Contracting on Credit Ratings: Adding Value to Public Information∗

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Abstract

We provide a rationale for the use for credit ratings even when ratings contain no new information about a risky security (such as in the case of sovereign debt). In our model, an investor contracts with an manager who invests in a risky bond. The bond’s return depends in part on the state. The state is known to both investor and manager, but unverifiable to a third party and therefore non-contractible. A credit rating on the bond provides a verifiable signal about the state. We show that the investor uses the credit rating to reward the manager for holding a riskless bond in a bad state. In an economy with a continuum of investor-manager pairs, the use of the rating in a contract in turn determines the equilibrium return of the risky asset. We establish that widespread use of credit ratings may increase asset volatility and lower returns, and discuss the determinants of optimal rating precision.

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1 Introduction

There are many contexts in which market participants use and react to the issuance of reports based on already known, publicly-available information; noteworthy examples are credit ratings on sovereign bonds, or insured municipal bonds. While many models of credit ratings on companies assume that the rating agency possesses information not already reflected in market prices, such a claim is difficult to make for government debt. A credit rating merely provides a summary of information already available, and yet markets react to it.¹

In this paper, we pursue a novel explanation for the existence of this (seemingly) redundant information aggregation and reporting: managers use ratings to improve incomplete contracts. Consider an investor who delegates the management of her portfolio and wants to provide incentives to a manager who is prone to moral hazard. An incentive contract based on portfolio outcomes may not be precise enough to ensure that the manager always acts in her best interests. However, an additional signal (such as a credit rating) can provide her with a useful tool to improve on the contract.

Our framework is (loosely) based on Aghion and Bolton (1992). Briefly, this is a stylized incomplete contracting model between an investor and an agent in which states are observable, but not verifiable. There are two states, good and bad, and two feasible actions: hold a risky bond or hold a riskless asset instead. The agent’s preferred action depends on the realization of a stochastic private benefit. Thus, in contrast to many contracting frameworks, the size of the private perquisites the manager can extract are unrealized at the time the contract is written and unknown to both parties. The potential inefficiency is that, due to these private benefits, the investor and manager may end up preferring different state-contingent actions. We interpret a credit rating as a verifiable signal about the state. Contracts may be written on this verifiable signal, potentially improving efficiency in the contracting relationship.

In our model, the investor chooses the wages to offer the manager based on the return delivered by the manager, and the credit rating of the risky bond. Each investor/manager pair is atomistic and takes the possible returns as given. We show that in general, the investor rewards the manager for holding the riskless bond when the state is bad and the bond has a low rating. If the state is bad and the bond has a high rating, the investor is more tolerant. The key intuition is that the investor must induce the manager to hold the risky bond when the state is good. Boosting the reward for holding the riskless bond in the bad state requires

¹For example, Brooks, et al., find that downgrades of sovereign debt adversely affect both the level of the domestic stock market and the dollar value of the country’s currency.
correspondingly increasing the reward for holding the risky bond in the good state, providing a trade-off for the investor. The optimal contract, of course, depends on the precision of the credit rating.

We then turn to the market-wide equilibrium implications of credit ratings. Investors face a cost of accessing the market directly and may, instead, choose to hire a manager. Because the contract does not perfectly align the incentives of the investor and the manager, the demands generated by the portfolio management sector may differ from the demand of investors who directly invest. The equilibrium return of the risky bond depends on the realization of these demands. Taking the return of the risky asset as given, each investor benefits from using the credit rating. However, when all investors do so, this affects the bond’s returns. In particular, even when fundamentals are fixed, the price of the risky bond now depends on its credit rating.

Our framework has implications for the effect of ratings on asset returns and the choice of an optimal rating precision. Asset prices are more volatile than justified by fundamentals when credit ratings are widely used in contracts. Further, ratings may lead to lower returns. The aggregate effects of contracting on credit ratings on the returns of the asset therefore imply that, even absent any direct cost to producing more precise credit ratings, it may be optimal for a rating to have some noise in it.

Our focus on the use of non-informative credit ratings to mitigate contracting frictions is novel. Other work on non-informative ratings includes Boot, Milbourn, and Schmeits (2006), who present a framework in which a firm’s funding costs depend on the market’s beliefs about the type of project being chosen. The credit rating agency, by providing a rating, allows infinitesimal investors to coordinate on particular beliefs when multiple equilibria are possible. Further the credit watch procedure provides a mechanism to monitor the firm if it can improve the payoff of its project. Manso (2014), also considers how a credit rating might have real effects, in a model with multiple equilibria self-fulfilling beliefs. In his framework changes in a firm’s credit rating affects its ability to raise capital, which then reinforce the original rating.

Much of the work in the literature considers credit ratings that communicate new information about the firm to the market. For example, Opp, Opp and Harris (2013) illustrate how the use of ratings by regulators might have pernicious effects, and Fulghieri, Strobl and Xia (2013) consider rating manipulation by the credit rating agency itself. Mathis, McAndrews and Rochet (2009) demonstrate that when the flow income from new transactions is high, a rating agency’s concern for future reputation no longer acts as a disciplining device.

Rating shopping has been examined by Bolton, Freixas and Shapiro (2012) in a world
with some boundedly rational consumers who trust the assigned rating. Competition between credit rating firms induces inefficiency, and ratings are more likely to be inflated in booms. Skreta and Veldkamp (2009) examine a similar friction and show that issuers have an incentive to produce complex assets when some consumers are naïve. Subsequent work by Sangiorgi and Spatt (2013) considers rating shopping when all consumers are rational, with the key friction being opacity about how many ratings an issuer has actually obtained. In equilibrium, too many ratings are obtained. While we have a single rating in our model, we consider the effect on different users of the rating, such as firms and portfolio managers.

Donaldson and Piacentio (2013) also consider the effect of credit ratings in contracts, and suggest that investment mandates based on ratings lead to inefficiency. Researchers have also considered the optimal degree of coarseness (see Goel and Thakor (2013) and Kartasheva and Yilmaz (2013)).

We build on the large literature on optimal contracts in a delegated portfolio problem. Bhattacharyya and Pfleiderer (1985) consider such a problem with asymmetric information. In a normal exponential framework, they find that quadratic contracts lead to true information revelation. Dybvig and Spatt (1986) demonstrates that if the manager and the investor have similar preferences then risk–sharing can be efficient. By contrast, we show that risk aversion on the part of the manager can lead to optimal contract to include risk limits. Similar to Grinblatt and Titman (1989), we impose limited liability on the manager. In this case, the manager has an incentive to take on excessive risk as he is not penalized in the bad states. They propose a contract that if a portfolio is constructed so that the loss to managers is larger than the gain, excessive risk taking can be curbed. Stoughton (1993) considers moral hazard in which the manager expends effort to get better information about the risk asset. He characterizes the effect of two types of incentive contracts: linear and quadratic in realized payoffs. He finds that as the investor approaches risk neutrality, the quadratic contract induces the first best.

In other work, Admati and Pfleiderer (1997), Lynch and Musto (1997), and Das and Sundaram (1998) consider the use of benchmark evaluation measures. Innes (1990) provides optimal contracts in a limited liability setting when there is moral hazard on the part of the investor. Finally, Palomino and Prat (2003) consider a delegated portfolio problem in which the manager can affect the risk of the portfolio. They find that the optimal contract is a fixed fee plus a bonus if the payout is above a certain threshold. Interestingly, the optimal contract may induce either insufficient or excessive risk.

There is little work on the equilibrium effects of delegated portfolio management. A notable exception is a recent paper by Guerrieri and Kondor (2012) who model fund managers
with career concerns where some managers have private information on bond risk. Managers get fired based on past performance, which means that when default risk is high, return on bonds is high to compensate managers for the risk of being fired. The reputational premium also amplifies price volatility.

We introduce our model in Section 2. In Section 3, we demonstrate the optimal contract for a single investor-manager pair, holding the price of the risky bond as fixed for each state and credit rating. We then step back to exhibit the equilibrium effects of the contract in Section 4. We provide some implications of our findings in Section 5, and discuss some features of the model and its interpretation in Section 6.

2 Model

The portfolio management sector of an economy comprises a continuum of investors and a continuum of portfolio managers. Investors seek to take a position in one of two assets, a risky bond or a risk-free one. Each investor may invest directly or delegate by hiring a manager. The mass of investors is one, and we assume the mass of portfolio managers is large, so that each investor who wishes to hire a portfolio manager is able to do so.

