Instability of Centralized Markets*

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Abstract

Centralized markets reduce the costs of search for buyers and sellers. Their 'thickness' increases the chance of order execution at competitive prices. In spite of the incentives to consolidate, some markets, securities markets and online advertising, being the most notable, are fragmented into multiple trading venues. We argue that fragmentation is an inevitable feature of any centralized market except in certain special circumstances.[1]

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1 Introduction

A centralized market reduces the costs of clearing, settlement and search compared to one consisting of multiple trading venues. Were these costs to decline because of technological innovation, a centralized market should still dominate a fragmented market because traders would prefer the venue that offers the highest probability of order execution and the most competitive prices. Each additional trader on an exchange reduces the execution risk for other potential traders, attracting more traders. This positive feedback should encourage trade to be concentrated in a single exchange.

In spite of the incentives to consolidate, many markets have spawned multiple trading venues. Securities markets are the most well known example. Traditional stock exchanges face a host of competitors such as ECNSs (electronic communication networks), ATSs (alternative trading systems) and the trading desks of broker-dealer firms. These alternative trading venues are not restricted to non-standardized assets and have large trading volumes. Securities markets are not unique in this respect. In on-line advertising there are 5 major exchanges that are 'open' and numerous others that are 'private'. These venues use a variety of pricing rules, need not broadcast the bids they receive, and, in some cases allow traders to restrict who they will transact with.[2] Madhavan (2000) calls this the *network externality puzzle* and writes: "Despite strong arguments for consolidation, many markets are fragmented and remain so for long periods of time."

A variety of explanations (not entirely mutually exclusive), summarized below, have been offered for why centralized markets fragment.

1. Regulation: Fragmentation enhances efficiency because competition between exchanges forces them to narrow their bid-ask spreads (e.g., Pagano (1989); Biais, Martimort, and Rochet (2000)). Fragmentation in securities markets can be traced to regulation in the 80s and 90s designed to limit the abuse of market power by operators of centralized exchanges.[3] Fragmentation can also enhance efficiency (total welfare) by limiting the market power of participants (Malamud and Rostek (2014)).[4]

[2]For fragmentation in labor markets see Roth and Xing (1994).

[3]Regulation National Market System in the US and the Market in Financial Instruments Directive (MiFID) in Europe.

[4]Malamud and Rostek (2014) provide examples where a market fracture can increase the total welfare of market participants, however, the payoff of some agents may be lower post fracture. We

2. Heterogenous Preferences: Alternative trading venues arise to cater to traders who differ in their preferences for order size, anonymity and likelihood of execution (Harris (1993), Ambrus and Argenziano (2009) and Petrella (2009)).

3. Congestion: As a market becomes thicker, the time to select, evaluate, and process offers lengthens, during which time prices may change. This encourages participants to transact earlier, fragmenting the market in time (see Roth and Xing (1994)).[5]

4. Informational: Traders seek out alternative venues so as to conceal private information (see Madhavan (1995)), other venues spring up to attract uninformed traders from the incumbent exchange (Easley (1996)) or competing venues affect the incentives to acquire information (Glode and Opp (2016)).

We don't consider the reasons listed above to be fundamental, because they can all be eliminated, *in principle*, by a suitable (but possibly impractical) mechanism. In the first case, the operator could be mandated to implement the constrained efficient mechanism. In the remaining cases, a mechanism that allowed agents to use a richer message space to communicate preferences could be employed. In addition, the particular explanations given are tied to the institutional details of the setting in which the asset is being traded.

This paper argues that centralized markets are inherently unstable and this is the cause of fragmentation. Instability is caused by the violation of the price taking assumption. Within the model in which we make this point, the reasons for fragmentation just enumerated don't apply. Our setting is the standard model of bilateral trade (Myerson and Satterthwaite (1983)) where each seller has one unit of a homogeneous good and each buyer is interested in purchasing at most one unit of the same good. The private type of each buyer is their marginal value for the good and the private type of each seller is the opportunity cost of their endowment. Thus, agents are all interested in the same order size. Holding prices equal, they are indifferent about who they trade with. There is no common values component in the private information of agents making them equally informed (or uninformed). Trade takes place in a single time period, so the timing of trades is irrelevant. Our argument does

focus on the incentives for a group of agents to break off from the centralized market.

[5]Congestion can cause fragmented markets to persist as agents tradeoff thickness in one venue for less competition in another (Ellison and Fudenberg (2003), Ellison, Fudenberg and Mobius (2004)).

not rely on the institutional details of either securities or advertising, in this sense we are making a *universal* claim.

Trade within the incumbent exchange is modeled as being conducted via an individually rational, weakly budget balanced and incentive compatible mechanism. The first condition is true of almost all observed trading mechanisms. The second prevents the operator of the incumbent exchange from subsidizing trades. The third recognizes that trading agents will act strategically. Even if the mechanism in the incumbent exchange were not incentive compatible, by the revelation principle, there would be a corresponding incentive compatible direct mechanism that would replicate the outcome of the mechanism in the incumbent exchange. We are interested in when the incumbent mechanism is stable, in the sense that no subset of agents has an incentive to deviate and trade among themselves using a different mechanism, called the blocking mechanism. Our message is that budget balance and incentive compatibility conspire to make this difficult if not impossible. Given that individual rationality, incentive compatibility, budget balance and efficiency are incompatible, we interpret this to mean that it is the violation of the price taking assumption that makes an exchange vulnerable to fragmentation.

Formalizing the idea of blocking raises two conceptual difficulties. First, the decision to participate in the blocking mechanism reveals something about one's type which should be incorporated into the beliefs of potential counterparties. Second, payoffs in the incumbent mechanism will depend on the equilibrium played in that mechanism, which can be affected by the presence of a blocking mechanism. For this reason, we consider two related notions of blocking that depend on the equilibrium being "played" in the incumbent mechanism. The first, from Peivandi (2013), assumes that agents in the incumbent mechanism play a dominant strategy equilibrium. The second, new to this paper, assumes that the agents play a Bayesian equilibrium of the incumbent mechanism. We distinguish between them by calling the first D-blocking and the second B-blocking. They differ from prior notions of blocking used in the theory of cooperative games by allowing agents to condition their beliefs about counterparties based on which mechanism they are participating in. Roughly speaking, an incumbent mechanism is *blocked* by a coalition of agents and a blocking mechanism if the blocking mechanism gives to each member of the blocking coalition, for a critical subset of their types, at least as much surplus as they would obtain if they remained in the incumbent mechanism. Furthermore, no agent with a type outside of their critical subset of types will participate in the blocking mechanism. One feature of this notion of blocking differentiate it from prior notions (see Section 5) is that each agent recognizes that a decision by a counterparty to defect from the incumbent mechanism (or not) reveals some information about the counterparty's type, which should be used. We argue that the conditions under which a mechanism is immune to blocking are both restrictive and fragile. From this, we conclude that centralized markets are unstable.

We offer two sets of results. In the first, we restrict attention to deterministic mechanisms that are ex-post (weakly) budget balanced (EBB), features enjoyed by many observed trading rules. We do not specify a particular mechanism but consider all mechanisms that are robust to the beliefs of agents. Like Hagerty and Rogerson (1987) we model this by requiring the mechanism to be dominant strategy incentive compatible (DSIC). This is often touted as a desirable feature for mechanisms. Dominant strategy incentive compatibility does not exclude the possibility that the mechanism can depend on the designer's beliefs. For example, the designer could select a single price at which all trade must take place *a priori,* which depends on the designer's beliefs about the distribution of types of the agents. We show that for any EBB, and DSIC mechanism, there is a distribution over types, for which this mechanism can be "D-blocked" by another ex-post individually rational (EIR), EBB, and DSIC mechanism.

