), let us consider \( x \) that satisfies \( \theta(1+i+x) \geq (1+i)\kappa \). In this case \( v_b = \theta(1+i+x) \). Now suppose the strict inequality \( \theta(1+i+x) > (1+i)\kappa \) holds. It follows that \( v_d > v_a \) and \( \phi = \alpha \). Then, we have

\[
\min\{\phi v_b, v_b - (1+i)\} = \begin{cases} v_b - (1+i) & \text{if } v_b \leq (1+i)/(1-\alpha) \\ \alpha v_b & \text{if } v_b > (1+i)/(1-\alpha). \end{cases}
\]

(69)

Notice that \((1+i)\kappa < (1+i)/(1-\alpha)\) if \( \kappa < 1/(1-\beta) \) and \( \beta \leq \alpha \).

For the case of \( v_b \leq (1+i)/(1-\alpha) \), we use the first part of equation (69) and substitute \( v_b = \theta(1+i+x) \) into equation (39) to obtain

\[
f = 1 + c\{i - i + \tau(1+i+x) + [1+i - (\tau + \theta)(1+i+x)](v_b c)^\lambda\}. \tag{70}
\]

Its partial derivative with respect to \( x \) is, using the definition of \( \theta \),

\[
f'_x = c\{\tau - [\tau + \lambda x/(1+i+x)](v_b c)^\lambda\}. \tag{71}
\]

In view of the formula of \( i'_c \), the partial derivative of \( f \) with respect to \( c \) is

\[
f'_c = 1+i - [1+i - \tau(1+i+x)][1-(v_b c)^\lambda] - \lambda x (v_b c)^\lambda
- \{ (1-\tau)[1-(v_b c)^\lambda] + [\lambda x/(1+i+x)](v_b c)^\lambda \} \lambda i/[1-(v_a c)^\lambda]. \tag{72}
\]

Using equation (71), we can write the above as

\[
f'_c = f'_x(1+i+x)/c + 1+i - (1+i)[1-(v_b c)^\lambda]
- \{ (1-\tau)[1-(v_b c)^\lambda] + [\lambda x/(1+i+x)](v_b c)^\lambda \} \lambda i/[1-(v_a c)^\lambda]. \tag{73}
\]
Then, optimality of $c$ relative to $x$ implies

\[
f'_x(1 + i + x)/c = -(1 + \iota) + (1 + \iota)[1 - (v_b c)^\lambda]
+ \{(1 - \tau)[1 - (v_b c)^\lambda] + [\lambda x/(1 + i + x)](v_b c)^\lambda\} \lambda i/[1 - (v_a c)^\lambda].
\] (74)

Since $x/(1 + i + x)$ is less than 1 and decreasing in $x$, we have

\[
f'_x(1 + i + x)/c < -\iota + i[1 - (v_b c)^\lambda] + \lambda i\{1 - \tau + \lambda (v_b c)^\lambda/[1 - (v_a c)^\lambda]\}. \] (75)

When $\kappa$ rises to $1/(1 - \beta)$, $\iota$ converges to zero, and $v_a$ approaches above $(1 + \iota)/(1 - \beta)$. Consequently, there exists $\kappa_1$ such that the right-hand side is negative for all $\kappa > \kappa_1$.

For the case of $v_b > (1 + \iota)/(1 - \alpha)$, substitution of $v_b = \theta(1 + i + x)$ and the second part of equation (69) into equation (39) gives

\[
f = 1 + c\{\iota - i + \tau (1 + i + x) - [\iota - i + (\tau + \alpha \theta)(1 + i + x)](v_b c)^\lambda\}. \] (76)

The partial derivative of $f$ with respect to $x$ is

\[
f'_x = c\{\tau - [(1 + \lambda)(\tau + \alpha \theta) + \lambda (\iota - i)/(1 + i + x)](v_b c)^\lambda\}. \] (77)