An investor may directly enter the market for the risky asset themselves, at a cost $c$. This cost may be interpreted as either the opportunity cost of time for the investor or the direct cost of access to certain securities. Investors are heterogeneous in the access cost, which is independent and identically distributed across investors, and is drawn from a distribution $G(\cdot)$ with support $[0, C]$. Alternatively, the investor may hire a portfolio manager. In the delegated portfolio management sector, investors and managers are randomly matched in pairs, and contract exclusively with each other. For simplicity, we assume that each investor or each manager may purchase one unit of either the risky or riskless asset.

If initiated, the investor-manager relationship continues over four periods, $t = 1, \ldots, 4$. At time $t = 1$, an investor knows her own access cost, $c$, and chooses whether to invest on her own or to hire a portfolio manager. In the latter case, she also offers the manager a contract that specifies a wage at time $t = 4$, conditional on the portfolio performance, and potentially on a credit rating for the risky bond.

At time $t = 2$, three pieces of information become available to market participants. First, a state, which affects the payoff to holding the risky bond, is realized. The risky security has two possible payoff states, $h$ and $\ell$, which correspond to the risk-return relationship offered

\footnote{For example, under SEC Rule 144A, only qualified institutional buyers may purchase certain private securities.}
by the bond. State $h$ corresponds to the “solvent” state for the bond, with a relatively high return. State $\ell$ corresponds to a “default” state, with a relatively low (possible negative) return.

Even though both parties know the state, it is not verifiable, and so not directly contractible. However, a contractible signal $\sigma$ is available, in the form of a credit rating on the risky bond. We do not model the source of the credit rating, but consider it to be imperfectly correlated with the state. Specifically, the signal takes on one of two values, $g$ or $b$, and is potentially informative, with $\text{Prob}(\sigma = g \mid s = h) = \text{Prob}(\sigma = b \mid s = \ell) = \psi \geq \frac{1}{2}$. Thus, if $\psi = \frac{1}{2}$, the rating is completely uninformative, which is equivalent to the investor and manager being able to contract only on the final value of the investment, and if $\psi = 1$, the rating perfectly informative, which is equivalent to the investor and manager being able to contract directly on the state.

The return to holding the riskless asset is $r^f$, regardless of state or signal on the risky bond. The return to the risky bond can depend both on the state and its credit rating. We denote its return in state $s$ when the credit rating is $\sigma$ as $r^s_\sigma$. These returns are determined in equilibrium based on the total demand for the risky bond on the part of investors and managers. In choosing whether to hire a manager (i.e., at $t = 1$) and the contract to offer a hired manager (at $t = 2$), an investor has rational expectations about the returns to the risky bond under different scenarios. That is, she knows $r^s_\sigma$ for each $s = h, \ell$ and $\sigma = g, b$.

We assume that $r^h_\sigma > r^f > r^\ell_\sigma$ for each $\sigma = g, b$, that is, the return on the risky bond is greater in state $h$ than in state $\ell$, regardless of the credit rating. Under these assumptions, an investor purchasing bonds directly would prefer to buy the risky bond in state $h$ (when the reward to bearing its risk is high) and the riskless bond in state $\ell$ (when the reward to bearing the risk on the risky bond is low). Let $a_h$ denote the action of buying the risky bond and $a_\ell$ the action of buying the riskless one. Then, for the investor, the optimal action is $a_h$ in state $h$ and $a_\ell$ in state $\ell$.

The manager obtains a private benefit $m$ from holding the risky bond in state $\ell$, which engenders an agency conflict. The private benefit corresponds to either synergies with his other funds (“soft money”) or side transfers that he obtains from a sell-side firm if he places the risky bond in an investor’s portfolio. The private benefit is random and is realized at time 2. It is drawn from a distribution $F(\cdot)$ with support $[0, M]$. The private benefit is independent across managers and is also independent of the corresponding investor’s access cost. As is customary, the private benefit $m$ is not verifiable, so cannot be contracted on.

At time $t = 3$, direct investors and portfolio managers each choose an action $a_h$ or $a_\ell$. Collectively, their determine the demand for the risky bond, and hence the actual return.
Let $q^p$ denote the demand from investors who hire portfolio managers, and $q^d$ the demand from investors who invest directly.\(^3\) The total demand for the risky asset from investors in the PM sector is then $q = q^p + q^d$. Let $r^s(q)$ denote the return when the state is $s$, the credit rating is $\sigma$, and the demand from the PM sector is $q$. We assume that $r^s(q)$ is decreasing in $q$. That is, a larger demand leads to a higher price and so a lower return.

Note that at $t = 3$, given the agency conflict, managers may sometimes take an inefficient action. Potentially, there are gains to trade from renegotiation between the investor and manager at that time. For now, we assume that renegotiation is costly enough to be infeasible, and return to a discussion of renegotiation in Section 6.

To summarize: There are four dates in the model, $t = 1$ through 4. Figure 1 shows the sequence of events in the model.

\[
\begin{array}{cccc}
 t = 1 & t = 2 & t = 3 & t = 4 \\
\hline
\text{Investors choose} & \text{(i) State $h$ or $\ell$ revealed} & \text{(i) Each direct investor} & \text{Output realized;} \\
\text{to hire manager} & \text{(ii) Contractible signal} & \text{and hired manager} & \text{Payoffs earned} \\
\text{or invest directly} & \text{$g$ or $b$ obtained} & \text{takes action $a_h$ or $a_\ell$} & \\
 & \text{(iii) Size of private benefit} & \text{(ii) Return of risky} & \\
 & \text{$m$ realized} & \text{bond determined} & \\
\end{array}
\]

![Figure 1: Sequence of Events](image-url)

It is important to note that the contract is written before the state and credit rating are realized. We have in mind a situation in which contracts are written on a periodic basis (say once a year), whereas the state (which could reflect other aspects of the investors’ portfolio) can change frequently in between. The credit rating need not be known as soon as the state is revealed, but it must be known before the manager takes an action. The private benefit of the manager reflects the effect of market events on other assets held by the manager or other payments he receives from his relationships, so is known only when the state itself is revealed.

There is no discounting, and we model both parties as risk-neutral. The payoff to the investor from this relationship is the net return generated by the manager less the total compensation paid to the manager. The payoff to the manager is the sum of the wage and any private benefits he may garner. The manager has limited liability that requires the wage

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\(^3\)As the state is known to all investors, the demand of direct investors does not depend on the rating.
in any state to be non-negative. His reservation utility is zero, so any contract that satisfies limited liability is also individually rational. When the outcome is realized at time 4, the manager is paid the wage specified by the contract signed at time 1 and the investor keeps any extra investment income.

Let \( \bar{r}^s = r^s(0) \) be the maximal return to the risky asset in state \( s \). This return is realized if the price of the risky asset is low; that is, its demand from the DPM sector is zero. Correspondingly, let \( r^s = r^s(1) \) be the minimal return to the risky asset in state \( s \), obtained when its price is high; specifically, when all investors (either directly or through their hired managers) wish to buy the risky asset, so that the demand from the DPM sector is one. The minimal return on a long position is -100%, so \( r^s \geq -1 \) for each \( s \).

We make the following assumptions.

**Assumption 1**

(i) \( r^h > r^f > \bar{r}^f \).

(ii) \( M \geq r^f - \bar{r}^f \).

(iii) \( \frac{f'(x)}{f(x)} < \frac{1}{r^f - \bar{r}^f} \) for all \( x \in [0, M] \).

The first part of the assumption is standard, and implies that action \( a_h \) (buying the risky bond) maximizes output in the high state \( h \), whereas action \( a_\ell \) (buying the riskless bond) maximizes output in the low state \( \ell \). Part (ii) ensures that for some realizations of \( m \), the agency conflict between investor and manager can be large. Finally, part (iii) is a technical condition that ensures a unique solution to the contracting problem. The condition implies that the prior distribution over \( m \) does not increase too rapidly at any point. Notice, for example, that the condition is readily satisfied by the uniform distribution, for which \( f'(x) = 0 \) at all \( x \).

### 3 Optimal Contract for a Single Investor-Manager Pair

We first consider the optimal contract for a single investor-manager pair. Suppose the investor has decided to hire a manager and has been matched up with one. The contract is entered into at time \( t = 1 \), before the state and credit rating are known. In addition, the extent of the moral hazard problem (i.e., the size of the realized private benefit \( m \)) is unknown to both parties. The demand of each investor and each manager is infinitesimal, so they take as given the return on the risky asset in each scenario (i.e., state-rating pair). The investor can write a compensation contract for the manager that depends on the possible investment returns and the credit rating. A contract is therefore characterized by \( W = \{ w^h_g, w^f_g, w^h_b, w^f_b \} \).
where \( w^j_\sigma \) denotes the compensation to the manager when the credit rating is \( \sigma \in \{g, b\} \) and the portfolio payoff is \( j \in \{h, \ell, f\} \).