D-blocking is accomplished by a particularly simple mechanism called a positive spread posted price mechanism. In this mechanism, two prices $p_1 \leq p_2$ are posted. If buyer and seller agree to trade, the seller is paid p_1 and the buyer pays p_2. The spread of a posted price mechanism is $p_2 - p_1$, and this is what the designer pockets. Thus, every EBB, and DSIC mechanism can be D-blocked by a mechanism that gives the operator of the blocking mechanism positive expected profit. In fact, the blocking mechanism can be implemented with one of the agents making a take it or leave it offer to a subset of agents. Thus, blocking does not rely on the presence of another party who is more informed than the operator of the incumbent mechanism.

If we restrict attention to the case of just one buyer and seller, we provide a characterization of all EIR, EBB and DSIC mechanisms that are immune to D-blocking. It gives rise to an easily interpretable sufficient conditions for immunity to D-blocking. For example, if the distribution of buyer and seller types satisfies the monotone hazard rate condition, there is only one EIR, EBB, and DSIC mechanism immune to

D-blocking. It is a posted price mechanism: a price p is fixed a priori, and a pair of buyer and seller who wish to transact do so at price p.[6] However, any fixed price p will *not* suffice. It must lie between the optimal monopsony price set by the highest type buyer and the optimal monopoly price set by the lowest type seller. Therefore, the only EIR, EBB and DSIC mechanisms immune to D-blocking must be *sensitive* to the underlying distribution of types. If one desires a mechanism to be independent of the beliefs of the designer as well, then, no mechanism (within the class considered) is immune to D-blocking.

Our second result focuses on the double bid auction. With one buyer and seller, the price is set between the bid and ask (provided they cross). It is observed in practice, satisfies EIR and EBB, but is not DSIC. It is not even Bayesian incentive compatible (BIC). It does satisfy a different notion of robustness: the mechanism's rules do not depend on the beliefs of the agents or designer. We show that there is a Bayesian equilibrium of the double auction that cannot be B-blocked by any positive spread posted price mechanism. This shows that the rules of the mechanism alone do not determine its stability but the equilibrium played. The intuition carries through the case of many buyers and sellers. Some Bayesian equilibria of the double auction feature no trade for some agents and these non-trading agents may form a block. We also furnish an example of a constrained efficient equilibrium of a double auction that can be blocked. This shows that efficiency does not, in general, prevent instability.

In the next section of this paper we introduce notation and give a precise definition of D-blocking. Subsequently we contrast D-blocking with prior notions of the core of games with incomplete information. The subsequent section states and proves the main results concerning D-blocking. In section 4 we introduce B-blocking and its application to the double bid auction. Section 6 concludes.

2 D-blocking

Let $N = \{1, 2, 3, ..., n\}$ be the set of agents. The value of agent i for a unit of the good is $v_i \in V_i$ where $V_i \subset \mathbb{R}^+$ is bounded. Each v_i is the private information of agent $i \in N$ and is independently distributed. Each agent i has an endowment $\omega_i \in \{0, 1\}$ of the good, which is common knowledge. If $\omega_i = 1$, then agent i is a seller, and if

[6] One can interpret this as a reason for why posted price mechanisms are widely used in practice, see Einav et al. (2013).

$\omega_i = 0$, agent i is a buyer. Preferences are quasilinear; that is, buyer (seller) i's payoff from receiving (giving up) a quantity q of the good (interpret as probability) for a monetary payment (compensation) of t is $qv_i - t$ $(t - qv_i)$.

A direct mechanism is defined by an allocation rule and a payment rule. The allocation rule maps profiles of reports of the private information of agents to an allocation of the good. If Q is the allocation rule, denote the component of Q that corresponds to agent i's allocation by q_i. Thus, $q_i : \prod_{i \in N} V_i \to \mathbb{R}^+$. As agent i has an endowment of ω_i, we require that an allocation rule be feasible in the sense that for all $i \in N$ and all profiles $\mathbf{v} \in \prod_{i \in N} V_i$ that

1. $1 \geq q_i(\mathbf{v}) + \omega_{\mathbf{i}} \geq \mathbf{0}$, and,
2. $\sum_{i \in N} q_i(v) = 0$.

The payment rule maps each profile $\mathbf{v} \in \prod_{i \in N} V_i$ to a *per-unit* price each agent must pay. If P is the payment rule, the component of P that corresponds to agent i's per-unit payment is denoted p_i. Thus, $p_i : \prod_{i \in N} V_i \to \mathbb{R}^+$.

We now define dominant strategy incentive compatibility. Let $\mathbf{v} = (v_i, v_{-i})$ and $\hat{\mathbf{v}} = (\hat{v}_i, v_{-i})$ be two profiles of valuations in $\prod_{i \in N} V_i$. Observe that $\hat{\mathbf{v}}$ differs from $\mathbf{v}$ in that agent i only changes the report of his marginal value. Agents can misreport their marginal value or opportunity cost but not their role as buyer or seller. Note that we only need to impose incentive compatibility on deviations from profiles that result in feasible outcomes. The mechanism (Q, P) is DSIC if for all $\mathbf{v}$ and $\hat{\mathbf{v}}$:

$$q_i(\mathbf{v})(v_i - p_i(\mathbf{v})) \geq q_i(\hat{\mathbf{v}})(v_i - p_i(\hat{\mathbf{v}})).$$

Mechanism (Q, P) is ex-post individually rational if for all profiles $\mathbf{v} \in \prod_{i \in N} V_i$ and all $i \in N$

$$q_i(\mathbf{v})(v_i - p_i(\mathbf{v})) \geq 0.$$

Mechanism (Q, P) is (weakly) ex-post budget balanced if for all $\mathbf{v} \in \prod_{i \in N} V_i$

$$\sum_{i \in N} p_i(\mathbf{v}) q_i(\mathbf{v}) \geq 0.$$

In mechanism (Q, P), the utility that agent $i \in N$ under profile $\mathbf{v}$ enjoys is

$$u_i(\mathbf{v}, Q, P) = q_i(\mathbf{v})(v_i - p_i(\mathbf{v})).$$

The expected utility that agent $i \in N$ enjoys when her type is v_i is

$$E_{v_{-i}}[u_i(\{v_i, v_{-i}\}, Q, P)].$$

Now suppose an alternative feasible, DSIC, EIR mechanism $(\hat{Q}, \hat{P})$:

$$\hat{p}_i : \prod_{i \in A} V_i \to \mathbb{R}^+$$
$$\hat{q}_i : \prod_{i \in A} V_i \to \mathbb{R}^+ \ \forall i \in A$$

We will give a definition of what it means for $(\hat{Q}, \hat{P})$ to D-block the incumbent mechanism (Q, P) by a subset $A \subseteq N$ of the agents. Imagine that before participating in the mechanism (Q, P), each agent in A (and only A) is invited to participate in $(\hat{Q}, \hat{P})$. If at least one of the agents in A declines the invitation, all agents are required to participate in (Q, P); in this case we say the D-block is unsuccessful. If every agent in A accepts the invitation, this becomes common knowledge among them, and they enjoy the outcome delivered by $(\hat{Q}, \hat{P})$. The agents now face a Bayesian game in which they must first decide which of the two mechanisms to participate in and subsequently what to report in their chosen mechanism. As each mechanism is DSIC, we assume truthful reporting. We say the set A D-blocks (Q, P) if there is Bayesian equilibrium of the game, where with positive probability all agents in A choose $(\hat{Q}, \hat{P})$. Formally, we need for each $i \in A$, a positive measure subset $V'_i \subseteq V_i$ and an equilibrium where each $i \in A$ chooses $(\hat{Q}, \hat{P})$ if their type is in V'_i and (Q, P) otherwise. Call V'_i the **critical** set of types for agent i and for each $i \in A$ let T_i be the event that each agent $j \in A \setminus \{i\}$ has a type in V'_j. The set A D-blocks (Q, P) with respect to $\Pi_{i \in A} V'_i$ if the five conditions listed below hold.