The partial derivative of $f$ with respect to $c$ is

\[
f'_c = (\iota - i)[1 - (v_b c)^\lambda]
+ (1 + i + x)\{\tau - [(1 + \lambda)\tau + \lambda \alpha(1 - \tau) + \lambda (\iota - i)/(1 + i + x)](v_b c)^\lambda\}
- \{(1 - \tau)[1 - (v_b c)^\lambda] + \lambda[\tau + \alpha(1 - \tau) + (\iota - i)/(1 + i + x)](v_b c)^\lambda\}
\cdot \lambda i/[1 - (v_a c)^\lambda]. \] (78)

Combining equation (77) and (78), we have

\[
f'_c = (\iota - i)[1 - (v_b c)^\lambda] + f'_x(1 + i + x)/c
- \{(1 - \tau)[1 - (v_b c)^\lambda] + \lambda[\tau + \alpha(1 - \tau) + (\iota - i)/(1 + i + x)](v_b c)^\lambda\}
\cdot \lambda i/[1 - (v_a c)^\lambda]. \] (79)

If $c$ is optimal relative to $x$, we have $f'_c = 0$ and thus

\[
f'_x(1 + i + x)/c = -(i - i)[1 - (v_b c)^\lambda]
+ \{(1 - \tau)[1 - (v_b c)^\lambda] + \lambda[\tau + \alpha(1 - \tau) + (\iota - i)/(1 + i + x)](v_b c)^\lambda\}
\cdot \lambda i/[1 - (v_a c)^\lambda]. \] (80)
It follows from \( v_b \geq v_a \) and \( \theta(1+i+x) > (1+i)/(1-\alpha) \) that

\[
f'_x(1+i+x)/c < -(\iota-i)[1-(v_bc)^\lambda]
+ \left\{ 1 - \tau + \lambda \left[ \tau + \alpha(1-\tau) + (\iota-i)(1-\alpha)/(1+i) \right] \right.
\left. (v_bc)^\lambda/[1-(v_bc)^\lambda] \right\} \lambda \iota. \tag{81}
\]

Then, there exists \( \kappa_2 < 1/(1-\beta) \) such that \( i \) is so small that the right-hand side is negative when \( \kappa > \kappa_2 \).

Now, let \( \bar{\kappa} = \max\{\kappa_1, \kappa_2\} \). If \( \kappa > \kappa^* \), we have \( f'_x < 0 \) for all \( x \) that satisfy \( \theta(1+i+x) > (1+i)\kappa \), if \( c \) is kept to be optimal relative to \( x \). Therefore, \( \theta(1+i+x) > (1+i)\kappa \) cannot be optimal because reducing \( x \) will add value to the bank. Consequently, the optimal choice for \( x \) must satisfy \( \theta(1+i+x) = (1+i)\kappa \), which implies \( v_d = (1+i)\kappa \) and thus \( v_a = v_d \).

Observing that \( i \) converges to zero uniformly for all \( c \) when \( \omega \) goes down to zero but \( \kappa \) is fixed, the proof of existence of \( \bar{\omega} \) for fixed \( \kappa \) follows similarly.

Since the optimality condition requires \( v_a = v_d \), the default and closure boundaries have the same probability to hit: \( \pi = (v_ac)^\lambda = (v_bc)^\lambda \). This means \( v_a = \pi^{1/\lambda}/c \). Then, in view of equation (33), we have \( (1+i)\kappa = \pi^{1/\lambda}/c \). The above gives equation (44). Equations (36) and (33) imply

\[ i = (1+i)\omega[1-(1-\beta)\kappa]^+\pi/(1-\pi) \tag{82} \]

which gives equation (43). It follows from equation (72) that

\[
f'_c = 1 + \iota - [1 + i - \tau(1+i)\kappa/\theta][1 - (1+\lambda)\pi]
- (1-\tau)\lambda\iota + \left[ \frac{\theta(1+i)}{(1+i)\kappa} - 1 \right] \lambda^2 i \frac{\pi}{1-\pi} \tag{83}
\]

Substituting equation (82) and multiplying by \( (1-\pi)^2 \), we obtain a polynomial of four degrees in \( \pi \), where the five coefficients of the polynomial are simple algebraic functions of \( \lambda, \iota, \kappa, \tau, \theta \) and \( \omega \). This completes the proof.