Suppose the state is \( h \) (the high reward state) and the rating is \( \sigma \). Notice that the manager takes action \( a_h \) if his payoff from doing so is higher than the payoff from investing in the risk-free bond; that is, if \( w^h_\sigma \geq w^f_\sigma \). He takes action \( a_\ell \) otherwise. Thus, the payoff to investing in the risk-free asset affects incentive compatibility in both the high and low reward states. This will allow us to characterize the optimal contract.

In state \( h \), the credit rating is \( g \) with probability \( \psi \) and \( b \) with probability \( 1 - \psi \). If the investor induces the action \( a_h \), her payoff is \( r^h_g - w^h_g \); if she induces the action \( a_\ell \), her payoff is \( r^f - w^f_\sigma \). Thus, her expected payoff in the high reward state \( h \) is:

\[
\pi^h = \psi \left( (r^h_g - w^h_g)1_{\{w^h_g \geq w^f_g\}} + (r^f - w^f_g)1_{\{w^h_g < w^f_g\}} \right) \\
+ (1 - \psi) \left( (r^h_b - w^h_b)1_{\{w^h_b \geq w^f_b\}} + (r^f - w^f_b)1_{\{w^h_b < w^f_b\}} \right),
\]

(1)

where \( I_{\{x\}} \) is an indicator function that takes on the value of 1 if the event \( x \) occurs, and 0 otherwise.

Next, consider the low reward state \( \ell \). The credit rating is \( g \) with probability \( 1 - \psi \) and \( b \) with probability \( \psi \). Given a signal \( \sigma \), the manager takes the action \( a_\ell \) if \( w^f_\sigma \geq w^f_\sigma + m \) and action \( a_h \) if \( w^f_\sigma < w^f_\sigma + m \). Of course, at the time the contract is established, neither party knows \( m \), the size of the manager’s private benefit, and so the investor has to take expectations over the possible values it may take. Overall, the investor’s expected payoff in the low state \( \ell \) is

\[
\pi^\ell = (1 - \psi) \left( (r^f - w^f_g)F(w^f_g - w^f_b) + (r^f - w^f_b)(1 - F(w^f_g - w^f_b)) \right) \\
+ \psi \left( (r^f - w^f_b)F(w^f_b - w^f_g) + (r^f - w^f_g)(1 - F(w^f_b - w^f_g)) \right).
\]

(2)

At time 1, when the contract is signed, the investor’s expected payoff is

\[
\Pi = \phi \pi^h + (1 - \phi)\pi^\ell.
\]

(3)

The investor chooses the various wage levels \( \{w^h_g, w^h_b, w^f_g, w^f_b, w^f_\sigma, w^f_\sigma\} \) to maximize \( \Pi \).

We start by showing two features of the optimal contract. First, \( w^h_\sigma = w^f_\sigma \); that is, for any rating \( \sigma \), the agent is paid the same compensation when the return is \( r^h \) as she earns by investing in the risk-free asset. Because she always has the choice of investing in the risk-free asset, an investor who wants to induce her to hold the risky asset must reward her at least
as much for the latter action when the state is high. To minimize the cost of providing this incentive, the investor set $w^h_\sigma$ as low as possible, that is, equal to $w^f_\sigma$. Second, $w^l_\sigma = 0$. That is, if the agent invests in the risky asset in the low reward state $\ell$, she obtains a zero payoff. As the investor does not want to hold the risky bond in this state, it cannot be worthwhile to reward an agent who does hold the risky bond in state $\ell$. Formally,

**Lemma 1** The optimal contract sets $w^h_\sigma = w^f_\sigma$ and $w^l_\sigma = 0$ for each credit rating $\sigma = g, b$.

Lemma 1 has two implications that we exploit going forward. First, all hired managers invest in the risky bond in state $h$, regardless of rating, because their payoff from doing so weakly exceeds the payoff from investing in the riskfree bond. All investors who invest directly will also buy the risky bond in state $h$. Therefore, in state $h$, the total demand from the portfolio management sector for the risky bond is one, and is invariant to the rating. This further implies that $r^h_g = r^h_b$. For simplicity, we denote this $r^h$.

Second, the lemma reduces the investor’s problem to two choice variables, $w^f_g$ and $w^f_b$. That is, the optimal contract is characterized by the compensation that the manager receives for investing in the risk-free asset, given the rating on the risky bond. Using Lemma 1, we can write the investor’s payoff as

$$
\Pi = \phi [r^h - \psi w^f_g - (1 - \psi)w^f_b] + (1 - \phi)(1 - \psi)[(r^f - w^f_g)F(w^f_g) + r^f_b(1 - F(w^f_g))] + (1 - \phi)\psi[(r^f - w^f_b)F(w^f_b) + r^f_b(1 - F(w^f_b))].
$$

(4)

The first-order conditions for an interior optimum are $\frac{\partial \Pi}{\partial w^f_\sigma} = 0$ for each $\sigma = g, b$. Taking the derivatives and simplifying, we obtain

$$
(r^f - w^f_g - r^f_b)f(w^f_g) - F(w^f_g) = \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi}
$$

(5)

$$
(r^f - w^f_b - r^f_b)f(w^f_b) - F(w^f_b) = \frac{\phi}{1 - \phi} \frac{1 - \psi}{\psi}
$$

(6)

Notice that when $\psi = \frac{1}{2}$ (i.e., the credit rating is completely uninformative about the state), the right-hand sides of the two conditions are equal. That is, the optimal contract is invariant to the rating on the risky bond. Conversely, when $\psi > \frac{1}{2}$, the right-hand side of equation (5) exceeds the RHS of equation (6).

Broadly, the optimal contract when a credit rating is available rewards the manager for avoiding the risky bond in the low return state, $\ell$, when its credit rating is low ($b$). If the signal embodied in the credit rating is sufficiently informative about the state (i.e., $\psi$ is sufficiently high), the manager receives a positive wage $w^f_b$ for buying the riskless asset when
the risky bond has a low credit rating, and a zero wage ($w^f_g = 0$) for the same action when the risky bond has a good credit rating. In other words, if the credit rating is sufficiently precise, the investor induces the manager to tilt toward the risky bond when it has a high credit rating and steer clear of the risky bond when it has a bad credit rating.

The exact ranges of signal precision under which different compensation levels are paid are outlined in Proposition 1. For each rating $\sigma = g, b$, define a threshold precision for the credit rating as follows:

$$\bar{\psi}_\sigma = \frac{1}{1 + \frac{1-\phi}{\phi} (r^f - r^f_\sigma) f(0)},$$  \hspace{1cm} (7)

Then,

**Proposition 1** In the optimal contract in the economy with a credit rating:

(i) If $\psi < 1 - \bar{\psi}_g$, then $w^f_g$ satisfies the first-order condition (5) and decreases in $\phi$, $\psi$, and $r^g$. Conversely, if $\psi \geq 1 - \bar{\psi}_g$, then $w^f_g = 0$.

(ii) If $\psi > \bar{\psi}_b$, then $w^f_b$ satisfies the first-order condition (6) and decreases in $\phi$ and $r^b$, but increases in $\psi$. Conversely, if $\psi < \bar{\psi}_b$, then $w^f_b = 0$.

First, consider the case that the risky bond obtains a bad credit rating $b$. Increasing $w^f_b$ has two effects: (i) In the low reward state, $\ell$, it induces the manager to hold the riskless bond more often (i.e., for a larger set of private benefit realizations), which increases the investor’s payoff (ii) In the high reward state $h$, the incentive compatibility constraint to induce the manager to hold the risky bond is $w^h_b \geq w^f_b$. Therefore, increasing $w^f_b$ implies that the investor has to pay the manager a higher amount $w^h_b$ to induce him to hold the risky bond in the high risk return state $h$, which reduces the investor’s payoff. When $\psi$ is high (say close to 1), the second effect is unimportant because the risky bond is highly unlikely to obtain a low credit rating when it is the high risk return state $h$. Therefore, $w^f_b$ is reasonably high. Conversely, when $\psi$ is low, the first effect is less important (the risky bond may get a high credit rating even in state $b$) and the second effect more important (the risky bond may get a low credit rating even in state $g$), so that $w^f_b$ is set to zero.