1. If $v_i \in V'_i$, then,

$$E_{-i}[u_i(\{v_i, v_{-i}\}, Q, P)|T_i] \leq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i] \ \forall i \in A \tag{1}$$

2. If $v_i \notin V_i'$ then,

$$E_{-i}[u_i(\{v_i, v_{-i}\}, Q, P)|T_i] \geq E_{-i}[u_i(\{v_i, v_{A\setminus\{i\}}\}, \hat{Q}, \hat{P})|T_i] \ \forall i \in A. \tag{2}$$

3. For all $\bar{v} \in \prod_{i \in A} V_i'$

$$\sum_{i \in A} \hat{q}_i(\bar{\mathbf{v}}) = 0. \tag{3}$$

4.

$$\forall \bar{\mathbf{v}} \in \prod_{i \in A} V_i' \quad \sum_{i \in A} \hat{q}_i(\bar{\mathbf{v}}) \hat{p}_i(\bar{\mathbf{v}}) \geq 0 \tag{4}$$

5.

$$E[\sum_{i \in A} \hat{q}_i(\bar{\mathbf{v}}) \hat{p}_i(\bar{\mathbf{v}}) | \bar{\mathbf{v}} \in \prod_{i \in A} V_i'] > 0. \tag{5}$$

Condition (1) states that if each $i \in A$ has a type in V_i', then every agent in A choosing to participate in $(\hat{Q}, \hat{P})$ is a best response to the other agents in A doing so. Condition (2) states that if $i \in A$ is the only member with a type not in V_i', then choosing to participate in (Q, P) is a best response for agent i. Condition (3) ensures that the sum of the net trades is zero. Condition (4) states that the mechanism is weakly ex-post budget balanced. Condition (5) requires that, on some profile, the D-blocking mechanism generates a positive surplus. There is a technical and a substantive reason for this condition. The strict inequality means that there is a strict incentive for someone to offer the D-blocking mechanism. In prior notions of blocking, the analogue of inequality (1) holds strictly for some agent $i$to prevent a mechanism from blocking itself. We eliminate the possibility of an exactly budget balanced mechanism being D-blocked by itself by imposing a strict budget balanced condition on the blocking mechanism.

We have assumed that if any agent in A declines the invitation, all agents must participate in the incumbent mechanism. This makes D-blocking harder. To see why, suppose one buyer and one seller only. If any agent who accepts the invitation must trade in the alternative mechanism, there would be two pure strategy equilibria: one where both agents always choose the incumbent mechanism and one where both always choose the alternative. We also assumed that once the agents choose the blocking mechanism, and this becomes common knowledge, the choice is irrevocable.

This is not essential because the incumbent mechanism is DSIC. Allowing agents to return to the incumbent mechanism after observing the participants in the blocking mechanism does not alter subsequent results.

We have considered the Bayesian equilibrium for the agent's decision game rather than a dominant strategy equilibrium. This is because the agents unlike the incumbent mechanism designer, share a common prior about the distribution of types.

3 Vulnerability to D-blocking

To provide some intuition, we restrict attention to one buyer and seller. In this case, the set A of agents that could possibly D-block an incumbent mechanism will be the set of all agents. We focus on positive (or zero) spread posted price mechanisms for the reason stated below.

Observation 1. *In the case of one buyer and one seller, every deterministic, dominant strategy incentive compatible, (weakly) ex-post budget balanced, and ex-post individually rational mechanism that generates positive expected profits can be implemented as a positive spread posted price mechanism.*

The proof follows Hagerty and Rogerson (1987) and can be found in Kuzmic and Steg (2016). We give two sufficient conditions on the distribution of types such that the only EIR, EBB and DSIC mechanism immune to D-blocking is a posted price mechanism. One of these will follow from a characterization of mechanisms immune to D-blocking. As the characterization is hard to interpret we do not emphasize it.

3.1 Bilateral Trade

Consider a positive spread posted price mechanism. It is easy to see that such a mechanism can always be D-blocked by a positive spread posted price mechanism with a smaller spread. Thus, the only mechanisms (within the class considered) that might be immune to D-blocking are posted price mechanisms. But, what should the posted price be? Let agent 1 be the seller with an opportunity cost of $c \in [0, 1]$ and $\omega_1 = 1$ and agent 2 the buyer with a value of $v \in [0, 1]$ and $\omega_2 = 0$. Assume c and v are private information distributed independently with atomless density functions $g(c)$ and $f(v)$ respectively. Denote the corresponding cumulative distribution functions by G and F. Endowments are common knowledge.

Theorem 2. *If $x(1-F(x))$ and $(1-x)G(x)$ are concave and arg* $\max_{x\in[0,1]} x(1-F(x)) \geq \arg\max_{x\in[0,1]}(1-x)G(x)$*, then, any posted price mechanism with a price*

$$p \in [\arg\max_{x\in[0,1]}(1-x)G(x), \arg\max_{x\in[0,1]} x(1-F(x))]$$

is immune to D-blocking. The left-hand endpoint of this interval is the optimal posted price set by a buyer whose value is 1; the optimal monopsony price of the highest type buyer. The right-hand endpoint is the optimal monopoly price set by the lowest type seller.

Proof. Let $p \in [\arg\max_{x\in[0,1]}(1-x)G(x), \arg\max_{x\in[0,1]} x(1-F(x))]$ be the posted price of the incumbent mechanism. Consider a positive spread posted price mechanism (p', p'') with $p' < p''$ as a possible D-blocking mechanism. As there are only two agents (one buyer and one seller), the D-blocking coalition will consist of just these two agents. It remains to identify a critical set of types. We can do this by "reverse" engineering. There are three cases:

1. **Case 1:** $p' < p'' < p:$ A buyer with type $v \geq p''$ strictly prefers the D-blocking mechanism conditioned on a seller being present. Thus, the critical set of types of the buyer will be $[1, p'']$. Now, we find the critical set of types for the seller that would make them prefer the D-blocking mechanism. A seller with type $c < p'$ will join the D-blocking mechanism only if:

$$(p-c)Pr(v \geq p | v \geq p'') \leq (p'-c) \ \Rightarrow\ \frac{1-F(p)}{1-F(p'')} \leq \frac{p'-c}{p-c}.$$

The right-hand side is maximized at $c = 0$; therefore, the posted price p *cannot* be D-blocked by the positive spread posted price mechanism (p', p'') if the following holds:

$$\frac{1-F(p)}{1-F(p'')} > \frac{p'}{p}.$$

This is equivalent to $p(1-F(p)) > p'(1-F(p''))$. Therefore, if for all $p' < p$,

$$p(1-F(p)) > p'(1-F(p')), \tag{6}$$

the posted price mechanism cannot be blocked with prices lower than p. This is clearly true given the choice of p.

2. **Case 2:** $p < p' < p''$: In this case, the seller with opportunity cost $c < p'$ joins the D-blocking mechanism conditional on a buyer being present. A buyer with type $v > p''$ joins the D-blocking mechanism if

$$(v - p)Pr(c \leq p | c \leq p') \leq v - p''.$$

   As in Case 1 this does not happen if:

$$\forall\, p' > p\ (1 - G(p)) > (1 - p')G(p'). \tag{7}$$

3. **Case 3:** $p' < p < p''$: In this case no agent will join the D-blocking mechanism.

□

Our second sufficient condition is based on the hazard rates of the distribution of types and is based on a characterization of the EIR, EBB, and DSIC mechanisms immune to D-blocking. Recall that the hazard rate of the buyer is defined as $v - \frac{1-F(v)}{f(v)}$ while the hazard rate of the seller is defined as $c + \frac{G(c)}{g(c)}$.