### A.5 Proof of Lemma 3

With \( i = 0, \phi = \alpha, \) and \( \kappa = 1/(1-\alpha) \), the scaled bank value is

\[
f(x,c) = 1 + [1 - (v_bc)^\lambda][\iota + \tau(1+x)]c - (v_bc)^\lambda \min\{\alpha v_b, v_b - (1+i)\}c. \tag{84}
\]

First, consider the case of \( \theta(1+x) > (1+i)/(1-\alpha) \). In this case, \( v_b = \theta(1+x) \). Equation (84) becomes

\[
f(x,c) = 1 + c\left\{ [\iota + \tau(1+x)][1 - (v_bc)^\lambda] - \alpha \theta(1+x)(v_bc)^\lambda \right\}. \tag{85}
\]
The partial derivatives of $f(x, c)$ are
\[
\begin{align*}
f'_x(x, c) &= c\left\{ \tau - [\lambda \mu/(1 + x) + (1 + \lambda)(\tau + \alpha \theta)](v_b c)^\lambda \right\} \\
f'_c(x, c) &= \mu + \tau(1 + x) - (1 + \lambda)[\mu + (\tau + \alpha \theta)(1 + x)](v_b c)^\lambda.
\end{align*}
\] (86) (87)

Let $c_1(x)$ be value of $c$ such that $f'_c = 0$ for given $x$. Then, $f'_c(x, c) > 0$ for $c < c_1(x)$ and $f'_c(x, c) < 0$ for $c > c_1(x)$.

\[
[v_b c_1(x)]^\lambda = \frac{1}{1 + \lambda} \frac{\mu + \tau(1 + x)}{\mu + (\tau + \alpha \theta)(1 + x)}.
\] (88)

Let $x^*$ be the value of $x$ such that $\theta(1 + x) = (1 + \mu)/(1 - \alpha)$. By Lemma 1, $(x^*, c_1(x^*))$ maximizes $f$.

The formula of leverage ratio becomes
\[
l(x, c) = c\left\{ \mu[1 - (v_b c)^\lambda] + (1 + x)[1 - (1 - (1 - \alpha)\theta)(v_b c)^\lambda] \right\}.
\] (89)

Let $c_2(x)$ be the value of $c$ such that $l(x, c) = 1 - \zeta$ for given $x$. Then,
\[
[v_b c_2(x)]^\lambda = \frac{\mu + 1 + x - (1 - \zeta)/c}{\mu + (1 - (1 - \alpha)\theta)(1 + x)}.
\] (90)

The partial derivatives of $l(x, c)$ are
\[
\begin{align*}
l'_x(x, c) &= c\left\{ 1 - [\lambda \mu/(1 + x) + (1 + \lambda)(1 - (1 - \alpha)\theta)](v_b c)^\lambda \right\} \\
l'_c(x, c) &= \mu + 1 + x - (1 + \lambda)[\mu + (1 - (1 - \alpha)\theta)(1 + x)](v_b c)^\lambda.
\end{align*}
\] (91) (92)

If $l(x, c_1(x)) < 1 - \zeta$, let $g_1(x) = f(x, c_1(x))$. It follows that
\[
g'_1(x) = f'_x(x, c_1(x)) + f'_c(x, c_1(x))c'_1(x) = -\mu/(1 + x)[1 - (v_b c)^\lambda] < 0.
\] (93)

and thus reducing $x$ increases $f$, which implies $x$ cannot be optimal. If $l(x, c_1(x)) \geq 1 - \zeta$. For any $c < c_1(x)$, by equation (92) we have
\[
l'_c(x, c) > l'_c(x, c_1(x)) = \{1 - [1 - (1 - \alpha)\theta]\}(1 + x) > 0.
\] (94)

Thus, $l(x, c) \leq 1 - \zeta$ implies $c < c_1(x)$, which further implies $f'_c(x, c) > 0$. This means $c_2(x)$ maximizes $f(x, c)$ subject to $l(x, c) \leq 1 - \zeta$. Now, let $g_2(x) = f(x, c_2(x))$. Then,
\[
g'_2(x) = l'_x(x, c_2(x)) + l'_c(x, c_2(x))c'_2(x).
\] (95)