The intuition for setting $w^f_g > 0$ is similar. On the one hand, in the low return state, $\ell$, it induces the manager to hold the riskless bond for a higher range of private benefit realizations. On the other, it requires the investor to increase $w^h_g$ correspondingly, which lowers her payoff in the high return state, $h$. When $\psi$ is high, the latter effect dominates, because the risky
bond is very likely to obtain the high credit rating in state $h$. Conversely, when $\psi$ is low, the first effect dominates, so the investor sets $w^f_h$ to a positive number.

### 3.1 Investor and Manager Payoffs

Consider the payoff to a manager who is hired and offered an optimal contract. In state $h$, the manager takes action $a_h$ and obtains $w^f_h \sigma$ when the credit rating is $\sigma$. Using Lemma 1, the manager’s expected payoff in state $h$ is $\psi w^f_h + (1 - \psi) w^f_b$. In state $\ell$, the time 1 contract induces the manager to take action $a_h$ if $m > w^f_h \sigma$ (recall that $w^f_\ell \sigma = 0$) and action $a_\ell$ if $m < w^f_\ell \sigma$. Thus, the manager’s expected payoff is

$$\Gamma = \phi \left\{ \psi w^f_g + (1 - \psi) w^f_b \right\} + (1 - \phi) \left\{ (1 - \psi) [w^f_g F(w^f_g) + (1 - F(w^f_g))] E(m | m \geq w^f_g) \right\} + \psi \left\{ w^f_b F(w^f_b) + (1 - F(w^f_b)) E(m | m \geq w^f_b) \right\}. \quad (8)$$

Given the manager’s actions, the payoff to an investor who hires a portfolio manager is

$$\Pi^p = \phi \left\{ r^h - \psi \sigma^f_h - (1 - \psi) \sigma^f_b \right\} + (1 - \phi) \left\{ (1 - \psi) [(r^f - \sigma^f_g) F(\sigma^f_g) + \sigma^f_b (1 - F(\sigma^f_g))] \right\} + \psi \left\{ (r^f - \sigma^f_b) F(\sigma^f_b) + \sigma^f_\ell (1 - F(\sigma^f_b)) \right\}. \quad (9)$$

Suppose the investor were to invest directly. Recall that she knows the state. She incurs her access cost $c$, and buys the risky bond in state $h$ and the riskless bond in state $\ell$. Therefore, her payoff is $\Pi^d = \phi r^h + (1 - \phi) r^f - c$. The surplus generated by the transaction between the investor and the portfolio manager is therefore $\Lambda = \Pi^p + \Gamma - \Pi^d$, or

$$\Lambda = c + (1 - \phi) \left\{ (1 - \psi) E(m | m \geq \sigma^f_g) - (r^f - \sigma^f_g) \right\} + \psi (1 - \sigma^f_g) E(m | m \geq \sigma^f_b) - (r^f - \sigma^f_\ell). \quad (10)$$

The overall benefit of the transaction consists of two factors: the investor saves on the direct access cost $c$, and in the low return state $\ell$, the manager captures the private benefit $m$ when it is sufficiently large. However, whenever the latter happens, the portfolio consists of the risky bond rather than the riskless bond, resulting in a loss of surplus equal to $r^f - \sigma^f_\ell$.

### 4 Market Equilibrium

Consider the equilibrium in the market for the risky asset. At time 3, the proportion of investors who have chosen to hire a manager is fixed at (say) $\rho$. In state $h$, all direct investors
buy the risky asset, and, as shown in the previous section, all managers also buy the risky asset. The demand for the risky asset from the PM sector is therefore just one. In state $\ell$, direct investors hold the riskless asset. Managers purchase the riskless asset if $m \leq w_f$, and hold the risky asset if $m > w_f$. Therefore, if the rating on the risky bond is $\sigma$, the demand for the asset from the PM sector in state $\ell$ is $\rho(1 - F(w_f^\ell))$.

Let $q_d$ be the demand for the risky asset from direct investors, and $q_p^\sigma$ the demand from portfolio managers. The total demand from the PM sector is $q = q_d + q_p^\sigma$. In state $s$, the return on the risky asset is given by $r^s(q)$, where $r^s(\cdot)$ is an increasing function.

We establish that in general, $w_g^f \leq w_b^f$. In the particular case that the rating precision $\psi$ exceeds both $\frac{1}{2}$ and the threshold $\bar{\psi}_b$, we show that $w_g^f < w_b^f$, and in turn that $r_g^\ell < r_b^\ell$.

**Proposition 2**

(i) The optimal contract satisfies $w_g^f \leq w_b^f$, and the return on the risky bond in state $\ell$ satisfies $r_g^\ell \leq r_b^\ell$. Both inequalities are strict when $\psi > \max\{\bar{\psi}_b, \frac{1}{2}\}$.

(ii) In state $h$, the net return of the risky bond is invariant to the credit rating, and is given by $r_g^h(1) = r_b^h(1) = r^h(1)$.

(iii) In state $\ell$, if the rating on the risky bond is $\sigma$, its return is equal to $r_g^\ell = r^\ell\left(\rho(1 - F(w_f^\ell))\right)$, where $\rho$ is the proportion of investors who hire a manager.

In a market equilibrium, therefore, the optimal contract has the following property: In the low return state $\ell$, the agent is rewarded for holding the riskless bond when the risky bond has a bad rating (i.e., $w_b^f$ is relatively high) and for holding the risky bond when it has a good rating (i.e., $w_g^f$ is relatively low). Although the rating contains no new fundamental information about the bond or the state of the world, investors find it beneficial to condition the agent’s compensation on the rating. Therefore, the demand for the risky bond is high when it has a good rating, leading to a low return. Conversely, when it has a bad rating, the demand is low, resulting in a high return. The use of the rating to determine the manager’s compensation therefore induces a volatility in the return of the risky bond in state $\ell$. Conversely, in state $h$, even though the manager’s compensation still depends on the rating (recall that $w_b^\sigma = w_f^\sigma$ for each $\sigma$), the manager always buys the risky bond. Direct investors too buy the risky bond, so that the return is invariant to rating.

From Proposition 2, there are two cases of interest to consider. Suppose first that the rating is sufficiently precise, specifically that $\psi > \max\{\bar{\psi}_b, \frac{1}{2}\}$. Then, $w_g^f > w_b^f \geq 0$. Here, the rating has bite — it affects the manager’s compensation, and in turn his action and the return on the risky bond in state $\ell$. An increase in the precision of the rating leads to a strict increase in the payoff to the investor. It also improves the total surplus, because better
decisions on asset holdings are made. The second case is when \( w^f_b = 0 \). In this case, we also have \( w^f_g = 0 \). In this case, the manager’s compensation does not depend on the rating, so in turn both his action and the return on the risky bond in state \( \ell \) are invariant to the rating. It follows that a small increase in rating precision that leaves \( w^f_b \) at zero will have no effect on payoffs.

**Proposition 3** Suppose \( \psi > \bar{\psi}_b \). Then, an increase in the precision of the rating, \( \psi \), strictly increases the payoff to an investor of hiring a portfolio manager, \( \Pi_p \), and the surplus generated by the transaction between the investor and manager, \( \Lambda \).

The surplus generated in the transaction between the investor and the manager strictly increases when the precision of the rating improves. Therefore, even though the payoff to the manager increases, it is not immediate that the manager is worse off. Indeed, we find in a numerical example in Section 4.2 that the manager’s payoff also increases when the rating is more precise.

### 4.1 Decision to Invest Directly or Hire a Manager

In equilibrium, an investor will hire a portfolio manager if it leads to a higher payoff than investing directly. The payoff to direct investing is \( \Pi^d = \phi r^h + (1 - \phi) r^f - c \). The payoff to investing through a portfolio manager is \( \Pi_m \) as shown in equation (9), and is independent of \( c \). Let the net benefit to an investor of hiring a manager be denoted as \( B(c) = \Pi_p - \Pi^d \).

Then, we can write

\[
B(c) = c - \phi \{ \psi w^f_g + (1 - \psi) w^f_b \} - (1 - \phi)(1 - \psi)(w^f_g F(w^f_g) + (r^f - r^f \sigma))(1 - F(w^f_g)) - (1 - \phi)\psi(w^f_b F(w^f_b) + (r^f - r^f \sigma - r^f))(1 - F(w^f_b)). \tag{11}
\]

The tradeoff in the hiring decision is therefore the following. If the investor hires a manager, she saves on the direct access cost \( c \). However, in the high return state \( h \), she has to induce the manager to buy the risky bond by ensuring that \( w^h \sigma \geq w^f_g \). In the low return state \( \ell \), she either pays the manager \( w^f \sigma \) (if the manager holds the risky bond), or loses the difference in returns \( r^f - r^\ell \sigma \) (if the manager holds the riskless bond).