Theorem 3. *Assume that the hazard rate of both buyer and seller are increasing. Suppose there exists $p \in [0, 1]$ such that $p - \frac{1-F(p)}{f(p)}$ and $1 - p - \frac{G(p)}{g(p)}$ are both non-positive. Then, a posted price mechanism with price p is immune to D-blocking by a positive spread posted price mechanism.*

Proof. Let $\mathcal{M}$ be any EBB and DSIC mechanism for the case of bilateral trade. Denote by $u_b(v, c)$ and $u_s(v, c)$ the buyer's and seller's payoff, respectively under $\mathcal{M}$. We first identify conditions under which $\mathcal{M}$ is immune to D-blocking by a positive spread posted price mechanism.

Lemma 4. *$\mathcal{M}$ is immune to D-blocking by a positive spread posted price mechanism if and only if for all $0 \leq y < x \leq 1$ the following holds:*

$$E[u_b(x, c)|c \leq y] + E[u_s(v, y)|v \geq x] \geq x - y. \tag{8}$$

If for some $0 \leq y < x \leq 1$ inequality (8) is violated, we construct a posted price blocking mechanism. Let $V_b = [x, 1]$ and $V_s = [0, y]$ be the critical set of types for buyer and the seller respectively. As inequality (8) is violated, there exists

$0 \leq p_1 < p_2 \leq 1$ such that the following holds:

$$E[u_b(x,c)|c \leq y] = x - p_2, \tag{9}$$

$$E[u_s(v,y)|v \geq x] = p_1 - y. \tag{10}$$

For a candidate D-blocking mechanism we choose the positive spread posted price mechanism with prices (p_1, p_2). This mechanism is clearly dominant strategy incentive compatible and budget balanced. We now verify that all types in the critical set weakly prefer the D-blocking mechanism to the mechanism $\mathcal{M}$.

Let $a(v,c)$ be the probability of trade in $\mathcal{M}$ when the the profile of types is (v,c). Recall, from Myerson and Satterthwaite (1983) that $u_b(\alpha,\beta) = \int_0^\alpha a(t,\beta)dt$ and $u_s(\alpha,\beta) = \int_\beta^1 a(\alpha,t)dt$. Therefore, for all $1 \geq v' \geq x$ and $y \geq c' \geq 0$ the following holds:

$$\begin{aligned} E[u_b(v',c)|c \leq y] &= E[u_b(x,c)|c \leq y] + \int_x^{v'} E[a(v,c)|c \leq y]dv \\ &\leq E[u_b(x,c)|c \leq y] + (v' - x) = v' - p_2, \end{aligned} \tag{11}$$

$$\begin{aligned} E[u_s(v,c')|v \geq x] &= E[u_s(v,y)|v \geq x] + \int_{c'}^{y} E[a(v,c)|v \geq x]dc \\ &\leq E[u_s(v,y)|v \geq x] + (y - c') = p_1 - c'. \end{aligned} \tag{12}$$

Equations (11) and (12) ensure that all types in the critical set weakly prefer the D-blocking mechanism to $\mathcal{M}$. It is straightforward to check that when an agent's type is outside the critical set, this agent does not prefer the blocking mechanism to $\mathcal{M}$.

To prove the reverse we show that if there is a positive spread posted price D-blocking mechanism, inequality (8) is violated for some $0 \leq y < x \leq 1$. Let $0 \leq p_1 < p_2 \leq 1$ be the prices in the D-blocking mechanism and V_b and V_s be the associated critical set of types. As the sets V_b and V_s have positive measure, there exists $x \geq p_2$ and $y \leq p_1$ such that $x \in V_b$ and $y \in V_s$. For all such x, y the following must hold:

$$E[u_b(x,c)|c \in V_s] \leq E[(x - p_2)I_{\{c \leq p_1\}}|c \in V_s]. \tag{13}$$

The left-hand side of (13) is the expected payoff to the buyer when she participates in $\mathcal{M}$ knowing that the seller has a type in the critical set V_s. The right-hand side is the expected payoff to the buyer when she chooses to participate in the D-blocking mechanism conditional on the seller's type being in the critical set and the seller participating in the D-blocking mechanism. A similar observation yields:

$$E[u_s(v,y)|v \in V_b] \leq E[(p_1 - y)I_{\{v \geq p_2\}}|v \in V_b]. \tag{14}$$

Thus, rewriting inequality (13) yields:

$$\begin{aligned}
&\frac{\int_{c \in V_s} u_b(x,c)g(c)dc}{Pr(c \in V_s)} \leq \frac{(x-p_2)Pr(V_s \cap [0,p_1])}{Pr(c \in V_s)} \\
\iff &\frac{\int_{c \in V_s} u_b(x,c)g(c)dc}{Pr(V_s \cap [0,p_1])} \leq x - p_2 \\
\iff &\frac{\int_{c \in V_s \cap [0,p_1]} u_b(x,c)g(c)dc}{Pr(V_s \cap [0,p_1])} \leq x - p_2 \\
\iff &E[u_b(x,c)|c \in V_s \cap [0,p_1]] \leq x - p_2
\end{aligned} \tag{15}$$

Similarly, the following inequality holds:

$$E[u_s(v,y)|v \in V_b \cap [p_2,1]] \leq p_1 - y. \tag{16}$$

Inequalities (15) and (16) allow us to assume $V_b \subseteq [p_2,1]$ and $V_s \subseteq [0,p_1]$. Let $x^* = \inf V_b$ and $y^* = \sup V_s$. As the distribution of types is atomless, we may assume $x^* \in V_b$ and $y^* \in V_s$. The following inequalities hold:

$$E[u_b(x^*,c)|c \in V_s] \leq x^* - p_2, \tag{17}$$

$$E[u_s(v,y^*)|v \in V_b] \leq p_1 - y^*. \tag{18}$$

Note that the payoff to a seller with type $c \in V_s \cap [p_1,1]$ is zero in the D-blocking mechanism. Therefore, if a seller has type in $c \in V_s \cap [p_1,1]$, it must receive a payoff of zero in $\mathcal{M}$, i.e., almost surely $\forall v \in V_b$ $u_s(v,c) = 0$.This is similar to a buyer whose type is in $V_b \cap [p_2,1]$. If $a(x^*,y)$ is constant for all $y \leq y^*$, then $u_b(x^*,c) = u_b(x^*,c')$ for any two $c,c' \in V_s$. It follows from (17) that $u_b(x^*,c) = x^* - p_2$ for all $c \leq y^*$. Hence

$$E[u_b(x^*,c)|c \leq y^*] \leq x^* - p_2, \tag{19}$$

Similarly, if $a(x, y^*)$ is constant for all $x \geq x^*$ we deduce that

$$E[u_s(v, y^*)|v \geq x^*] \leq p_1 - y^*. \tag{20}$$

Thus, if $a(x, y)$ is constant in the relevant ranges, the proof is complete. Suppose, for a contradiction, this is not true. Consider the case $x > x^*$ (a similar argument applies when $y < y^*$). For all $x > x^*$ the following holds:

$$\begin{aligned} E[u_b(x, c)|c \in V_c] &= E[u_b(x^*, c)|c \in V_c] + \int_{x^*}^{x} E[a(s, c)|c \in V_c]ds \\ &\leq (x^* - p_2) + (x - x^*) = x - p_2 \end{aligned} \tag{21}$$

If inequality (21) holds with equality for any $\bar{x} > x^*$, it must be the case that for all $x > x^*$ and almost all $c \in V_c$, $a(x, c) = 1$. To see why, note that equality for $x = \bar{x}$ implies that $a(x, c) = 1$ for all $x^* < x \leq \bar{x}$. However, $a(\cdot, c)$ is monotone in its first component by dominant strategy incentive compatibility. Therefore, $a(x, c) = 1$ for all $x > x^*$. This means that $a(x, c)$ is constant and (19) applies.