Differentiating the constraint (89) with respect to $x$, we obtain
\[
0 = c'_2(x)\left\{ \mu + (1 + x)[1 - (1 + \lambda)(v_b c_2(x))^\lambda] + (1 - \alpha)\theta(1 + x)(1 + \lambda)(v_b c)^\lambda \right\}
\]
\[
+ c_2(x)\left\{ 1 - (1 + \lambda)(v_b c_2(x))^\lambda + (1 - \alpha)\theta(1 + \lambda)(v_b c_2(x))^\lambda \\
- [\lambda \mu/(1 + x)](v_b c_2(x))^\lambda \right\}.
\] (96)
The above is equivalent to

\[
\left[(1 + x)c'_2(x) + c_2(x)\right]\left[[1 + \nu/(1 + x)][1 - (1 + \lambda)(v_bc_2(x))^\lambda]
+ (1 - \alpha)\theta(1 + \lambda)(v_bc)^\lambda\right] = c'_2(x)\left[\nu/(1 + x)[1 - (v_bc_2(x))^2]\right].
\] (97)

Subtracting equation (96) from equation (95), we obtain

\[
g'_2(x) = -[c + (1 + x)c'_3]\left\{(1 - \tau)[1 - (1 + \lambda)(v_bc)^\lambda] + \theta(1 + \lambda)(v_bc)^\lambda\right\}.
\] (98)

Substituting the definition of \(\theta\), we simplify the above into

\[
g'_2(x) = -[c + (1 + x)c'_3](1 - \tau)[1 - (1 + \lambda)(v_bc)^\lambda].
\] (99)

Notice that equation (97) implies \(c_2(x) + (1 + x)c'_2(x) > 0\). It then follows that \(g'_2(x) < 0\). Therefore, decreasing \(x\) while keep the capital requirement binding increases \(f\), and thus \(x\) cannot be optimal.

Now, consider the case of \(\theta(1 + x) < (1 + \nu)/(1 - \alpha)\). In this case, \(v_b = v_a = (1 + \nu)/(1 + \alpha)\). It follows that \([(1 - \alpha)v_b - (1 + \nu)]^+ = 0\) and simplifies equation (84) to

\[
f(x, c) = 1 + c\left\{[\nu + \tau(1 + x)][1 - (v_ac)^\lambda] - (\alpha v_a)(v_ac)^\lambda\right\}.
\] (100)

The derivatives of \(f\) are

\[
f'_x(x, c) = c\tau[1 - (v_ac)^\lambda]
\] (101)

\[
f'_c(x, c) = [\nu + \tau(1 + x)][1 - (1 + \lambda)(v_ac)^\lambda] - (\alpha v_b)\lambda(v_ac)^\lambda
\] (102)

In this case, the formula of leverage ratio (45) is simplified to

\[
l(x, c) = c\left\{1 + \nu + x[1 - (v_ac)^\lambda]\right\}.
\] (103)

The derivatives of \(l(x, c)\) are

\[
l'_x(x, c) = c[1 - (v_ac)^\lambda] > 0
\] (104)

\[
l'_c(x, c) = 1 + \nu + x[1 - (1 + \lambda)(v_ac)^\lambda].
\] (105)

If \(\theta(1 + x) < v_a\) and \((x, c)\) is optimal in this case, the capital requirement must be binding because \(l_x(x, c) > 0\) and otherwise we can increase \(f\) by increasing \(x\). Let \(c_3(x)\) be the value of \(c\) such that \(l(x, c_3(x)) = 1 - \zeta\) for give \(x\) with \(\theta(1 + x) < v_a\). Differentiate the binding constraint with respect to \(x\), we obtain

\[
c_3(x)[1 - (v_ac_3(x))^\lambda] + c'_3(x)\left\{1 + \nu + [1 - (1 + \lambda)(v_ac_3(x))^\lambda]\right\} = 0.
\] (106)
Let $g_3(x) = f(x, c_3(x))$. We have

$$
g'_3(x) = f'_x(x, c_3(x)) + f'_c(x, c_3(x))c'_3(x)
= c'_3(x)\{(1-\tau)v - [\tau + va](1+\lambda)(va c_3(x))^\lambda\}. \quad (107)$$