As \( \Pi^d \) is decreasing in \( c \), it is immediate that there is a threshold cost \( \underline{c} \) such that when \( c < \underline{c} \), the investor directly invests directly, and when \( c > \underline{c} \), the investor delegates to a portfolio manager. Further, \( \underline{c} \) is decreasing in the precision of the credit rating, \( \psi \). That is, a more precise credit rating leads to greater delegation to managers.
Proposition 4 There is a threshold $c$, so that when $c > c$, an investor will hire a portfolio manager, whereas for $c < c$, she will invest directly. Further, when $\psi > \bar{\psi}_b$, $c$ is decreasing in $\psi$.

An improvement in rating precision therefore leads to an increase in delegation. As portfolio managers are compensated in part based on the rating of the risky bond, if $\psi > \bar{\psi}_b$, it further follows that there is an increase in the volatility of returns in state $\ell$, depending on whether the bond has a good or bad rating. Further, as more managers are drawn into the delegated portfolio management sector, the compensation of each manager increases due to incentive effects alone.

4.2 Example

We now present an example to illustrate our results. Let $F$, the distribution of manager private benefits, be the uniform distribution over $[0, M]$. Then, $F(x) = \frac{x}{M}$, $f(x) = \frac{1}{M}$, and $f'(x) = 0$ for all $x \in [0, M]$. Further, let $G$, the distribution of direct access costs for investors, be the uniform distribution over $[0, C]$.

Consider the first-order conditions for an interior solution to $w^f$, equations (5) and (6). Substitute in for $f(\cdot)$ and $F(\cdot)$, and solve for $w^f_g$ and $w^f_b$. Further imposing limited liability to ensure that the wages are non-negative, we have

\begin{align*}
  w^f_g &= \max \left\{ \frac{1}{2} \left( r^f - r^\ell_g - \phi \frac{\psi}{1 - \phi} M \right), 0 \right\}, \\
  w^f_b &= \max \left\{ \frac{1}{2} \left( r^f - r^\ell_b - \phi \frac{1 - \psi}{1 + \psi} M \right), 0 \right\}.
\end{align*}

Observe that as $\psi \to 1$ (i.e., the signal becomes fully revealing), we have $w^f_g = 0$ and $w^f_b \to \frac{1}{2}(r^f - r^\ell_b)$.

Now, consider the return on the risky asset in state $b$. Suppose the return distribution is linear in $q$, the total demand for the risky asset from the portfolio management sector. That is, the return is $r^\ell(q) = r^\ell - q(r^\ell - r^\ell)$. Here, $q = \rho(1 - F(w^f_b))$, where $\rho$ is the proportion of investors who delegate to managers.

We illustrate the effects of a changing precision of the credit rating in the context of a numeric example. We set $\phi = 0.8$, $r^f = 0$, $r^h = 0.1$, $M = 0.1$, and $C = 0.08$. We set $r^\ell = -0.5$ and $r^\ell = -0.2$. We vary $\psi$ from $\frac{1}{2}$ to 1. For each value of $\psi$, we solve for $c$, the threshold access cost above which investors delegate, the optimal contract $\{w^f_g, w^f_b\}$, the demand for the risky asset from the portfolio management sector, and the return on the risky asset in...
state $\ell$. The results are shown in Figure 2.

Figure 2: Effects of changing rating precision $\psi$

In the figure, the top-left graph shows the threshold $c$ decrease as $\psi$ increases. When $c > \underline{c}$, the investor hires a portfolio manager; when $c < \underline{c}$, she invests directly. The top-right figure shows the optimal wages. For low values of $\psi$ (given our parameters, $\bar{\psi}_b \approx 0.58$), both $w^f_g$ and $w^f_b$ are zero. However, as $\psi$ increases above $\bar{\psi}_b$, the wage $w^f_b$ is strictly positive and increasing in $\psi$. In the limit when $\psi = 1$, the rating is almost fully revealing about the state. Therefore, a high $w^f_b$ has no cost in state $h$, and induces the right action from the manager in state $\ell$.

The bottom figures show the demand from the portfolio management sector and the
return on the risky bond, given the rating. For $\psi \leq \bar{\psi}_b$, neither the demand nor the return vary with rating precision. As $\psi$ increases above $\bar{\psi}_b$, the demand given the bad rating $b$ falls, and the demand given the good rating $g$ increases due to the increased delegation by investors. This leads to the return increasing in $\psi$ for the bad rating and decreasing in $\psi$ for the good rating.

![Figure 3: Volatility of prices in state $\ell$ as $\psi$ changes](image)

Figure 3 shows the volatility of risky bond returns in state $\ell$ in equilibrium. That is, we compute the standard deviation of the distribution that places mass $1 - \psi$ on $r_{\ell}^l$ and $\psi$ on $r_{\ell}^b$. Of course, when $\psi < \bar{\psi}_b \approx 0.58$, the volatility is zero, as $r_{\ell}^b = r_{\ell}^l$. Similarly, it is zero at $\psi = 1$, because the distribution collapses to a single point. For $\psi$ between 0.58 and 1, volatility first increases and then decreases. The maximal volatility is equal to approximately 15% of the absolute value of the mean return in state $\ell$.

5 Implications of the Incomplete Contracts Approach

The incomplete contracts approach has implications for returns, optimal precision of ratings, empirical implications of changing ratings, and the price impact of trades.
5.1 Implications of credit ratings on returns, and return volatility.

We first observe that, because credit ratings are useful in incentive contracts, they have an effect first on price levels and second on price volatility. To see this, we compare asset returns with and without ratings. From proposition 3, we know that the equilibrium \( c \) that determines if investors invest directly or through managers, is decreasing in \( \psi \). It thus reaches its maximal value when \( \psi = \frac{1}{2} \). Therefore, if there are no ratings then more investment is direct. This of course implies that if there are no ratings, then the demand for the risky asset is lower in the low state, as all the direct investors put their money into the risk-free asset. Thus,

**Implication 1** Ceteris Paribus, assets with credit ratings will have lower returns than assets without credit ratings.

Similar logic applies to the volatility of the returns. Above and beyond variations that come from the realization of the state, assets that are held through the portfolio management sector are exposed to extra uncertainty namely the realization of the credit rating. Specifically, the demand varies depending on the realization of the credit rating, (see Proposition 2) and the returns are more volatile.

**Implication 2** Ceteris Paribus, assets with credit ratings will exhibit more volatile returns than assets without credit ratings.

In a world in which credit rating provide information, it is reasonable suppose that providing more ratings might be beneficial. If ratings are used as a contractible signal, they will have price effects which may not be in a regulator’s objective.

5.2 Optimal precision of ratings

In the usual framework in which a credit rating provides information about a risky corporate bond, the natural notion of efficiency relates to changes in real output. The goal of a planner is to facilitate investment in positive NPV projects. In our framework, in which the rating provides no new information, and the bond is potentially a government bond, a standard notion of efficiency is total surplus in the transaction between the investor and the manager. However, policy-makers have also focused on other measures specifically, a regulator interested in the informativeness of prices might wish to minimized excess return volatility.

We first observe, that if ratings perfectly match the state (i.e., \( \psi = 1 \)), then investors can extract maximal surplus from the managers and there is no excess return volatility. The
model that we have present is a simple two state model, and the rating can take on two values. However, consistent with the incomplete contracting approach, in reality one could image the state comprising many complex variables, while the credit rating summarizes a subset of these variables. Thus plausibly, a perfectly informative credit rating is not possible.

If a regulator cannot choose a perfectly informative rating, then a regulator interested in an optimal rating might err by making ratings too precise. Specifically, there are ranges over which an increase in ratings precision leads to increased return volatility.

**Implication 3** *If a rating is not perfectly informative, then increasing the precision may increase return volatility.*

The problem with improving rating efficiency is most clearly seen in Figure 3. Here, there are ranges for which increasing precision will increase return volatility. This comes about because changes the rating precision affects the set of investors who choose the delegated portfolio management sector. As these trades are typically correlated, this affects the returns as a function of the realization of the signal.