Suppose then that inequality (21) is strict for all $x > x^*$. Therefore, $E[u_b(x, c)|c \in V_c] < x - p_2$ for all $x > x^*$. Hence, $x \in V_b$ for all $x > x^*$. A similar argument shows that $y \in V_s$ for all $y < y^*$. This proves the lemma.

Consider a posted price mechanism that selects a price according to density $h(p)$. Lemma 8 implies that this mechanism is D-blocked by a positive spread posted price mechanism if for all $1 \geq x > y \geq 0$ the following holds:

$$\begin{aligned} &\int_y^x (x - p)h(p)dp + \frac{\int_0^y \int_0^p (x - p)g(c)h(p)dcdp}{G(y)} \\ &+ \int_y^x (p - y)h(p)dp + \frac{\int_x^1 \int_p^1 (p - y)h(p)f(v)dvdp}{1 - F(x)} \\ &\geq x - y \end{aligned} \tag{22}$$

The right-hand side of inequality (22) can be rewritten as follows:

$$(x - y)(H(x) - H(y)) + \int_x^1 (p - y)\frac{1 - F(p)}{1 - F(x)}h(p)dp + \int_0^y (x - p)\frac{G(p)}{G(y)}h(p)dp$$

Using integration by parts inequality (22) can be written as follows:

$$\int_0^y \int_x^1 H(v)(v - \frac{1-F(v)}{f(v)} - y)f(v)g(c)dvdc +$$
$$\int_0^y \int_x^1 H(c)(c + \frac{G(c)}{g(c)} - x)f(v)g(c)dvdc \geq (x-y)G(y)(1-F(x)) \quad (23)$$

Inequality (23) provides a necessary and sufficient condition for immunity of a trade mechanism to D-blocking by a positive spread posted price mechanism.

To prove this, consider a randomized posted price mechanism that randomizes only over prices for which both hazard rates are negative. Note that if $H(v) = 1$ for all $v \geq x$ and $H(v) = 0$ for all $v \leq y$, then inequality (23) holds with equality. Such a randomized posted price mechanism sets $H(v) < 1$ in the first part of the integral only if $v - \frac{1-F(v)}{f(v)} \leq 0$ and it sets $H(v) > 0$ in the second integral only if $\frac{G(c)}{g(c)} - 1 \geq 0$. Note that

$$v - \frac{1-F(v)}{f(v)} \leq 0 \Rightarrow v - \frac{1-F(v)}{f(v)} - y \leq 0 \text{ and } \frac{G(c)}{g(c)} - 1 \geq 0 \Rightarrow \frac{G(c)}{g(c)} - x \geq 0.$$

Therefore, inequality (23) holds for this mechanism. This proves the theorem. □

3.2 The General Case

We now allow for more than one buyer and seller.

Theorem 5. *Fix a EIR, EBB and DSIC mechanism that is robust to the beliefs of the designer. For this mechanism there is an atomless distribution over types under which the mechanism can be D-blocked by a group of agents.*

Proof. Suppose the mechanism cannot be D-blocked under any atomless distribution over types. We show that such a mechanism must be ex-post efficient. The theorem follows from the fact that such a mechanism does not exist. Let $I \subset N$ be the set of sellers and $J \subset N$ be the set of buyers. Consider a profile of valuations $x = (x_I, x_J) \in \prod_{i \in N} V_i$. Let $I' \subseteq I$ and $J' \subseteq J$ be the subset of the sellers and buyer that should trade in an efficient allocation. Note that $|I'| = |J'|$. Let W_i be the event that the types of the sellers in $I' \setminus \{i\}$ are below the $x_{I' \setminus i}$ and the type of buyers in

$J' \setminus i$ are below $x_{J' \setminus i}$s for all agents in $I' \cup J'$. Formally,

$$W_i = \{v \in \prod_{i \in N} V_i | \forall k \in I' \setminus \{i\} \; v_k \leq x_k \text{ and } \forall k \in J' \setminus \{i\} \; v_k \geq x_k\}.$$

If the following inequality is violated one can construct a D-blocking mechanism as in the proof of theorem (3).

$$\sum_{i \in I' \cup J'} E[u_i(x_i, v_{-i}) | W_i] \geq \sum_{k \in J'} x_k - \sum_{k \in I'} x_k. \tag{24}$$

Inequality (24) must hold for all possible atomless distributions. Consider a sequence of the atomless distributions that converge to the distribution that puts probability one on the event that the type profile is x. Therefore, the following must hold:

$$\sum_{i \in I' \cup J'} E[u_i(x) | W_i] \geq \sum_{k \in J'} x_k - \sum_{k \in I'} x_k. \tag{25}$$

Inequality (25) implies that the mechanism must be efficient. $\square$

If we have the same number of buyers and sellers and choose the posted price p so that $1 - F(p) = G(p)$, as the number of agents increases we converge to the Walrasian outcome. As the Walrasian outcome is in the core, this appears to contradict Theorem 5. It does not. As the number of agents increases, the expected profit of the blocking mechanism will decrease but still be positive. It is only in the continuum limit that the expected profit of a blocking mechanism falls to zero. We interpret this to mean that D-blocking can only take place if the price-taking assumption is violated.

4 Non-DSIC Mechanisms

In this section we examine two different approaches to stability when the incumbent mechanism is not DSIC. The first mimics D-blocking and is almost the same as the credible core of Dutta and Vohra (2005).

4.1 C-blocking

The notion of blocking mimics D-blocking. Call it C-blocking. First, select a Bayesian equilibrium of the incumbent mechanism, call it the chosen equilibrium. If a block fails, the agents in the putative blocking coalition A are assumed to play the chosen equilibrium of the incumbent mechanism. This is analogous to the definition of D-blocking where the agents in the putative blocking coalition continue to play the initial truthful equilibrium of the incumbent mechanism. It differs slightly from blocking in the credible core in that we continue to assume that the blocking mechanism be DSIC. In the credible core, the blocking mechanism is BIC.

Denote the incumbent mechanism, not necessarily direct, by (M, P, Q) where $M = \prod_{i \in N} M_i$ is the message space. Denote the putative blocking mechanism by $(\hat{P}, \hat{Q})$. The payoff of agent i with type v_i when all agents send message profile m is denoted $u_i(v_i, \{m\}, Q, P)$. We use the notation $(m_i(v_i))_{i \in N}$ for the chosen Bayesian Equilibrium of the incumbent mechanism. As in the case of D-blocking, before participating in the incumbent mechanism (Q, P), each agent in the putative blocking coalition A (and only A) is invited to participate in $(\hat{Q}, \hat{P})$. If at least one of the agents in A declines the invitation, all agents are required to participate in (Q, P); in this case the block fails. The definition of C-blocking is as follows:

The agents in A and a mechanism $(\hat{P}, \hat{Q})$ block the incumbent mechanism with equilibrium $(m_i(.))_{i \in N}$ if there exists a non-zero measure subset of types $(V_i')_{i \in A}$ (the critical subset) such that the following inequalities hold,

1. For all $i \in A$, if $v_i \in V_i'$, then,

$$E_{-i}[u_i(v_i, \{m_i(v_i), v_{-i})\}, Q, P)|T_i] \leq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i] \quad (26)$$

$$T_i = \{(v_k)_{k \in A} | v_k \in V_k', \ \forall k \in A \setminus \{i\}\} \quad (27)$$

2. For all $i \in A$, if $v_i \notin V_i'$ then,

$$E_{-i}[u_i(v_i, \{m_i(v_i), v_{-i}\}, Q, P)|T_i] \geq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i]. \quad (28)$$

3. For all $\bar{v} \in \prod_{i \in A} V_i'$,

$$\sum_{i \in A} \hat{q}_i(\bar{v}) = 0. \quad (29)$$

4.

$$E[\sum_{i \in A} \hat{q}_i(\bar{v}) \hat{p}_i(\bar{v}) | \bar{v} \in \prod_{i \in A} V_i'] > 0. \quad (30)$$

A mechanism is immune to C-blocking if it has a Bayesian equilibrium for which no blocking occurs.