The first-order condition $g'_3(x) = 0$ implies $(v_b c)^\lambda = \pi$, where $\pi$ is given in equation (48). It follows that $c_3(x) = \pi^{1/\lambda}(1-\alpha)/(1+\iota)$, which leads to equation (52) in view of equation (32). Substituting this $c$ into constraint (103), we solve for $x^\dagger$ and obtain equation (50). Function $g_3(x)$ achieves maximum at $x^\dagger$ because it is in fact increasing for $x < x^\dagger$ and decreasing for $x > x^\dagger$. The reason is as follows. Equation (106) actually indicates that $c'_3(x) < 0$, i.e., $c_3(x)$ is a decreasing function of $x$. Then, $(v_b c_3(x))^\lambda > \pi$ for $x < x^\dagger$ and $(v_b c_3(x))^\lambda < \pi$ for $x > x^\dagger$. In view of equation (107) and $c_3(x) < 0$, we know $g'_3(x) > 0$ for $x < x^\dagger$ and $g'_3(x) < 0$ for $x > x^\dagger$.

If $g'_3(x) > 0$ for all $x$ such that $\theta(1+x) < v_a$, the optimal $x^\dagger$ must satisfy $\theta(1+x^\dagger) = v_a$, which gives equation (51). Multiplying $v_a$ through $l(x^\dagger, c_3(x^\dagger)) = 1 - \zeta$ and letting $z = v_a c_3(x^\dagger)$, we obtain equation (49). Finally, $v_a c_3(x^\dagger) = \pi$ gives equation (52) and completes the proof.

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Table 1: Exogenous Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Range</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset value</td>
<td>$V$</td>
<td>$(0, \infty)$</td>
<td>1.00 $billion</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$r$</td>
<td>$(0, \infty)$</td>
<td>3.00 %</td>
</tr>
<tr>
<td>Asset volatility</td>
<td>$\sigma$</td>
<td>$(0, \infty)$</td>
<td>10.00 %</td>
</tr>
<tr>
<td>Asset cash flow</td>
<td>$\delta$</td>
<td>$[0, \infty)$</td>
<td>2.00 %</td>
</tr>
<tr>
<td>Bank service income</td>
<td>$\eta$</td>
<td>$(0, r)$</td>
<td>0.75 %</td>
</tr>
<tr>
<td>Corporate tax rate</td>
<td>$\tau$</td>
<td>$(0, 1)$</td>
<td>30.00 %</td>
</tr>
<tr>
<td>Court bankruptcy cost</td>
<td>$\alpha$</td>
<td>$(0, 1)$</td>
<td>25.00 %</td>
</tr>
<tr>
<td>FDIC liquidation cost</td>
<td>$\beta$</td>
<td>$(0, 1)$</td>
<td>25.00 %</td>
</tr>
<tr>
<td>Bank closure rule</td>
<td>$\kappa$</td>
<td>$(0, \infty)$</td>
<td>102.00 %</td>
</tr>
<tr>
<td>Insurance subsidy</td>
<td>$\omega$</td>
<td>$[0, 1]$</td>
<td>100.00 %</td>
</tr>
<tr>
<td>Capital requirement</td>
<td>$\zeta$</td>
<td>$(0, 1)$</td>
<td>7.00 %</td>
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Table 2: Endogenous Variables in Optimal Structure.

Numerical values of endogenous variables in optimal liability structures are calculated for give exogenous parameters in Table 1. The first two columns list the definitions of endogenous variables. For a bank in each regulatory environment, we report the optimal values (in percentage points). For each endogenous variable under FDIC, capital requirement (CR), and both (FDICR), we show its difference from the corresponding value in unregulated (free) market.