5.3 Implications of market access costs for return impact

We specified that the total investor sector is of mass one with different market access costs that are distributed on $[0,C]$. It is plausible to consider different assets as having different market access costs (for example, bond ETFs versus obscure munis), and also for market access costs to have changed over time. To see the effect of changes in access costs, consider the degenerate case in which access costs are zero. In this case, all investors will directly invest and choose the risky asset in the high state and the risk free asset in the low state. Compare this to any positive access costs, in which some investors choose to hire a manager which is the case that we have characterized in this paper.

**Implication 4** *High market access costs lead to higher asset volatility and lower returns.*

We stress that the causality runs through inefficiency in the managed portfolio sector.

5.4 Implications of the Incomplete contracts approach for Manager’s compensation

The model presents relationships between the manager’s compensation and asset returns. In the incomplete contracts framework, compensation is promised for actions that, in equilibrium, lead to different return patterns. It is useful to consider the relationship between
returns and manager’s payoffs; to do this, we focus on the manager’s received compensation (i.e., wage). We interpret this as a population. The probability distribution over the manager’s compensation is as follows.

<table>
<thead>
<tr>
<th>Wage</th>
<th>Return</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_g^f)</td>
<td>(r^h)</td>
<td>(\psi\phi)</td>
</tr>
<tr>
<td>(w_b^f)</td>
<td>(r^h)</td>
<td>((1 - \psi)\phi)</td>
</tr>
<tr>
<td>(w_b^f)</td>
<td>(r^f)</td>
<td>((1 - \phi)\psi F(w_b^f))</td>
</tr>
<tr>
<td>0</td>
<td>(r_b^f)</td>
<td>((1 - \phi)\psi (1 - F(w_b^f)))</td>
</tr>
<tr>
<td>(w_g^f)</td>
<td>(r^f)</td>
<td>((1 - \phi)(1 - \psi)F(w_g^f))</td>
</tr>
<tr>
<td>0</td>
<td>(r_g^f)</td>
<td>((1 - \phi)(1 - \psi)(1 - F(w_g^f)))</td>
</tr>
</tbody>
</table>

Table 1: Probability Distribution over Manager’s Compensation

**Implication 5** Different managers will earn different compensation for the same returns, and the performance fee (as a percentage of returns) will vary across managers.

The implication of this is that different managers will appear to generate different alpha, even though all investors who choose to engage a manager are better off doing so because of their market access fees.

### 6 Discussion

We interpret the verifiable signal in our model as a credit rating, and demonstrate its usefulness in contracting. There are many verifiable signals that may potentially be used in contracts. In this section, we briefly discuss why credit ratings may have become a more popular signal to contract on than some of the other possibilities. We also point out that renegotiation (frequently a feature in incomplete contracts) may affect the quantitative but not the qualitative results of our model.

#### 6.1 Interpreting the Verifiable Signal as a Credit Rating

If the investor and manager cannot directly contract on a state, then they may wish to use a contractible signal that is correlated with some states or subset of states. To determine the best contractible signal to use, both the investor and manager must weigh different factors. First, it is not often clear what states will be relevant so a signal has to be forward looking: For example, a firm that undertakes a new line of business after a contract has been signed
may have payoffs that depend on states that were not obvious when the investor and manager agreed on the contract. Second, the signal cannot be to “volatile.” This is because contracts have to be enforceable, if signals change at too high a frequency relative to the actions of the manager it is difficult to determine if he (or she) behaved appropriately given the contract.

Various public signals might be of use in contracts: first, prices, price differences, or price indices; second, professional reports such as auditor reports or analysts’ assessments; and third, government indices or variables such as non-farm payroll and finally credit ratings. For the purposes of this discussion, we’ll consider a state or a set of states as ones pertaining to the payoff to an investor in a particular firm.

The relationship between price levels and payoff-relevant states is tenuous as the abject failure of asset pricing to establish prices levels attests. Price changes or relative price changes are potentially useful. The primary drawback from both of these is volatility.

Price volatility means that enforcing contracts written on prices is difficult: prices that fall and rise could give different actions to prices that rise and fall. Further, how do you know what the manager knew when he took the action? One can imagine ways to control for volatility in contracts. In a CAPM world, price changes are driven by market movements in addition to idiosyncratic risk. Unless the investor strips out the systematic volatility (i.e., by also looking at the price change on an index) a contract that only looks at price changes will conflate the two sources of volatility. Relative prices i.e., the (perhaps beta adjusted) difference between an index and a firm’s returns could be used to condition on a firm’s performance. However, such measures will impose a significant computational burden on the contracting parties. It requires an estimate of betas, and potentially a conditional CAPM. Further, the maintained assumption has to be that the underlying firms’ returns are drawn from a stationary distribution. That is, any asset pricing model (which is imperative to distinguish between systematic and idiosyncratic risk) is inherently backward looking.

Another set of potentially contractible signals are generated by professional organizations such as auditors or analysts. Auditors provide information that is inherently backward looking. That is they verify cash flows and expenditures. The benefit of auditors is that the processes are very transparent but they do not provide guidance as to the future prospects of the firm. To some extent, an auditor is useful to determine if the correct action was taken in the past, but not to specify the correct future action. By contrast, analysts do provide an assessment of future prospects and could thus be useful in contracts that govern investments. However, analysts’ coverage tends to focus on equities and of those, larger issues. Thus, the investor who uses analysts’ recommendations has to restrict their investment universe.

Various government agencies provide information that can be used as a signal of the
macro-state. They are somewhat less useful for use in investment contracts that concern firms or industries. First, the mapping between the macro-state and performance of any given firm is not clearly defined. Second, the government data aggregates a tremendous amount of information and the figures are randomly and frequently revised.

Credit ratings (at least the US model) have a few characteristics that make them extremely useful in contracts. First, they are stable in that they change relatively infrequently and are cycle neutral. Second, they are also forward looking, that is they are provide a business analysis of all the states that could be relevant to a particular investment: they can anticipate movements into a new industry, or out of an existing one and changes in the competitive environment. Third, most issues are rated by more than one agency. Fourth, the rating agencies do not sell investment services or actively participate in intermediation markets, therefore they do not present an obvious conflict of interest such as might arise with analysts. It is interesting to note that in comparison to firms, ratings on assets owned by SIVs are less useful than (say) auditor reports. This is because the underlying assets do not change.

Our model implies that credit ratings are used only because they improve contracting efficiency between investors and managers. In the absence of credit ratings, managers sometimes take an inefficient action. In addition, the threat to take an inefficient action results in loss of payoff to the investors. In short, if credit ratings did not exist, investors would have to invent them. Therefore, moves such as those in the European Union in 2012 to ban the use of credit ratings are short-sighted at best.

6.2 Renegotiation

Suppose the state is \( \ell \) and the credit rating is \( \sigma \). Then, a manager with a private benefit in the range \( (w^f_\sigma, r^f - r^f_\sigma) \) will take an inefficient action, by buying the risky bond when it would be efficient to hold the riskless asset. Thus far, we have ignored the possibility of renegotiation between investor and manager. It is usual in incomplete contracts models to consider renegotiation. However, in the delegated portfolio management problem, one suspects that renegotiation is infrequent. After all, an investor delegates her investment decisions because she does not want to monitor her portfolio closely. However, it is worthwhile exploring the effects of renegotiation in particular because this both increases the “pie” and affects how the surplus is split between the manager and investor.

To incorporate the notion of renegotiation, consider the following amendment to the model. At time 3, given his contract and knowledge of the state and signal, the manager may renegotiate the contract. Sometimes, the contract will not induce the outcome that the
investor wants (i.e., holding the risky bond in state $\ell$), and yet $m$ will be low enough that there are gains to renegotiation. We assume that the opportunity to renegotiate is stochastic, and occurs with probability $\lambda$ (so with probability $1 - \lambda$, there is no renegotiation). We expect $\lambda$ to be high, for example, if the manager is a private wealth manager, and negotiates separate contracts with each of his clients. Conversely, if the manager is a bond fund manager with dispersed investors all signing the same contract, $\lambda$ will be zero.

When renegotiation is feasible, the manager has all the bargaining power. The manager makes a take-it-or-leave-it offer to the investor that specifies both the action the manager will take and a new wage contract for the manager. If the investor accepts, the old contract is torn up and the new one holds. If the investor rejects, the old contract remains in force. Any gains to trade at the renegotiation stage are therefore captured by the manager.\footnote{Suppose, instead, we gave all bargaining power at the renegotiation stage to the investor. This would be equivalent to allowing the investor to write a contract after the state were known, going against the spirit of the idea that contracts are revised only at periodic intervals, whereas the state may change rapidly in between contract revisions.} After any possible renegotiation, the manager invests by taking action $a_h$ or $a_\ell$.