A well studied and widely used instance of a non-DSIC mechanism is the buyer's bid double auction. With m buyers and m sellers, buyers and sellers submit bids and asks, and the mechanism designer sorts the bids and asks in an increasing order, the price is the $m+1$'th number in the order, sellers with an opportunity cost strictly less than the price and buyers with a valuation greater or equal to the price trade. Satterthwaite and Williams (1989) prove that the buyer's bid double auction has an equilibrium in which the seller submits his opportunity cost and the buyer with valuation v submits bid $b(v)$, where $b(v)$ is increasing and continuous in v.

Theorem 6. *Assume there is one buyer and one seller and that types are drawn independently from an atomless distribution with continuous probability density function. Then, the Bayesian Nash Equilibrium of the buyer's bid double auction described in Satterthwaite and Williams (1989) is immune to C-blocking by positive-spread posted price mechanisms.*

Proof. We show that this equilibrium cannot be blocked by any positive-spread posted price mechanism. Assume, on the contrary, that a block exists. A similar argument as in the proof of theorem (8) shows that the critical sets should be $V_b' = [x, 1]$ and $V_s' = [0, y]$, for the buyer and the seller respectively, for some $1 \geq x > y \geq 0$. Given seller's opportunity cost c and buyer's valuation v and buyer's bid b, $u_b(v, \{b, c\}, Q, P))$ and $u_s(c, \{b, c\}, Q, P)$ denote the buyer's and seller's payoff from the double auction, respectively. We show the following inequality holds, preventing a block from forming.

$$E_s[u_b(x, \{b(x), c\}, Q, P)|c \in [0, y]] + E_v[u_s(y, \{b(v), y\}, Q, P)|v \in [x, 1]] \geq x - y. \quad (31)$$

We show inequality (31) is violated. We consider two cases:

1. $b(x) \geq y$**:**
   Note that $u_b(x, \{b(x), c\}, Q, P))$ is decreasing in c and $u_s(y, \{b(v), y\}, Q, P)$ is increasing in v, therefore,

$$E_c[u_b(x, \{b(x), c\}, Q, P)|c \in [0, y]] + E_v[u_s(y, \{b(v), y\}, Q, P)|v \in [x, 1]] \geq$$
$$u_b(x, \{b(x), y\}, Q, P) + u_s(y, \{b(x), y\}, Q, P).$$

Buyer with type x trades with seller with type y with price $b(x)$, therefore,

$$u_b(x, \{b(x), y\}, Q, P) + u_s(y, \{b(x), y\}, Q, P) = x - y.$$

2. $b(x) < y$:
Note that the buyer with valuation x does not trade with seller whose opportunity cost exceeds y. Therefore,

$$E_c[u_b(x, \{b(x), s\}, Q, P)|c \in [0, y]] = \frac{E_c[u_b(x, \{b(x), c\}, Q, P)]}{Pr(c \in [0, y])}. \tag{32}$$

Note that a bid equal to $b(x)$ maximizes the surplus of a buyer with valuation x. In particular, the buyer weakly prefers to bid $b(x)$ instead of y; therefore,

$$E_c[u_b(x, \{b(x), s\}, Q, P)] \geq (x - y)Pr(c \in [0, y]). \tag{33}$$

Equations (32) and (33) imply:

$$E_c[u_b(x, \{b(x), s\}, Q, P)|s \in [0, y]] \geq x - y. \tag{34}$$

$\square$

Another class of well studied double bid auctions are the mid-point double auctions defined as follows: Assume there are n buyers and m sellers. Each buyer i reports bid, b_i, and each seller j reports an ask, c_j. Bids and asks are positive real numbers. Index the agents so that $b_1 \geq b_2 \geq b_3 \geq ... \geq b_n$ and $c_1 \leq c_2 \leq c_3 \leq ... \leq c_m$. Let k be the largest index where $b_k \geq c_k$. All buyers with bids larger or equal to b_k and all sellers with asks smaller or equal to c_k trade at the price of $\frac{b_k + c_k}{2}$. Thus, the price is allowed to depend on the profile of reported bids. If the number of trading buyers (sellers) is more than the number of trading sellers (buyers), then sellers (buyers) must be rationed.

Assume one buyer and one seller with types selected from the uniform distribution over $[0, 1]$. Consider the double bid auction that selects the mid-point between the bid and the ask. There is a Bayesian equilibrium of this double bid auction that is constrained efficient. The buyer with valuation v bids $b(v) = \frac{2}{3}v + \frac{1}{12}$ and the seller with opportunity cost c asks $a(c) = \frac{2}{3}c + \frac{1}{4}$.

Proposition 7. *When there is one buyer and one seller, the constrained efficient equilibrium of the mid-point double auction can be C-blocked by a randomized positive spread posted price mechanism.*

Proof. We design a randomized positive spread posted price mechanism with prices $\frac{1}{2}-\epsilon$ and $\frac{1}{2}+\epsilon$, and probability of trade π. Set of types who participate in the blocking mechanism is $[\frac{1}{2}+\epsilon, \frac{5}{8}+\delta]$ and $[\frac{3}{8}-\delta, \frac{1}{2}-\epsilon]$. For the B-blocking to work, we find ϵ, δ and π such that the following holds:

1. For $v \in [\frac{1}{2}+\epsilon, \frac{5}{8}+\delta]$,
$$E[u_b(v,c)|c \in [\frac{3}{8}-\delta, \frac{1}{2}-\epsilon]] \leq \pi(v - \frac{1}{2} - \epsilon)].$$

2. For $v \notin [\frac{1}{2}+\epsilon, \frac{5}{8}+\delta]$,
$$E[u_b(v,c)|c \in [\frac{3}{8}-\delta, \frac{1}{2}-\epsilon]] \geq \pi(v - \frac{1}{2} - \epsilon)].$$

3. For $c \in [\frac{3}{8}-\delta, \frac{1}{2}-\epsilon]$,
$$E[u_s(v,c)|v \in [\frac{1}{2}+\epsilon, \frac{5}{8}+\delta]] \leq \pi(\frac{1}{2} - \epsilon - c)],$$

4. For $c \notin [\frac{3}{8}-\delta, \frac{1}{2}-\epsilon]$,
$$E[u_s(v,c)|v \in [\frac{1}{2}+\epsilon, \frac{5}{8}+\delta]] \leq \pi(\frac{1}{2} - \epsilon - c)],$$

It is easy to check that $\epsilon = \delta = \frac{1}{32}$ and $\pi = \frac{7}{12}$ satisfy the above conditions above.

□

This example shows that constrained efficiency is not an antidote to instability.

4.2 B-Blocking

When a block fails under C-blocking , the agents are assumed to play the chosen equilibrium of the incumbent mechanism. However, failure of the block reveals information to members of the putative blocking coalition that will change their beliefs about the types of the other agents. We assert that this should change the equilibrium played in the incumbent mechanism. Furthermore, the precise equilibrium will depend on what information is revealed to which agent. Here we explore one such possibility.