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>formula</th>
<th>Free market</th>
<th>FDIC value</th>
<th>CR value</th>
<th>FDICR value</th>
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<td>Charter value</td>
<td>$(F - V)/V$</td>
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<td>35.26</td>
<td>2.65</td>
<td>31.01</td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>$D/V$</td>
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<td>55.34</td>
<td>12.38</td>
<td>35.85</td>
</tr>
<tr>
<td>Debt ratio</td>
<td>$D_1/V$</td>
<td>62.44</td>
<td>51.80</td>
<td>-10.64</td>
<td>57.15</td>
</tr>
<tr>
<td>Equity ratio</td>
<td>$E/V$</td>
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<td>28.12</td>
<td>0.91</td>
<td>38.01</td>
</tr>
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<td>Bankruptcy</td>
<td>$V_b/V$</td>
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<td>47.80</td>
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<td>0.09</td>
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</table>
Table 3: Trade Off between Deposit and Subordinated Debt

Numerical values of endogenous variables in optimal liability structures are calculated for given exogenous parameters in Table 1, except one parameter being perturbed higher or lower as indicated in each panel. For a bank under each regulatory environment, we report the optimal values (in percentage points), and their changes, of endogenous variables after a parameter is perturbed.

A. Higher Bankruptcy Cost ($\alpha : 25\% \rightarrow 30\%$)

<table>
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<th>Endogenous variable</th>
<th>Free</th>
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<th>CR</th>
</tr>
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<tbody>
<tr>
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<td>31.51</td>
<td>34.73</td>
<td>30.37</td>
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<td>39.40</td>
<td>54.49</td>
<td>33.84</td>
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<td>Debt ratio</td>
<td>63.81</td>
<td>51.16</td>
<td>59.16</td>
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<tr>
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<td>28.30</td>
<td>29.08</td>
<td>37.37</td>
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<tr>
<td>Bankruptcy</td>
<td>56.29</td>
<td>55.57</td>
<td>48.34</td>
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<td>1.64</td>
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<td>Credit spread</td>
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<td>0.38</td>
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B. Lower Corporate Tax Rate ($\tau : 30\% \rightarrow 25\%$)

<table>
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<tr>
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<th>Free</th>
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<th>CR</th>
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<tr>
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<td>51.76</td>
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<tr>
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<td>0.08</td>
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C. Higher Account Service Income ($\eta : 0.75\% \rightarrow 1.25\%$)

<table>
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</table>
Numerical values of endogenous variables in optimal liability structures are calculated for given exogenous parameters in Table 1, except one parameter being perturbed higher or lower as indicated in each panel. For a bank under each regulatory environment, we report the optimal values (in percentage points), and their changes, of endogenous variables after a parameter is perturbed.

### A. Higher Asset Volatility (σ : 10% → 15%)

<table>
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<tr>
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<th></th>
<th>CR</th>
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### B. Higher Asset Cash Flow (δ : 2% → 3%)

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<th>CR</th>
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<td>chg</td>
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<td>Bankruptcy</td>
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<td>-6.68</td>
<td>49.52</td>
<td>-6.93</td>
<td>46.91</td>
<td>-0.89</td>
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<tr>
<td>Loss in value</td>
<td>3.24</td>
<td>0.55</td>
<td>3.04</td>
<td>0.50</td>
<td>2.58</td>
<td>1.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit spread</td>
<td>1.03</td>
<td>0.34</td>
<td>0.97</td>
<td>0.32</td>
<td>0.85</td>
<td>0.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDIC premium</td>
<td>0.00</td>
<td>0.00</td>
<td>0.11</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5: Optimal Response to FDIC Regulation

Numerical values of endogenous variables in optimal liability structures are calculated for given exogenous parameters in Table 1, except one parameter about FDIC being perturbed higher or lower as indicated in each panel. For a bank under FDIC without capital requirement and with capital requirement (FDICR), we report the optimal values (in percentage points) of endogenous variables before and after a parameter is perturbed. We also report the changes in response to the parameter perturbation.

A. Lower FDIC Liquidation Cost ($\beta : 25\% \rightarrow 15\%$)

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>FDIC before</th>
<th>FDIC after</th>
<th>chg</th>
<th>FDICR before</th>
<th>FDICR after</th>
<th>chg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charter value</td>
<td>35.26</td>
<td>36.45</td>
<td>1.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>55.34</td>
<td>57.39</td>
<td>2.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt ratio</td>
<td>51.80</td>
<td>53.19</td>
<td>1.39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity ratio</td>
<td>28.12</td>
<td>25.87</td>
<td>-2.25</td>
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<td></td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>56.45</td>
<td>58.53</td>
<td>2.08</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Loss in value</td>
<td>2.54</td>
<td>1.76</td>
<td>-0.78</td>
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<td></td>
</tr>
<tr>
<td>Credit spread</td>
<td>0.66</td>
<td>0.75</td>
<td>0.09</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>FDIC premium</td>
<td>0.09</td>
<td>0.06</td>
<td>-0.03</td>
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<td></td>
</tr>
</tbody>
</table>