In such a set-up, the investor’s payoff is not affected by the possibility of renegotiation, because the manager captures all gains from renegotiation. Thus, renegotiation has no effect on the optimal contract, on the principal’s payoff, or on the decision to hire the manager. Therefore, most of our results are robust to this amendment.

What does change is the payoff to the manager. The manager now earns not just what was promised in the contract at time 1, but also captures any extra surplus he can garner from renegotiation at time 3. Therefore, an improvement in the precision of the rating may actually hurt the manager, since it reduces the need to renegotiate and therefore reduces the amount he can capture at time 3.

7 Conclusion

When contracts are incomplete, credit ratings have value even when they contain no new information about the issuer or the security being rated. They enable contracts to be written on a noisy signal about known but unverifiable states, improving efficiency when asset prices are given, and also increasing the payoff to the investor in the contract. However, when credit ratings are used in contracts economy-wide, there is a feedback effect leading to increased volatility of prices of risky assets.

The incomplete contracting approach suggests that credit ratings are not necessarily the most appropriate way for investors and managers to use in contracts governing investments
in structured finance vehicles. Consider either a company or a state. Using a forward looking business model, a credit rating might provide a useful summary of states in which a government might change tax or monetary policies. Further, they might provide a useful summary of states in which a company obtains refinancing or sells assets to ensure its financial solvency. However, in the case of structured finance, vehicles comprise pools of assets, for which the servicer does not take analogous actions. It therefore suggests that for these types of assets, whose quality is sunk at the time of origination and no action can be taken to improve quality or viability, an auditor or a business entity that specializes in backward-looking analysis is most appropriate.
Appendix: Proofs

Proof of Lemma 1

Suppose that the credit rating on the risky bond is $g$. As observed in the text, in state $h$ the manager takes action $a_h$ if $w^h_g \geq w^f_g$ and action $a_\ell$ otherwise. In state $\ell$, the manager takes action $a_h$ if $w^f_g < w^\ell_g + m$, and action $a_\ell$ if $w^f_g \geq w^\ell_g + m$. It is immediate to see that it cannot be optimal to set $w^h_g > w^f_g$: Reducing $w^h_g$ to $w^f_g$ does not change the action in either state, and strictly reduces the amount paid to the manager in the $h$.

Suppose $w^h_g < w^f_g$. Then, in state $h$, the manager takes action $a_\ell$, so that the investor’s payoff in this state is $r^f - w^f_g$. If the investor increases $w^h_g$ to set it equal to $w^f_g$, the manager switches to action $a_h$. The investor’s payoff in that state becomes $r^h_g - w^f_g$. Under Assumption 1 part (i), $r^h_g > r^f$, so this strictly improves the investor’s payoff. Therefore, it must be that $w^h_g = w^f_g$.

Now, increasing $w^\ell_g$ above zero has two effects: (i) the probability that the manager takes the inefficient action $a_h$ in state $\ell$ is $F(w^f_g - w^\ell_g)$; this probability increases in $w^\ell_g$ (ii) conditional on action $a_h$ being taken in state $\ell$, the investor’s payoff in that state is $r^h_g - w^\ell_g$, which decreases in $w^\ell_g$. Therefore, it must be optimal to set $w^\ell_g = 0$ (i.e., for the limited liability constraint to bind when the output is $r^f$).

A similar argument applies when the rating is $b$.

Proof of Proposition 1

The first-order conditions for interior values of $w^f_g$ and $w^f_b$ are $\frac{\partial \Pi}{\partial w^f_g} = 0$ and $\frac{\partial \Pi}{\partial w^f_b} = 0$. Take these derivatives and substitute in the wage levels for Lemma 1. We obtain

\[
-\phi \psi + (1 - \phi)(1 - \psi)[(r^f - w^f_g - r^\ell_g)f(w^f_g) - F(w^f_g)] = 0 \quad (14)
\]

\[
-\phi(1 - \psi) + (1 - \phi)\psi[(r^f - w^f_b - r^\ell_b)f(w^f_b) - F(w^f_b)] = 0. \quad (15)
\]

(i) It is immediate that any $w^f_g > r^f - r^\ell_g$ cannot be optimal; by simply setting $w^f_g = 0$, the investor strictly improves payoff in both states $h$ and $\ell$ when signal $g$ is realized. Consider equation (14). If $w^f_g = r^f - r^\ell_g$, the LHS is strictly negative, so it is optimal to reduce $w^f_g$. That is, it must be that at the optimum $w^f_g < r^f - r^\ell_g$.

The second derivative is $\frac{\partial^2 \Pi}{\partial w^f_g^2} = (1 - \phi)(1 - \psi)[(r^f - w^f_g - r^\ell_g)f'(w^f_g) - 2f(w^f_g)]$, which, given Assumption 1 part (iii), is strictly negative. Therefore, when a solution exists to the first-order condition (14), it provides the optimal level of $w^f_g$.

Note that the LHS of equation (14) is strictly decreasing in $w^f_g$. Therefore, for a non-negative solution to exist, it must be that the LHS is weakly positive when evaluated at
$w_f^g = 0$. Evaluating it at $w_f^g = 0$, the LHS reduces to $-\phi \psi + (1 - \phi)(1 - \psi)(r_f^f - r_f^\ell) f(0)$, which is weakly positive if and only if $\psi \leq 1 - \bar{\psi}_g$. If $\psi > 1 - \bar{\psi}_g$, the LHS is negative when evaluated at $w_f^g = 0$, so it is optimal to set $w_f^g = 0$.

Consider the comparative statics of $w_f^g$ as $\phi, \psi$, or $r_f^\ell$ change. Rewrite the first-order condition in equation (14) as shown in equation (5) in the text:

$$(r_f^f - w_f^g - r_f^\ell) f(w_f^g) - F(w_f^g) = \frac{\phi}{1 - \phi} \frac{\psi}{1 - \bar{\psi}_g}.$$

Since the second derivative $\frac{\partial^2 \Pi_2}{\partial w_f^g}$ is negative, it follows that the LHS is strictly decreasing in $w_f^g$. Now, observe that the RHS is strictly increasing in $\psi$ and $\phi$. Therefore, when $\psi$ or $\phi$ increase, $w_f^g$ must fall for the equality to be maintained. Similarly, suppose $r_f^\ell$ increases. All else equal, the LHS falls, so to maintain equality, $w_f^g$ must decrease.

(ii) Follow the same steps as in part (i), considering $w_b^f$ throughout. Evaluating the LHS of equation (15) at $w_b^f = 0$, we have $-\phi (1 - \psi) + (1 - \phi) \psi (r_f^f - r_f^\ell) f(0)$, which is weakly positive if and only if $\psi \geq \bar{\psi}_b$. Therefore, if $\psi \geq \bar{\psi}_b$, the optimal value of $w_b^f$ is given by the solution to equation (15), or equivalently to equation (6). Conversely, if $\psi < \bar{\psi}_b$, it is optimal to set $w_b^f = 0$.

The RHS of equation (6) is strictly decreasing in $\psi$ and strictly increasing in $\phi$. A similar argument as at the end of part (i) establishes that $w_b^f$ is strictly increasing in $\psi$ and strictly decreasing in $\phi$ and $r_f^\ell$.

Proof of Proposition 2

(i) Consider the first-order conditions for interior solutions to $w_f^g$ and $w_f^b$, as shown in equations (5) and (6) in the text. Note that the return in the low state is a function $r^\ell(\cdot)$ that depends in part on the demand from hired managers. In particular, the total demand for the asset from the portfolio management sector given credit rating $\sigma$ is $q = q_d + q_p$, where $q_p = \rho(1 - F(w_f^g))$. As $w_f^g$ increases, $q_p$ decreases, so $r^\ell$ increases.

Define $\gamma(w) = (r_f^f - w - r^\ell(\cdot)) f(w) - F(w)$. Then,

$$\gamma'(w) = (r_f^f - w - r^\ell(\cdot)) f'(w) - \left(1 + w + \frac{dr^\ell}{dw}\right) f(w) < (r_f^f - r^\ell(\cdot)) f'(w) - f(w) < 0,$$

where the last inequality follows from Assumption 1, part (iii).
Now, we can re-write the first-order conditions for an interior optimum for \( w_g^f \) and \( w_b^f \) as

\[
\gamma(w_g^f) = \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} \tag{16}
\]

\[
\gamma(w_b^f) = \frac{\phi}{1 - \phi} \frac{1 - \psi}{\psi} \tag{17}
\]

Notice that when \( \psi = \frac{1}{2} \), the right-hand sides of these two equations are the same, and the RHS of equation (17) strictly exceeds the RHS of equation (16) when \( \psi > \frac{1}{2} \). There are two cases to consider:

(a) Suppose \( w_b^f > 0 \) in the optimal contract. Then, \( \gamma(w_b^f) = \frac{\phi}{1 - \phi} \frac{1 - \psi}{\psi} \leq \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} \). As \( \gamma'(w) < 0 \), it must be that \( w_g^f \leq w_b^f \), with strict inequality if \( \psi > \frac{1}{2} \).