As before, if at least one of the agents in the putative blocking coalition A declines the invitation, all agents must participate in (Q, P); in this case the block fails. If every agent in A accepts the invitation, this selection is revealed to all agents. At this point, each agent in A is given the option of returning to (Q, P). If any one of them chooses to return, all agents in A must return. If all agents in A elect not to return, they enjoy the outcome delivered by $(\hat{Q}, \hat{P})$. This return feature serves to make the event that all agent's types are in their respective critical sets common knowledge. Therefore, any Bayesian equilibrium of the incumbent mechanism should be consistent with this updated belief and hence, must satisfy,

$$\forall i \in N \text{ and } v_i \in V_i \;\; m_i(v_i) = argmax_{m_i \in M_i} E[u_i(v_i, \{m_i, m_{-i}(v_{-i})\}, Q, P)|T_i].$$

The agents in A and a mechanism $(\hat{P}, \hat{Q})$ B-block the incumbent mechanism for a non-zero measure subset of types $(V_i')_{i \in A}$ (the critical set) and consistent Bayesian equilibrium of the incumbent mechanism, $m_i(v_i)$ if the following conditions are satisfied:

1. For al $i \in A$, if $v_i \in V_i'$, then,

$$E_{-i}[u_i(v_i, \{m_i(v_i), m_i(v_{-i}))\}, Q, P)|T_i] \tag{35}$$
$$\leq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i] \tag{36}$$
$$T_i = \{(v_k)_{k \in A} | v_k \in V_k', \forall k \in A \setminus \{i\}\} \tag{37}$$

2. For al $i \in A$, if $v_i \notin V_i'$ then,

$$\max_{m_i \in M_i} E_{-i}[u_i(v_i, \{m_i, m_i(v_{-i})\}, Q, P)|T_i] \geq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i] \;\forall i \in A. \tag{38}$$

3. For all $\bar{v} \in \prod_{i \in A} V_i'$,

$$\sum_{i \in A} \hat{q}_i(\bar{v}) = 0. \tag{39}$$

4.
$$E[\sum_{i\in A} \hat{q}_i(\bar{v})\hat{p}_i(\bar{v})|\bar{v} \in \prod_{i\in A} V_i'] > 0. \tag{40}$$

We can interpret these conditions in terms of the following "exit" game: agents in A simultaneously decide to join $(\hat{P}, \hat{Q})$ or not. Equations (35) and (38) imply that the "exit" game has a Bayesian equilibrium where agents in A choose the exit option if their types are in their respective critical set of types. Equation (39) is the market clearing condition. Finally, (40) requires that that the blocking mechanism generate positive expected surplus (conditional on types being in the critical set). The difference between B-blocking (with return) and D-blocking is that agents may report messages different from those reported when there was no alternative mechanism. When an agent with a type in the critical set learns that the block has failed, she or he has no choice about which mechanism to participate in, therefore what strategy is adopted in the incumbent mechanism is moot.

We extend the notion of B-blocking to allow the blocking mechanism to execute trades even when a strict subset of the blocking coalition participate in the blocking mechanism. Assume all agents have a null message such that when a buyer (seller) sends that message, the incumbent mechanism does not give (get) the object to (from) that buyer (seller). When agents participate in a blocking mechanism we assume a null message is sent to the incumbent mechanism. Denote the null message by $\emptyset$. We permit the blocking mechanism to execute trades between the subset of the blocking coalition that participate in. Let A be the set of agents who are invited to block the incumbent mechanism with blocking mechanism $\hat{\mathcal{M}}$. For each agent $i \in A$ and subset $B \subseteq A$, V_i^B denotes the set of types who participate in the blocking mechanism when i observes the set B of agents has participated in the blocking mechanism. Denote by $(m_i^B(v_i))_{i\in N}$ the equilibrium of the incumbent mechanism when the type of agent $i \in B$ are in their respective critical sets and types of agents in $A \setminus B$ are not. Set $u_i(\{v_i, v_{B\setminus\{i\}}\}, \hat{\mathcal{M}}_B)$ to be the payoff of agent i from the blocking mechanism when agents in B participate in the blocking mechanism. For a block to take place the following must hold for all $i \in A$ and sets B that satisfy $i \in B \subseteq A$:

1. For al $i \in A$, if $v_i \in V_i'$, then,

$$E_{-i}[u_i(v_i, \{m_i^B(v_i), m_{-i}^B(v_{-i})\}, \hat{\mathcal{M}}_B)|T_i^B] \tag{41}$$
$$\leq E_{-i}[u_i(\{v_i, v_{A\setminus\{i\}}\}, \hat{\mathcal{M}}_B)|T_i^B] \tag{42}$$
$$T_i^B = \{(v_k)_{k\in A}|v_k \in V_k^B,\ \forall k \in B \setminus \{i\}, v_k \notin V_k^B,\ \forall k \in A \setminus B\} \tag{43}$$

2. For al $i \in A$, if $v_i \notin V_i'$ then,

$$\max_{m_i \in M_i} E_{-i}[u_i(v_i, \{m_i, m_{-i}^B(v_{-i})\}, \hat{\mathcal{M}}_B)|T_i^B] \geq E_{-i}[u_i(\{v_i, v_{A\setminus\{i\}}\}, \hat{\mathcal{M}}_B)|T_i^B]. \tag{44}$$

3. For all $\bar{v} \in \prod_{i\in B} V_i'$,

$$\sum_{i\in B} \hat{q}_i(\bar{v}) = 0. \tag{45}$$

4.

$$E[\sum_{i\in A} \hat{q}_i(\bar{v})\hat{p}_i(\bar{v})|\bar{v} \in \prod_{i\in A} V_i'] > 0. \tag{46}$$

Consider now any member of the class of double auctions. For clarity, it may be helpful to focus on the midpoint double auction, but our results below hold for all members of the class of double auctions.

Theorem 8. *Assume there is one buyer and one seller, then for every positive-spread posted price mechanism, there exists a consistent equilibrium for the midpoint double auction and subset of types such that the midpoint double auction is immune to B-blocking by that positive-spread posted price mechanisms.*

Proof. Consider a potential positive-spread posted-price mechanism with prices p and p' such that $p < p'$. Suppose, for a contradiction, that it B-blocks the double auction. We show the set of buyer types who visit the blocking mechanism is $[x, 1]$ and the set of seller types is $[0, y]$, for some x and y such that $p' < x$ and $y < p$. Set V_b' and V_s' to be the type of agents. Let $x = \inf\{v|v \in V_b'\}$ and $y = \sup\{s|s \in V_s'\}$. If $V_b' \neq [x, 1]$, there exists $x' > x$ such that $x' \notin V_b'$. In that case, the following inequalities must hold:

$$E_s[u_i(x, \{m_b(x), m_s(s)\}, Q, P)|s \in V_s'] \leq x - p', \tag{47}$$
$$E_s[u_i(x', \{m_b(x'), m_s(s)\}, Q, P)|s \in V_s'] > x' - p. \tag{48}$$

Inequalities (47) and (48) imply:

$$E_s[u_i(x', \{m_b(x'), m_s(s)\}, Q, P)|s \in V_s'] - \tag{49}$$
$$E_s[u_i(x, \{m_b(x), m_s(s)\}, Q, P)|s \in V_s'] > x' - x. \tag{50}$$

Set $a(v)$ to be the probability that buyer with type v trades when the seller's type is in V_s'. The right-hand side of inequality (49) is equal to $\int_x^{x'} a(v)dv$. Note that since $a(v) \leq 1$, $\int_x^{x'} a(v)dv \leq x' - x$ which contradicts with (49). The contradiction proves $V_b' = [x, 1]$, similar proof shows $V_s' = [0, y]$.

Consider a Bayesian equilibrium where the buyer and the seller both report the same bid p'', such that $p < p'' < p'$. Note that this equilibrium is consistent, however, equation (35) is violated.

□

Theorem (8) and (6) show that despite Theorem (5), some equilibria of the double auction are immune to B-blocking by positive-spread posted price mechanisms. The incumbent mechanism is not immune to blocking if agents coordinate on the wrong equilibrium. For example, if the no trade equilibrium is played, all agents prefer to participate in the blocking mechanism. Thus, immunity to B-blocking depends on the equilibrium selected in the incumbent mechanism. This means that the rules of trade alone do not suffice to tells us if an incumbent mechanism is stable or not.