B. More Subsidized Insurance Premium ($\omega : 100\% \rightarrow 60\%$)

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>FDIC before</th>
<th>FDIC after</th>
<th>chg</th>
<th>FDICR before</th>
<th>FDICR after</th>
<th>chg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charter value</td>
<td>35.26</td>
<td>36.35</td>
<td>1.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deposit ratio</td>
<td>55.34</td>
<td>57.21</td>
<td>1.86</td>
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<td></td>
</tr>
<tr>
<td>Debt ratio</td>
<td>51.80</td>
<td>53.08</td>
<td>1.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity ratio</td>
<td>28.12</td>
<td>26.06</td>
<td>-2.05</td>
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<tr>
<td>Bankruptcy</td>
<td>56.45</td>
<td>58.35</td>
<td>1.90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loss in value</td>
<td>2.54</td>
<td>2.90</td>
<td>0.36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit spread</td>
<td>0.66</td>
<td>0.74</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDIC premium</td>
<td>0.09</td>
<td>0.10</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C. Tougher Bank Closure Rule ($\kappa : 102\% \rightarrow 110\%$)

<table>
<thead>
<tr>
<th>Endogenous variable</th>
<th>FDIC before</th>
<th>FDIC after</th>
<th>chg</th>
<th>FDICR before</th>
<th>FDICR after</th>
<th>chg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charter value</td>
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<td>34.44</td>
<td>-0.82</td>
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<td>Deposit ratio</td>
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<td>51.33</td>
<td>-4.02</td>
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<td></td>
</tr>
<tr>
<td>Debt ratio</td>
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<td>55.00</td>
<td>3.20</td>
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</tr>
<tr>
<td>Equity ratio</td>
<td>28.12</td>
<td>28.11</td>
<td>-0.01</td>
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</tr>
<tr>
<td>Bankruptcy</td>
<td>56.45</td>
<td>56.46</td>
<td>0.01</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Loss in value</td>
<td>2.54</td>
<td>2.54</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit spread</td>
<td>0.66</td>
<td>0.66</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDIC premium</td>
<td>0.09</td>
<td>0.06</td>
<td>-0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: An Illustration of Bank Liability Structure

<table>
<thead>
<tr>
<th>Asset Side</th>
<th>Liability Side</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets:</strong> V</td>
<td>Deposit: D</td>
</tr>
<tr>
<td>Volatility: σ</td>
<td>Benefit: deduct tax τ</td>
</tr>
<tr>
<td>Cash flow: δ</td>
<td>Benefit: account service η</td>
</tr>
<tr>
<td></td>
<td>Cost: bankruptcy α or β</td>
</tr>
<tr>
<td>Charter value: F − V</td>
<td>Cost: insurance premium I</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subordinated Debt: D₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefit: deduct tax τ</td>
</tr>
<tr>
<td>Cost: bankruptcy α or β</td>
</tr>
</tbody>
</table>

**Tangible equity:** V − (D + D₁)

**Equity:** E

**Bank value:** F = D + D₁ + E
Figure 2: Bankruptcy of Unregulated Banks

Bankruptcy Boundary

- Bank–run in fixed structure
- Default in fixed structure
- Bank–run in optimal structure
- Default in optimal structure
Figure 3: Bankruptcy under FDIC

- Dashed line: Closure in fixed structure
- Circle: Default in fixed structure
- Solid line: Closure in optimal structure
- Asterisk: Default in optimal structure

Bankruptcy Boundaries vs. Charter Authority Closure Rule
Figure 4: Insurance Premium, Closure Rule, and Volatility
Figure 5: Incentives of Risk Shifting

- Constant premium, Omega = 1.0
- Constant premium, Omega = 0.5
- Varying premium, Omega = 1.0
- Varying premium, Omega = 0.5