(b) Suppose \( w_b^f = 0 \) in the optimal contract. Then, \( \gamma(0) \leq \frac{\phi}{1 - \phi} \frac{1 - \psi}{\psi} \leq \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} \). Therefore, it must be that \( w_g^f = 0 \).

Therefore, we have \( w_g^f \leq w_b^f \), and since \( r^f(\cdot) \) is increasing in \( w \), we have \( r_g^f \leq r_b^f \). Further, we have shown in Proposition 1 that when \( \psi > \tilde{\psi}_b \), it follows that \( w_b^f > 0 \). As argued in case (a) above, if in addition \( \psi > \frac{1}{2} \), then \( w_g^f < w_b^f \). Since \( r^f(\cdot) \) is increasing in \( w \), it also follows that in this case \( r_g^f < r_b^f \).

This proves part (i) of the proposition. Part (iv) follows from case (a), noting in addition that we need \( \psi > \tilde{\psi}_b \) to ensure that \( w_b^f > 0 \). In addition, note that as argued above, \( r^f(\cdot) \) is increasing in \( w \), so that \( w_g^f < w_b^f \) implies that \( r_g^f < r_b^f \).

(ii) This part is argued in the text and is immediate.

(iii) The total demand from the PM sector is \( q = q^d + q^b \), where in turn \( q^b = \rho(1 - F(w_b^f)) \). Further, \( q^d = 0 \) in state \( \ell \). Therefore, \( q = \rho(1 - F(w_b^f)) \), from which it follows that \( r_g^f = r^f \left( \rho(1 - F(w_b^f)) \right) \). As \( w_g^f \leq w_b^f \), it follows that \( r_g^f \leq r_b^f \).

\section*{Proof of Proposition 3}

(i) Consider the payoff of the investor who hires a portfolio manager, shown in equation (9). Let \( \frac{d\Pi}{d\psi} \) denote the derivative with respect to \( \psi \) when \( w_g^f \) and \( w_b^f \) are acknowledged as functions of \( \psi \), and let \( \frac{\partial \Pi}{\partial \psi} \) denote the same derivative holding \( w_g^f \) and \( w_b^f \) fixed. Then,

\[
\frac{d\Pi}{d\psi} = \frac{\partial \Pi}{\partial \psi} + \frac{\partial \Pi}{\partial w_g^f} \frac{\partial w_g^f}{\partial \psi} + \frac{\partial \Pi}{\partial w_b^f} \frac{\partial w_b^f}{\partial \psi}, \tag{18}
\]

and

\[
\frac{\partial \Pi}{\partial \psi} = \phi(w_b^f - w_g^f) + (1 - \phi)[(r^f - w_g^f)F(w_b^f) + r_b^f(1 - F(w_b^f))] - (r^f - w_g^f)F(w_b^f) - r_b^f(1 - F(w_b^f)). \tag{19}
\]
Consider the derivative $\frac{\partial \Pi}{\partial w_g}$. There are two cases: (i) the first-order condition for an interior optimum, equation (5), holds with equality, in which case $\frac{\partial \Pi}{\partial w_g} = 0$, or (ii) $w^f_g = 0$, and further a small increase in $\psi$ has no effect on $w^f_g$, so here $\frac{\partial w^f_g}{\partial \psi} = 0$. A similar argument holds to show that either $\frac{\partial \Pi}{\partial w_b} = 0$ or $\frac{\partial w^f_b}{\partial \psi} = 0$. Therefore, in an argument similar to what is used to prove the Envelope Theorem, we have $\frac{d\Pi}{d\psi} = \frac{\partial \Pi}{\partial \psi}$.

Now, consider the expression for $\frac{d\Pi}{d\psi}$ in equation (??). When $\psi > \bar{\psi}_b$, we have $w^f_b > w^f_g$, so it is immediate that the first term, $\phi(w^f_b - w^f_g)$ is positive. Consider the term $(1 - \phi)(r^f - w^f_b)F(w^f_b) + r^b_f(1 - F(w^f_b)) - (r^f - w^f_g)F(w^f_g) - r^b_h(1 - F(w^f_g))]. Denote

$$\delta(w) = (r^f - w)F(w) + r^f(1 - F(w)).$$ (20)

Then, we have

$$\delta'(w) = (r^f - w - r^f(\cdot))f(w) - F(w) + (1 - F(w)) \frac{dr^f}{dw}. (21)$$

When $\psi \geq \bar{\psi}_b$, the first-order condition for an interior value of $w^f_b$ (equation (6) holds, so that $(r^f - w^f_b - r^f(\cdot))f(w^f_b) - F(w^f_b) = \phi \frac{1}{1 - \phi} \frac{1 - \psi}{\psi} > 0$. Further, as argued in the proof of Proposition 2, $\frac{dr^f}{dw} > 0$. Therefore, $(1 - F(w)) \frac{dr^f}{dw} > 0$, as $w < M$. Putting these together, we have $\delta'(w) > 0$ when evaluated at $w = w^f_b$. Therefore, if $w^f_g < w^f_b$, we have $\delta(w^f_b) - \delta(w^f_g) > 0$.

That is, in this case, we have

$$(1 - \phi)(r^f - w^f_b)F(w^f_b) + r^b_f(1 - F(w^f_b)) - (r^f - w^f_g)F(w^f_g) - r^b_h(1 - F(w^f_g))]$$

$$= (1 - \phi)(\delta(w^f_b) - \delta(w^f_g)) > 0.$$ Putting this together with $\phi(w^f_b - w^f_g) > 0$, we have $\frac{\partial \Pi}{\partial \psi} > 0$, and hence $\frac{d\Pi}{d\psi} > 0$. That is, the investor’s payoff on hiring a manager, $\Pi^p$, is strictly increasing in $\psi$.

(ii) Consider the effect of increasing $\psi$ on the total surplus. In state $h$, given the optimal contract, the manager takes the efficient action $a_h$, so in this state maximal surplus is realized regardless of the value of $\psi$. In state $\ell$, the contract induces a manager with a private benefit $m \in (w^f_b, r^f - r^f_b)$ to take the inefficient action $a_h$. When $\psi > \bar{\psi}_b$, an increase in $\psi$ has two effects. First, $w^f_b$ increases. All else equal, this reduces the demand for the risky asset when the state is $\ell$ and the rating is $b$. This leads to an increase in $r^f_b$ (the second effect). Both effects reduce the range of $m$ for which the manager takes the inefficient action. Therefore, the increase in $\psi$ strictly increases the total surplus $\Lambda$. 

\[\blacksquare\]
Proof of Proposition 4

An investor is indifferent between direct investing and hiring a manager if $B(c) = 0$, and strictly prefers to hire a manager if $B(c) > 0$. Consider the expression for $B(c)$ in equation (11). Suppose $c = 0$. It is immediate that $B(0) < 0$, as every term other than $c$ is negative. Next, suppose that $c = C$, the maximal value of the access cost. In the expression for $B(c)$, every term other than $c$ is finite. Therefore, if $C$ is large enough, it follows that $B(C)$ is strictly positive.

Now, suppose $\Psi$ is fixed. Then, $B(c)$ is strictly increasing in $c$. Therefore, it follows that there is a threshold $c^*$ such that the investor invests directly if $c < c^*$ and hires a manager if $c > c^*$.

Now, consider an increase in $\psi$. From Proposition 3, it follows that holding fixed the set of investors who hire a portfolio manager, the payoff to hiring a manager strictly improves. That is, if $w^f_d$ and $r^f_d$ are held fixed, then $c^*$ must fall. Of course, an increase in $\psi$, as argued earlier, leads to an increase in $w^f_b$ and a fall in $r^f_b$, and a weak decrease in $w^f_g$ and weak increase in $r^f_b$. By a similar argument to that made in part (i) of Proposition 3, we can show that $B(c)$ increases when $\psi$ improves. That is, $c^*$ increases. ■
References