5 Prior Notions of Blocking

Immunity to D-blocking (or B-blocking) can be interpreted as a notion of the core of a cooperative game of incomplete information. Forges, Minelli, and Vohra (2002) provides a brief survey of various notions of the core for cooperative games of incomplete information. They differ on two dimensions. First, is the decision to block made at the ex-ante or interim stage? Second, are incentive constraints relevant? In our case, the decision to block is made at the interim stage and incentive constraints are certainly relevant. For this reason, we don't discuss either the ex-ante core or the coarse core.[7] The corresponding incentive versions of these core concepts and their drawbacks are summarized in Dutta and Vohra (2005). In response to these

[7]The notion of durable decision rules due to Holmstrom and Myerson (1983) is concerned with blocking by the grand coalition only.

drawbacks, Dutta and Vohra (2005) propose the credible core and this is the notion most relevant to this paper. We first contrast the credible core with the notion of D-blocking.

In the credible core, the incumbent mechanism can be any BIC mechanism, not just DSIC. By restricting ourselves to DSIC mechanisms we avoid a problem not addressed in Dutta and Vohra (2005). Specifically, Bayesian incentive compatibility is a function of the mechanism and the beliefs of the agents. When a block fails and this event becomes known to the agents in the putative blocking coalition A, it changes their beliefs about the types. Truthful reporting in the incumbent mechanism need not be an equilibrium anymore. This issue does not arise when the incumbent mechanism is DSIC. The second difference is that in Dutta and Vohra (2005) the alternative mechanism is only BIC assuming types lie in their respective critical set. It is enforced by barring participation by types not in the critical set. This is accomplished by choosing 'no-trade' in the event than an agent in A reports a type outside their critical set. In our case the alternative mechanism does not rely on such restrictions because it is DSIC.

We now discuss the differences between the credible core and B-blocking. In the credible core, agents do not update their beliefs when a block fails. B-blocking, however, assumes that the event that a block fails becomes common knowledge. Agents then play a Bayes equilibrium in the incumbent mechanism consistent with this common knowledge event. Therefore, agents in the putative blocking coalition compare the payoff from the blocking mechanism with the payoff from a consistent equilibrium of the incumbent mechanism. We also, allow for the possibility that trades will be executed in the alternative mechanism even if if a strict subset of the putative blocking coalition show up. In this case all agents update their beliefs according to the participation decision of agents in the putative blocking coalition.

Dutta and Vohra (2005) is not the last word on the subject. We briefly summarize subsequent contributions highlighting differences. Myerson (2007), using the virtual utility construct, proposes a blocking notion that, in addition to the credibility requirements, considers random coalition formation and random allocations for each coalition. Serrano and Vohra (2007) use coalitional voting in an incomplete information environment to incorporate endogenous information transmission among members of a coalition. Finally, Liu et al. (2014) study the implications of common knowledge of stability of a two-sided match when one side of the market has incomplete information about the other side. The literature on competing mechanisms (common agency) has also focused on how agents will choose between two alternative mechanisms. However, the setting is one-sided in that agents are all buyers (or all workers), and they are choosing between alternative mechanisms in which to purchase something. The need for budget balance, for example, is absent. See Peters (2014) for a survey.

6 Conclusion

We have shown that only when a posted price is tailored to the distribution of types and those distributions are well-behaved it is immune to D-blocks by a positive spread posted price mechanism. When the type distributions are unknown, there is no mechanism that is robust against D-blocks by positive spread posted price mechanisms. When we consider B-blocking, we show that double auctions are immune to B-blocks by a positive spread posted price mechanism, however, the immunity is present only if agents play the "right" Bayesian Equilibrium in the incumbent mechanism. Our analysis shows that when the price taking assumption is violated, the conditions under which a mechanism is immune to blocking are both restrictive and fragile. For this reason we argue that centralized markets are unstable.

References

[1] Ambrus, Attila and Rossella Argenziano (2009). "Asymmetric Networks in Two-sided Markets," **American Economic Journal: Microeconomics**, 1(1), 17-52.

[2] Biais, Bruno; David Martimort and Jean-Charles Rochet (2000). "Competing Mechanisms in a Common Value Environment," **Econometrica**, 68 (4), 799-837.

[3] Dutta, Bhaskar and Rajiv Vohra (2005). "Incomplete Information, Credibility and the Core," **Mathematical Social Sciences,** 50 (2), 148-165.

[4] Ellison, Glenn, and Drew Fudenberg (2003). "Knife-Edge or Plateau: When do Market Models Tip?," **Quarterly Journal of Economics**, 118, 1249-1278.

[5] Ellison, Glenn, Drew Fudenberg and Markus Mobius (2004). "Competing Auctions," **Journal of the European Economic Association,** 2, 30-66.

[6] Forges, Francois, Enrico Minelli and Rajiv Vohra (2002). "Incentives and the Core of and Exchange Economy: A Survey," **Journal of Mathematical Economics**, 38 (1-2), 1-41.

[7] Glode, Vincent and Christian Opp (2016). "Can Decentralized Markets Be More Efficient?", manuscript.

[8] Kuzmic, Christoph and Jan-Henrik Steg (2016). "On Public Good Provision Mechanisms with Dominant Strategies and Balanced Budget", **Journal of Economic Theory**, forthcoming.

[9] Holmstrom, Bengt and Roger Myerson (1983). "Efficient and Durable Decision Rules with Incomplete Information", **Econometrica**, 51 (6), 1799-1819.

[10] Hagerty, Kathleen and William Rogerson (1987). "Robust Trading Mechanisms," **Journal of Economic Theory**, 42: 94-107.

[11] Liu, Qingmin, George J. Mailath, Andrew Postlewaite and Larry Samuelson (2014). "Stable Matching with Incomplete Information," **Econometrica**, 82 (2), 541–587.

[12] Madhavan, Anath (2000). “Market Microstructure: A Survey,” **Journal of Financial Markets**, 3 (3), 205-258.

[13] Madhavan, Anath (1995). “Consolidation, Fragmentation, and the Disclosure of Trading Information,” **Review of Financial Studies,** 8 (3): 579-603.

[14] Malamud, Semyon and Marzena Rostek (2014). “Decentralized Exchange”, manuscript.

[15] Myerson, Roger B. and Mark A. Satterthwaite (1983). “Efficient Mechanisms for Bilateral Trading,” **Journal of Economic Theory**, 29: 265-281.

[16] Myerson, Roger B. (2007). “Virtual Utility and the Core for Games with Incomplete Information,” **Journal of Economic Theory**, 136 (1), 260-285.

[17] Pagano, Michael (1989). “Endogenous Market Thinness and Stock Price Volatility,” **The Review of Economic Studies**, 56, 269-287.

[18] Peters, Michael (2014). “Competing Mechanisms,” **Canadian Journal of Economics**, 47 (2), 373-397.

[19] Petrella, Giovanni (2009). “MiFID, Reg NMS and Competition Across Trading Venues in Europe and United States,” manuscript.

[20] Roth, Alvin E., and Xiaolin Xing (1994).“ Jumping the Gun: Imperfections and Institutions Related to the Timing in Market Transactions,” **American Economic Review**, Vol. 84, No. 4, 992-1044.

[21] Satterthwaite, Mark A. and Steven R. Williams (1989). “The Rate of Convergence to Efficiency in the Buyer’s Bid Double Auction as the Market Becomes Large,” **The Review of Economic Studies**, 56, 477-498.

[22] Serrano, Roberto and Rajiv Vohra (2007). “Information Transmission in Coalitional Voting Games,” **Journal of Economic Theory**, 134 (1), 117-137.